

# Phase equilibria: solubility limit

## Introduction

- **Solutions** – solid solutions, single phase
- **Mixtures** – more than one phase

Adapted from Fig. 9.1,  
*Callister 7e*.

- **Solubility Limit:**

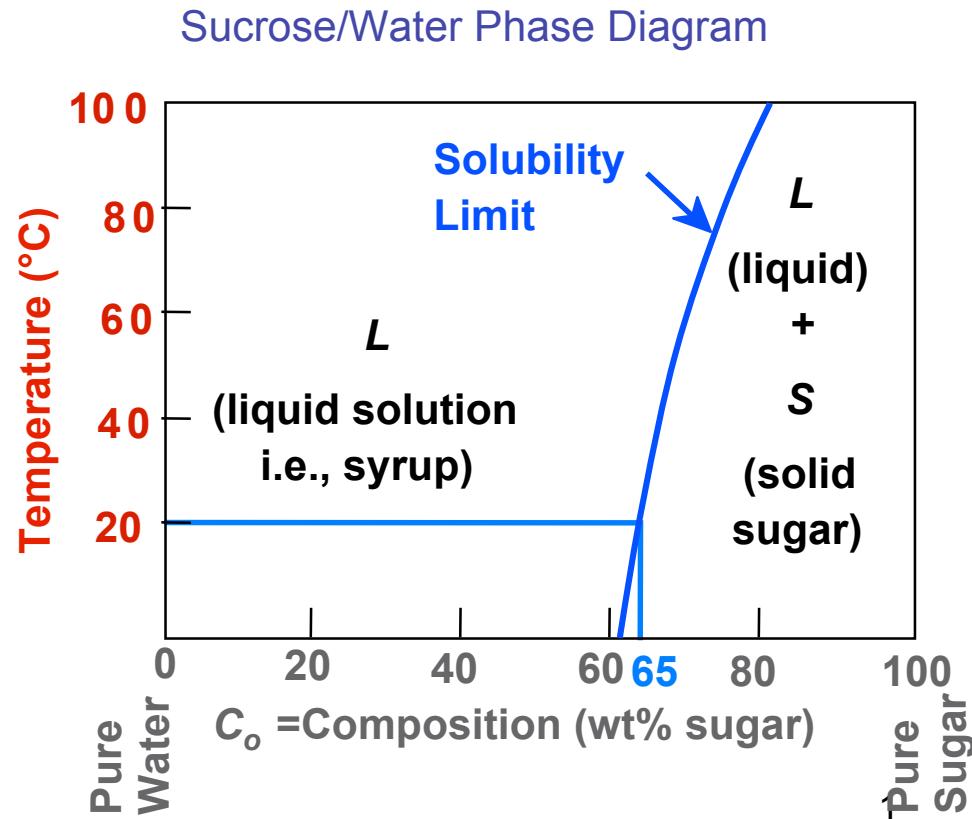
Max concentration for which only a single phase solution occurs.

Question: What is the solubility limit at 20°C?

Answer: 65 wt% sugar.

If  $C_o < 65$  wt% sugar: syrup

If  $C_o > 65$  wt% sugar: syrup + sugar.



# Components and phases

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- **Components:**

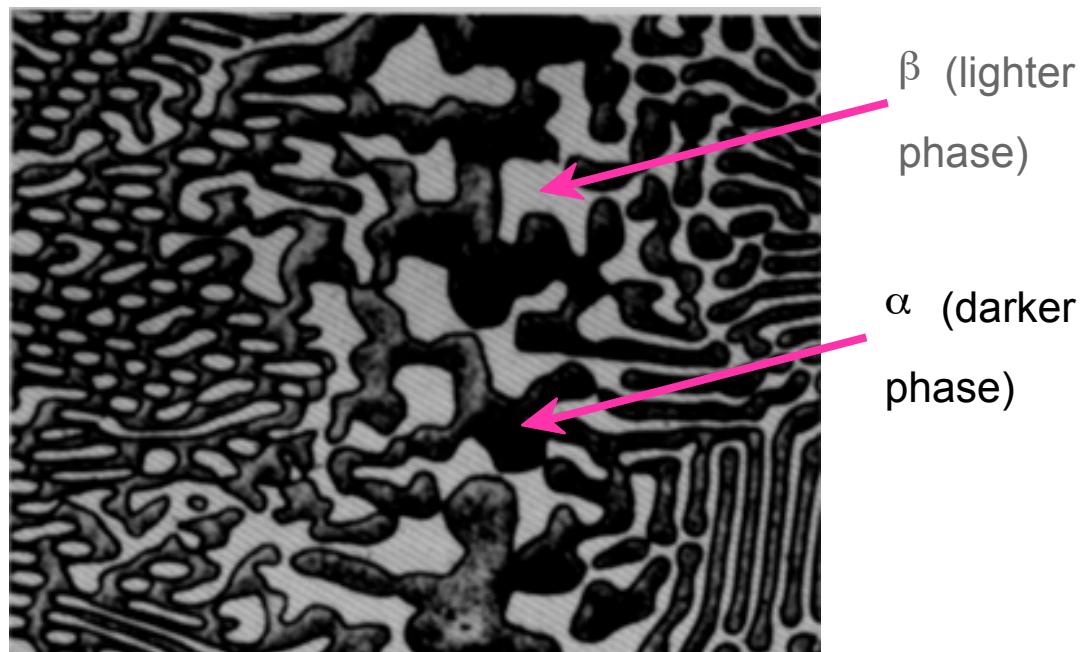
The elements or compounds which are present in the mixture  
(e.g., Al and Cu)

- **Phases:**

The physically and chemically distinct material regions  
that result (e.g.,  $\alpha$  and  $\beta$ ).

Aluminum-Copper Alloy

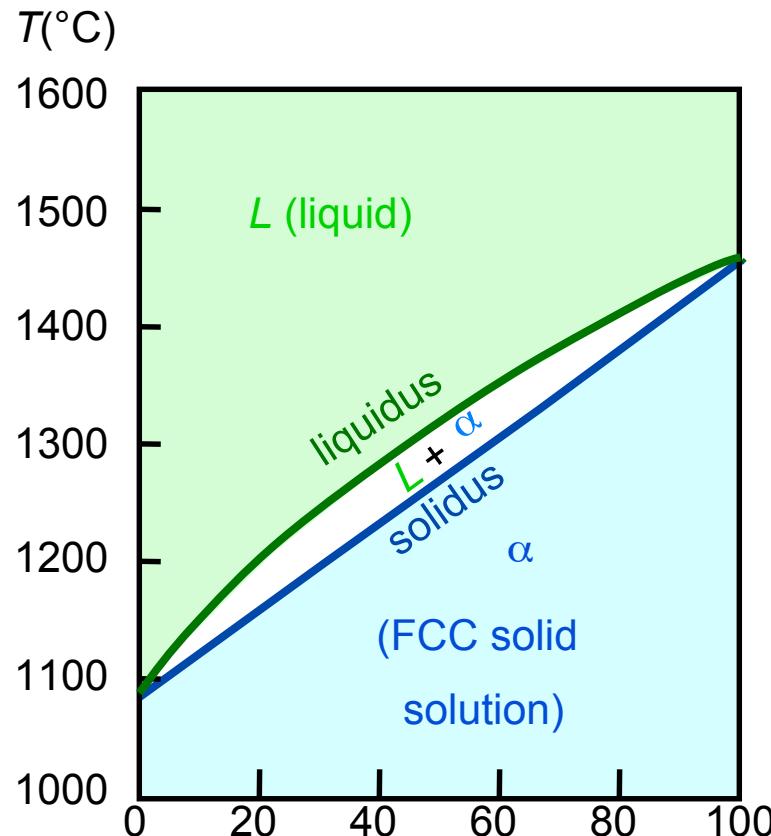
Adapted from chapter-opening photograph,  
Chapter 9,  
*Callister 3e*.



# Phase diagrams

- Indicate phases as function of  $T$ ,  $C_o$ , and  $P$ .
- For this course:
  - binary systems: just 2 components.
  - independent variables:  $T$  and  $C_o$  ( $P = 1 \text{ atm}$  is almost always used).

- Phase Diagram for Cu-Ni system



- 2 phases:
  - $L$  (liquid)
  - $\alpha$  (FCC solid solution)
- 3 phase fields:
  - $L$
  - $L + \alpha$
  - $\alpha$

Adapted from Fig. 9.3(a), Callister 7e.  
(Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH (1991)).

# Phase diagrams

- Rule 1: If we know  $T$  and  $C_o$ , then we know:  
--the # and types of phases present.

- Examples:

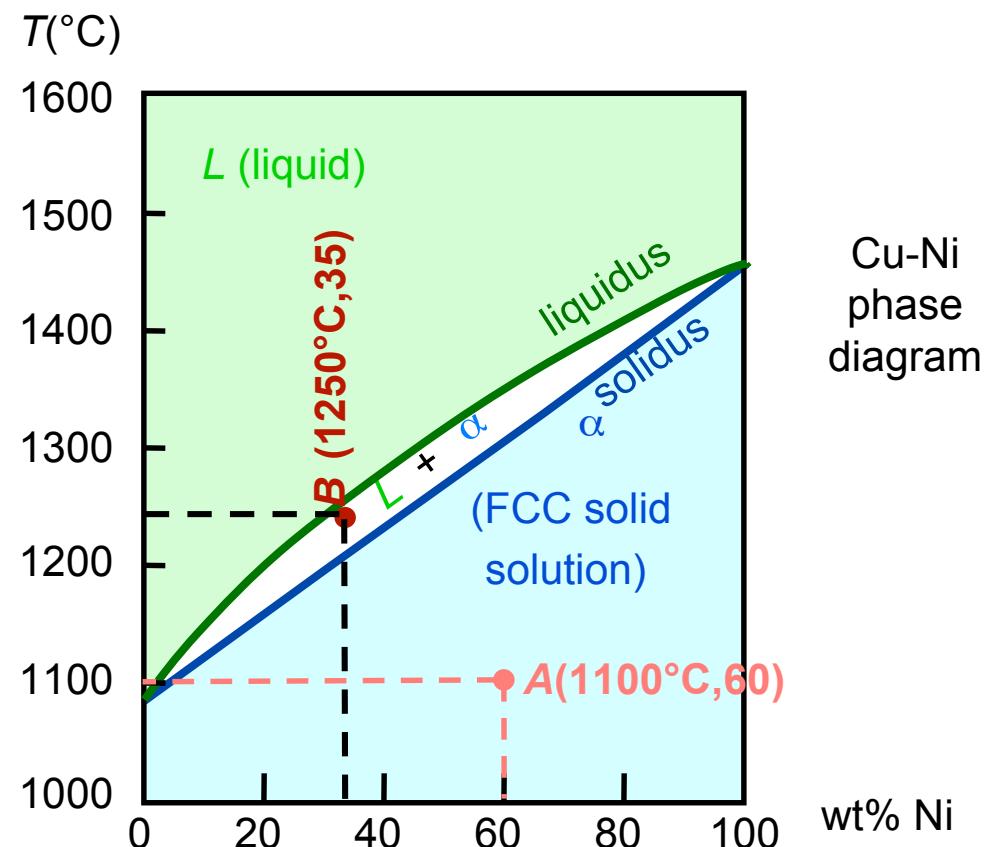
*A*(1100°C, 60):

1 phase:  $\alpha$

*B*(1250°C, 35):

2 phases:  $L + \alpha$

Adapted from Fig. 9.3(a), Callister 7e.  
(Fig. 9.3(a) is adapted from Phase  
Diagrams of Binary Nickel Alloys, P. Nash  
(Ed.), ASM International, Materials Park,  
OH, 1991).



# Phase diagrams

- Rule 2: If we know  $T$  and  $C_o$ , then we know:  
--the composition of each phase.

- Examples:

$$C_o = 35 \text{ wt\% Ni}$$

At  $T_A = 1320^\circ\text{C}$ :

Only Liquid ( $L$ )

$$C_L = C_o \quad (= 35 \text{ wt\% Ni})$$

At  $T_D = 1190^\circ\text{C}$ :

Only Solid ( $\alpha$ )

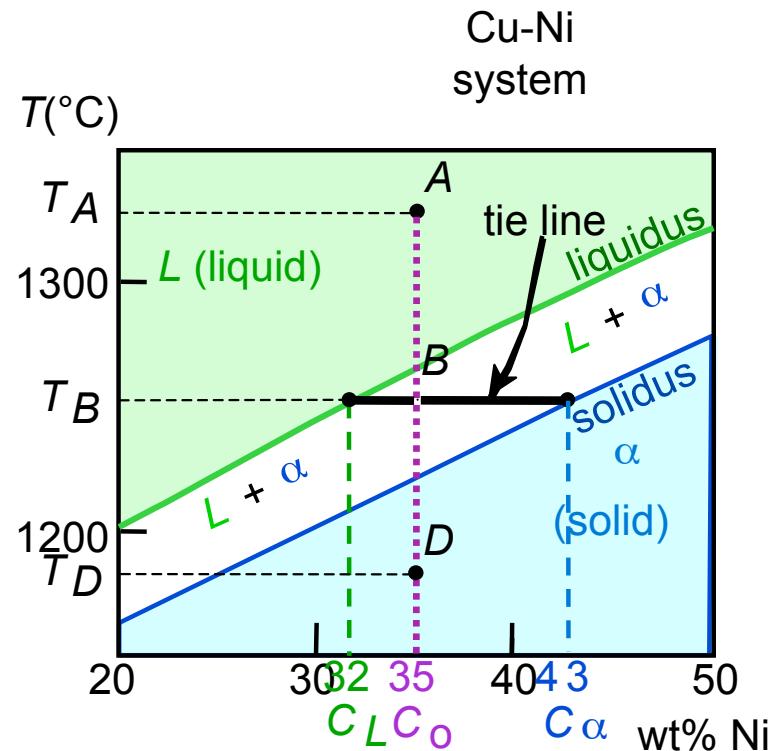
$$C_\alpha = C_o \quad (= 35 \text{ wt\% Ni})$$

At  $T_B = 1250^\circ\text{C}$ :

Both  $\alpha$  and  $L$

$$C_L = C_{\text{liquidus}} \quad (= 32 \text{ wt\% Ni here})$$

$$C_\alpha = C_{\text{solidus}} \quad (= 43 \text{ wt\% Ni here})$$



Adapted from Fig. 9.3(b), Callister 7e.  
(Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)

# Phase diagrams

- Rule 3: If we know  $T$  and  $C_o$ , then we know:  
--the amount of each phase (given in wt%).
- Examples:

$$C_o = 35 \text{ wt% Ni}$$

At  $T_A$ : Only Liquid (L)

$$W_L = 100 \text{ wt\%, } W_\alpha = 0$$

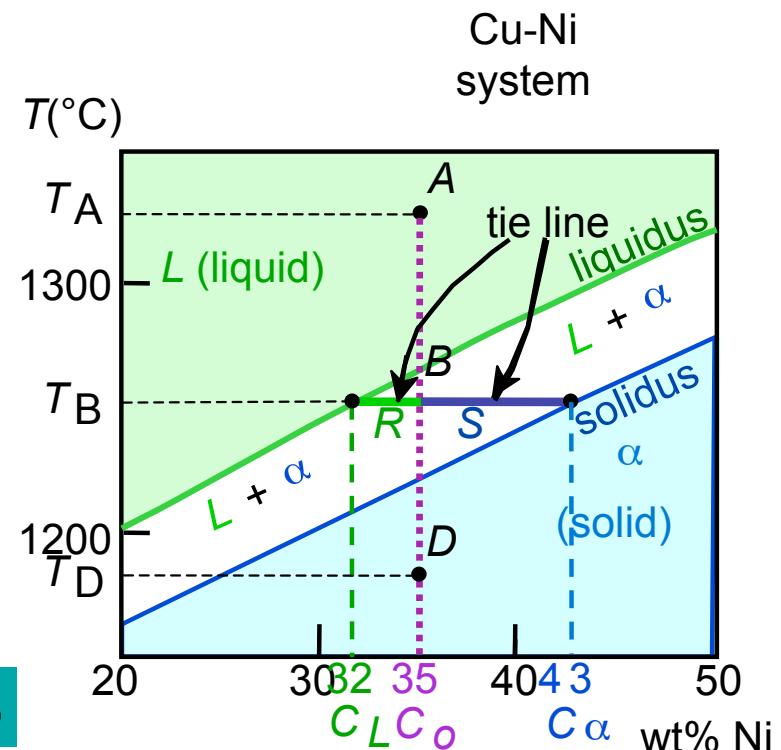
At  $T_D$ : Only Solid ( $\alpha$ )

$$W_L = 0, W_\alpha = 100 \text{ wt\%}$$

At  $T_B$ : Both  $\alpha$  and L

$$W_L = \frac{S}{R + S} = \frac{43 - 35}{43 - 32} = 73 \text{ wt\%}$$

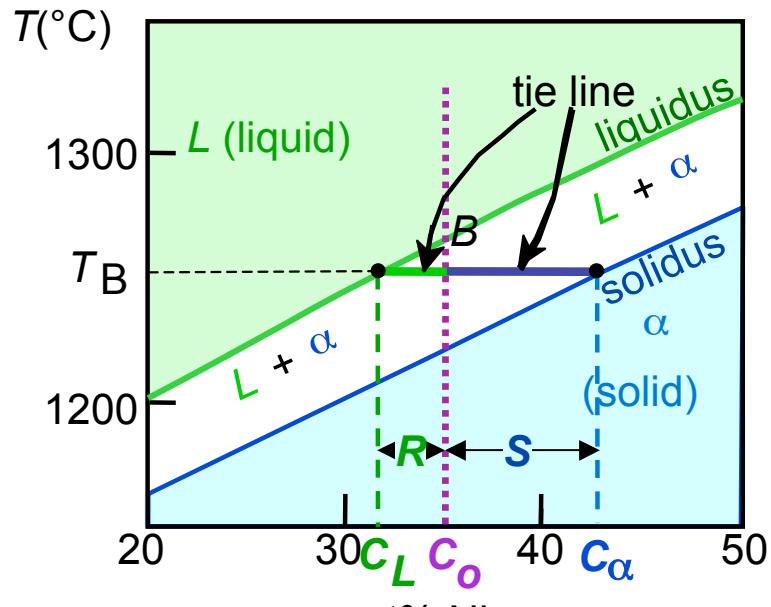
$$W_\alpha = \frac{R}{R + S} = 27 \text{ wt\%}$$



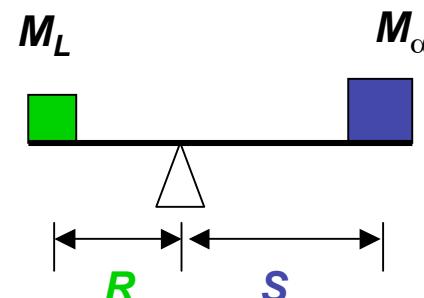
Adapted from Fig. 9.3(b), Callister 7e.  
(Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)

# The lever rule

Tie line – connects the phases in equilibrium with each other - essentially an isotherm



How much of each phase?  
Think of it as a lever (teeter-totter)



$$M_\alpha \cdot S = M_L \cdot R$$

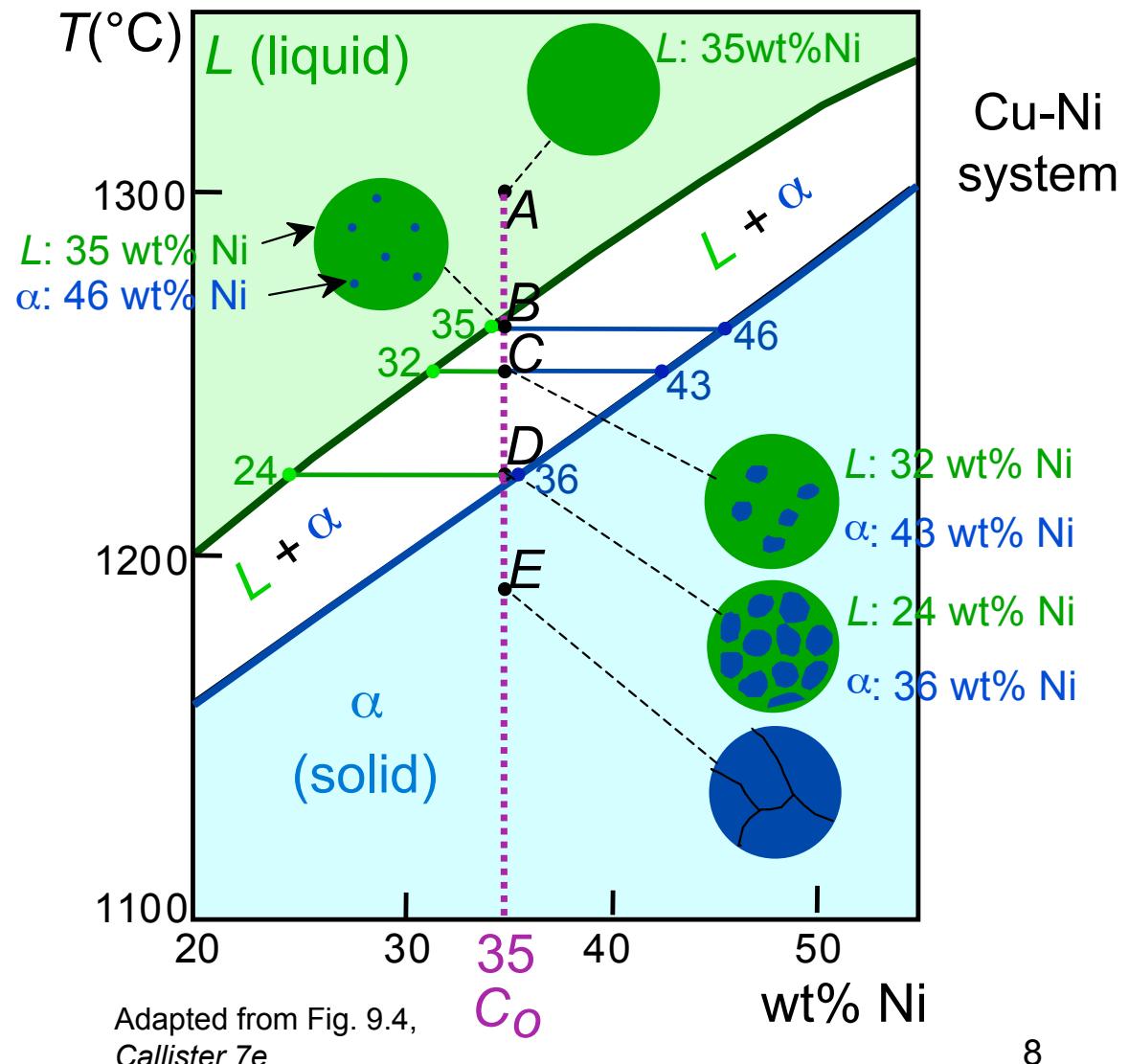
Adapted from Fig. 9.3(b),  
Callister 7e.

$$W_L = \frac{M_L}{M_L + M_\alpha} = \frac{S}{R + S} = \frac{C_\alpha - C_0}{C_\alpha - C_L}$$

$$W_\alpha = \frac{R}{R + S} = \frac{C_0 - C_L}{C_\alpha - C_L}$$

# Cooling

- Phase diagram: Cu-Ni system.
- System is:
  - binary**  
i.e., 2 components: Cu and Ni.
  - isomorphous**  
i.e., complete solubility of one component in another;  $\alpha$  phase field extends from 0 to 100 wt% Ni.
- Consider  $C_o = 35$  wt% Ni.

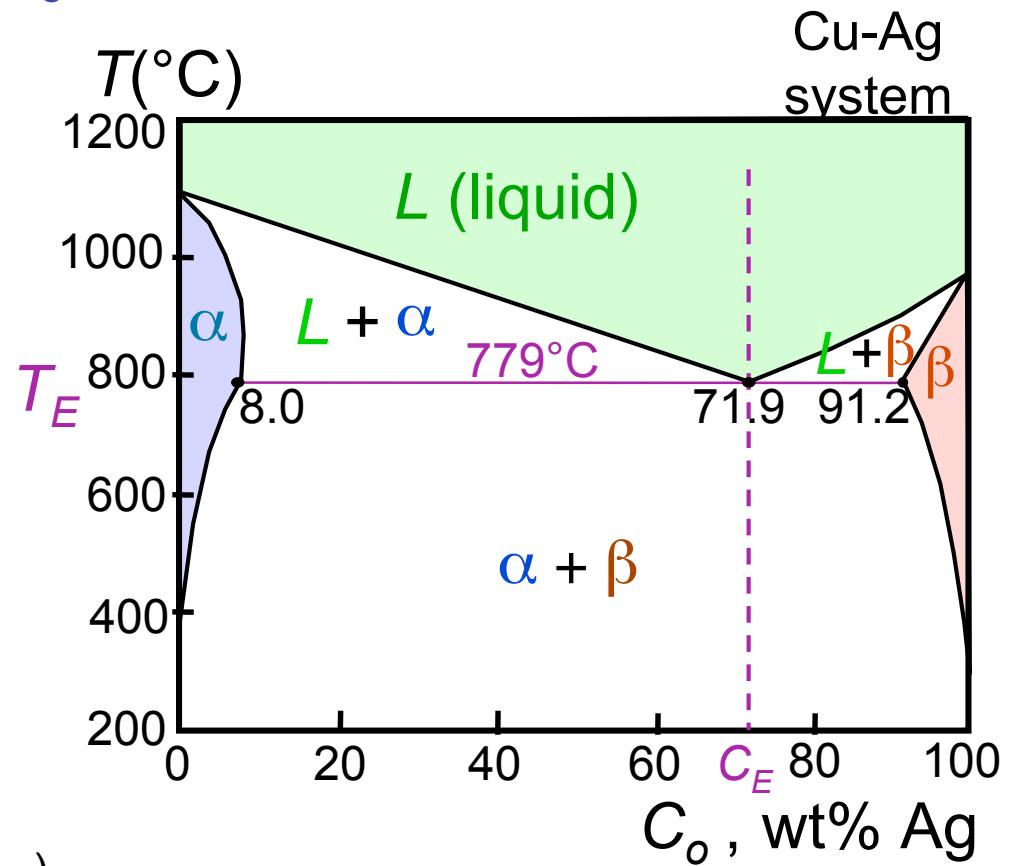


# Binary eutectic systems

2 components  
has a special composition with a min. melting T.

Ex.: Cu-Ag system

- 3 single phase regions ( $L$ ,  $\alpha$ ,  $\beta$ )
- Limited solubility:
  - $\alpha$ : mostly Cu
  - $\beta$ : mostly Ag
- $T_E$  : No liquid below  $T_E$
- $C_E$  : Min. melting  $T_E$  composition
- **Eutectic transition**
$$L(C_E) \rightleftharpoons \alpha(C_{\alpha E}) + \beta(C_{\beta E})$$



Adapted from Fig. 9.7,  
Callister 7e.

# Binary eutectic systems

- For a 40 wt% Sn-60 wt% Pb alloy at 150°C, find...

--the phases present:  $\alpha$  +  $\beta$

--compositions of phases:

$$C_O = 40 \text{ wt\% Sn}$$

$$C_\alpha = 11 \text{ wt\% Sn}$$

$$C_\beta = 99 \text{ wt\% Sn}$$

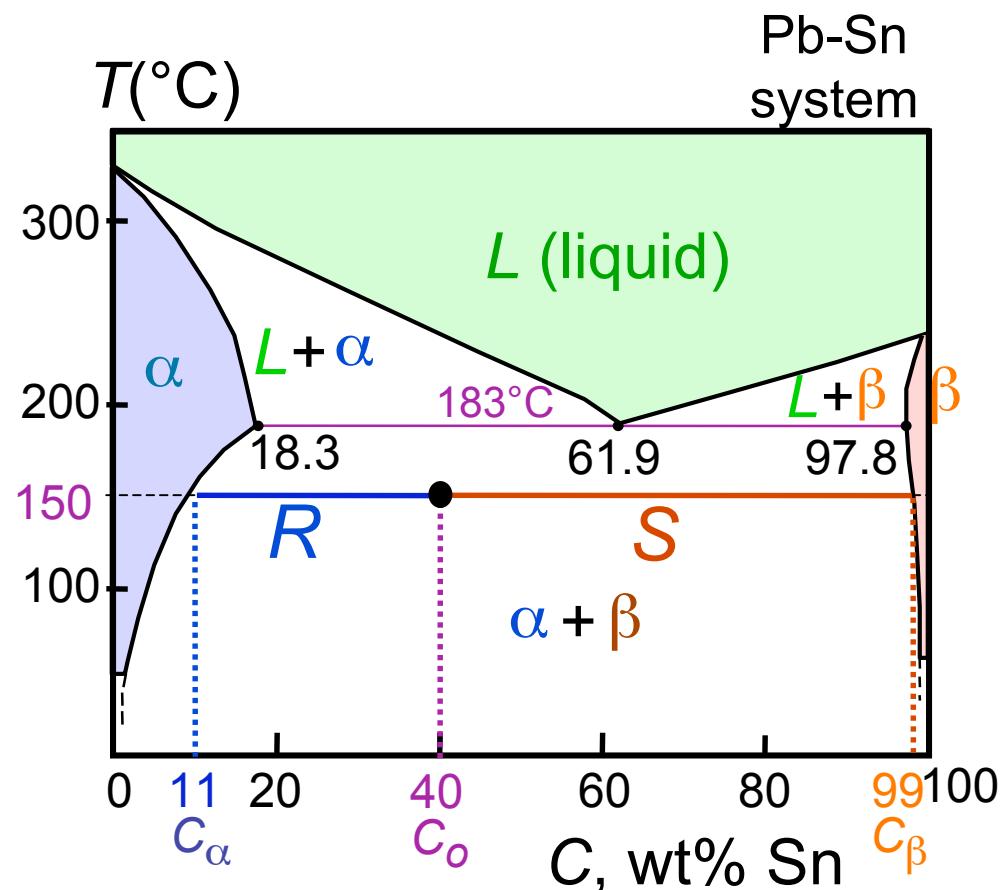
--the relative amount of each phase:

$$W_\alpha = \frac{S}{R+S} = \frac{C_\beta - C_O}{C_\beta - C_\alpha}$$

$$= \frac{99 - 40}{99 - 11} = \frac{59}{88} = 67 \text{ wt\%}$$

$$W_\beta = \frac{R}{R+S} = \frac{C_O - C_\alpha}{C_\beta - C_\alpha}$$

$$= \frac{40 - 11}{99 - 11} = \frac{29}{88} = 33 \text{ wt\%}$$



Adapted from Fig. 9.8,  
Callister 7e.

# Binary eutectic systems

- For a 40 wt% Sn-60 wt% Pb alloy at 200°C, find...

--the phases present:  $\alpha + L$

--compositions of phases:

$$C_O = 40 \text{ wt\% Sn}$$

$$C_\alpha = 17 \text{ wt\% Sn}$$

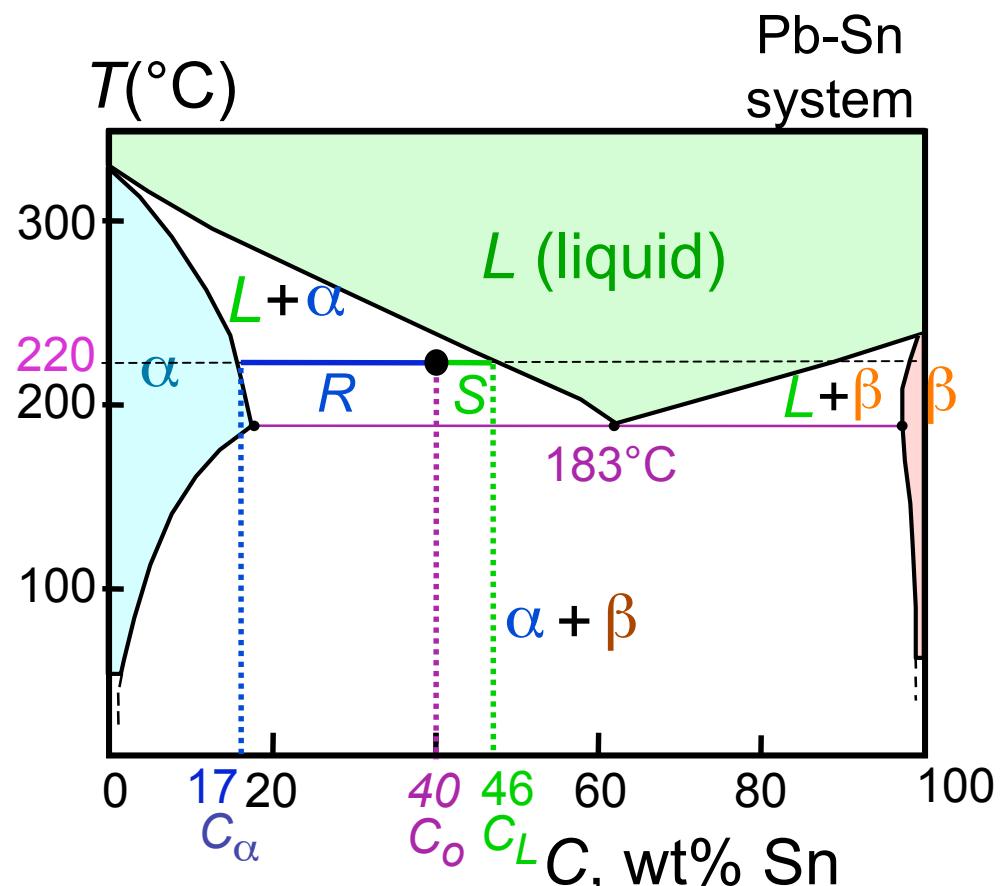
$$C_L = 46 \text{ wt\% Sn}$$

--the relative amount of each phase:

$$W_\alpha = \frac{C_L - C_O}{C_L - C_\alpha} = \frac{46 - 40}{46 - 17}$$

$$= \frac{6}{29} = 21 \text{ wt\%}$$

$$W_L = \frac{C_O - C_\alpha}{C_L - C_\alpha} = \frac{23}{29} = 79 \text{ wt\%}$$

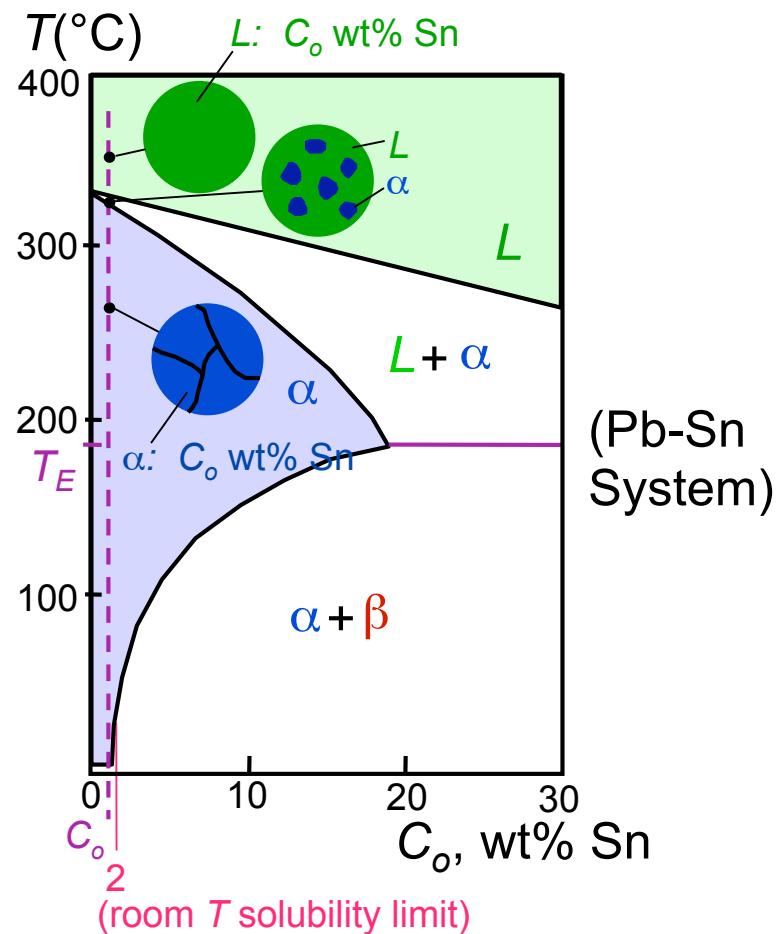


Adapted from Fig. 9.8,  
Callister 7e.

# Microstructures in eutectic systems

- $C_o < 2 \text{ wt\% Sn}$
- Result:
  - at extreme ends
  - polycrystal of  $\alpha$  grains  
i.e., only one solid phase.

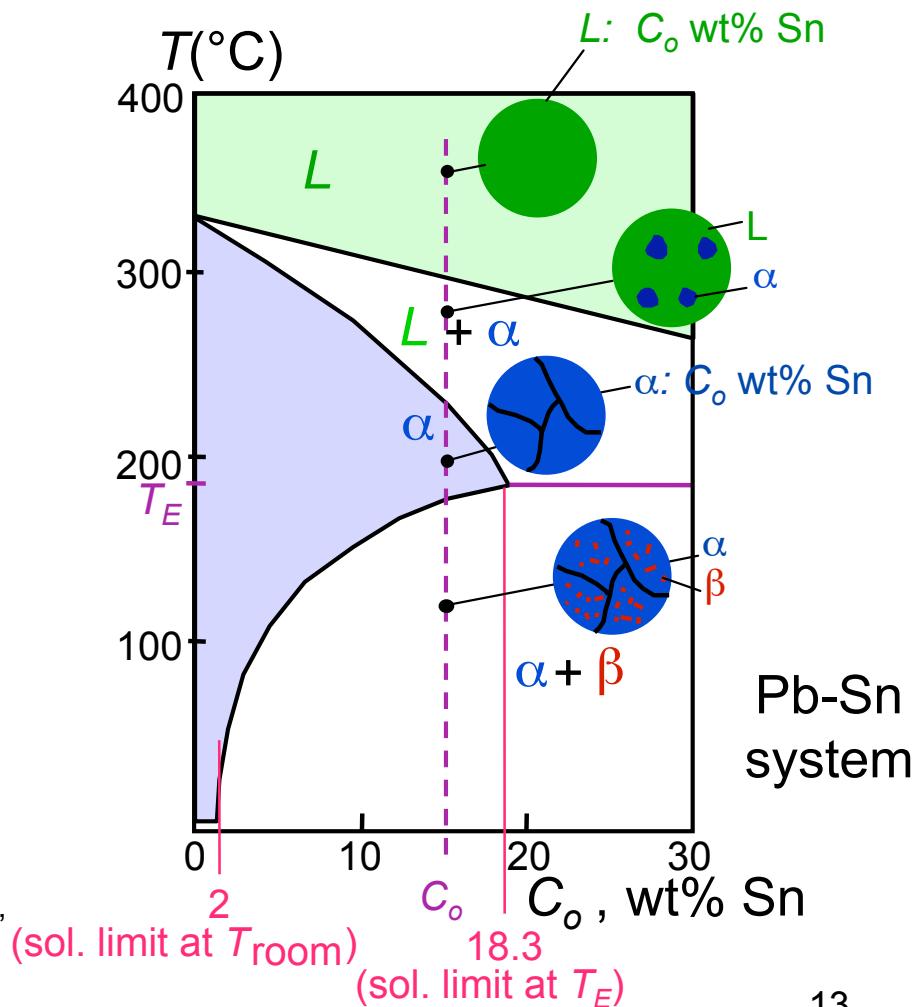
Adapted from Fig. 9.11,  
*Callister 7e.*



# Microstructures in eutectic systems

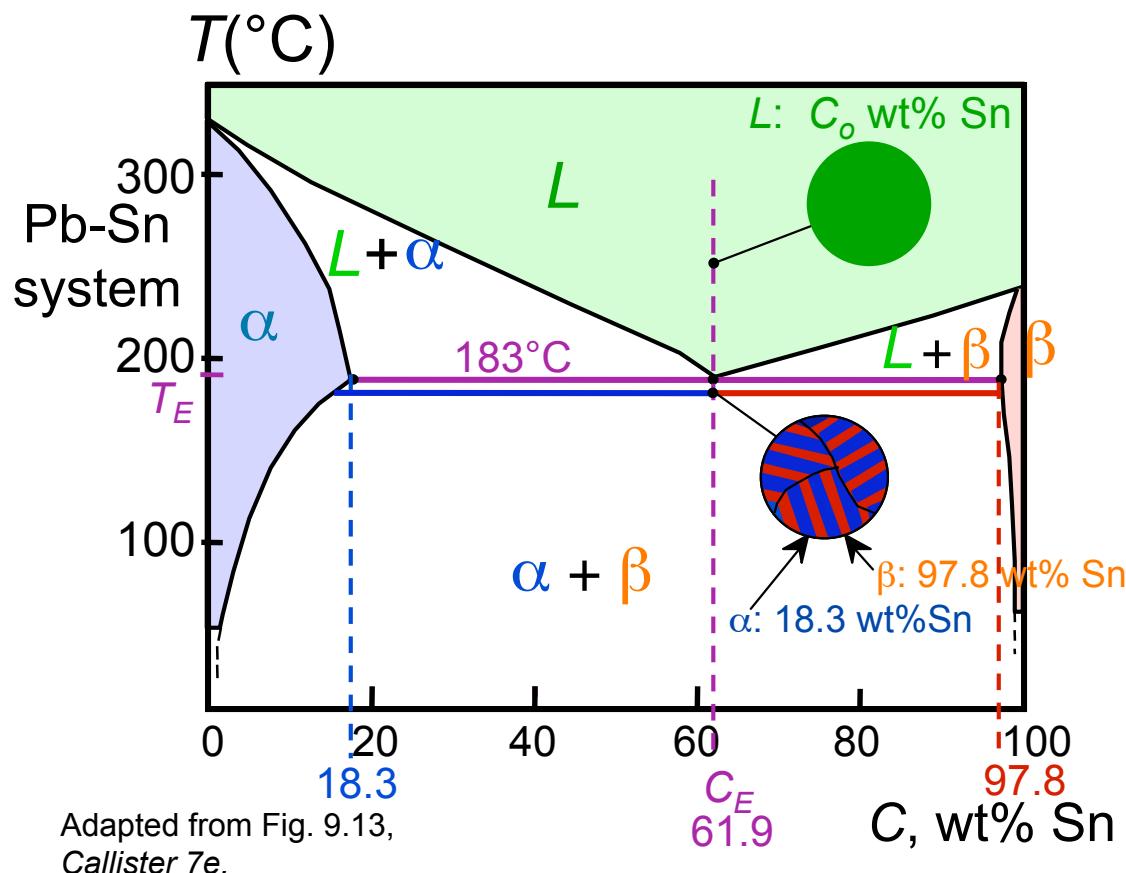
- $2 \text{ wt\% Sn} < C_o < 18.3 \text{ wt\% Sn}$
- Result:
  - Initially liquid +  $\alpha$
  - then  $\alpha$  alone
  - finally two phases
    - $\alpha$  polycrystal
    - fine  $\beta$ -phase inclusions

Adapted from Fig. 9.12,  
Callister 7e.



# Microstructures in eutectic systems

- $C_o = C_E$
- Result: Eutectic microstructure (lamellar structure)  
--alternating layers (lamellae) of  $\alpha$  and  $\beta$  crystals.



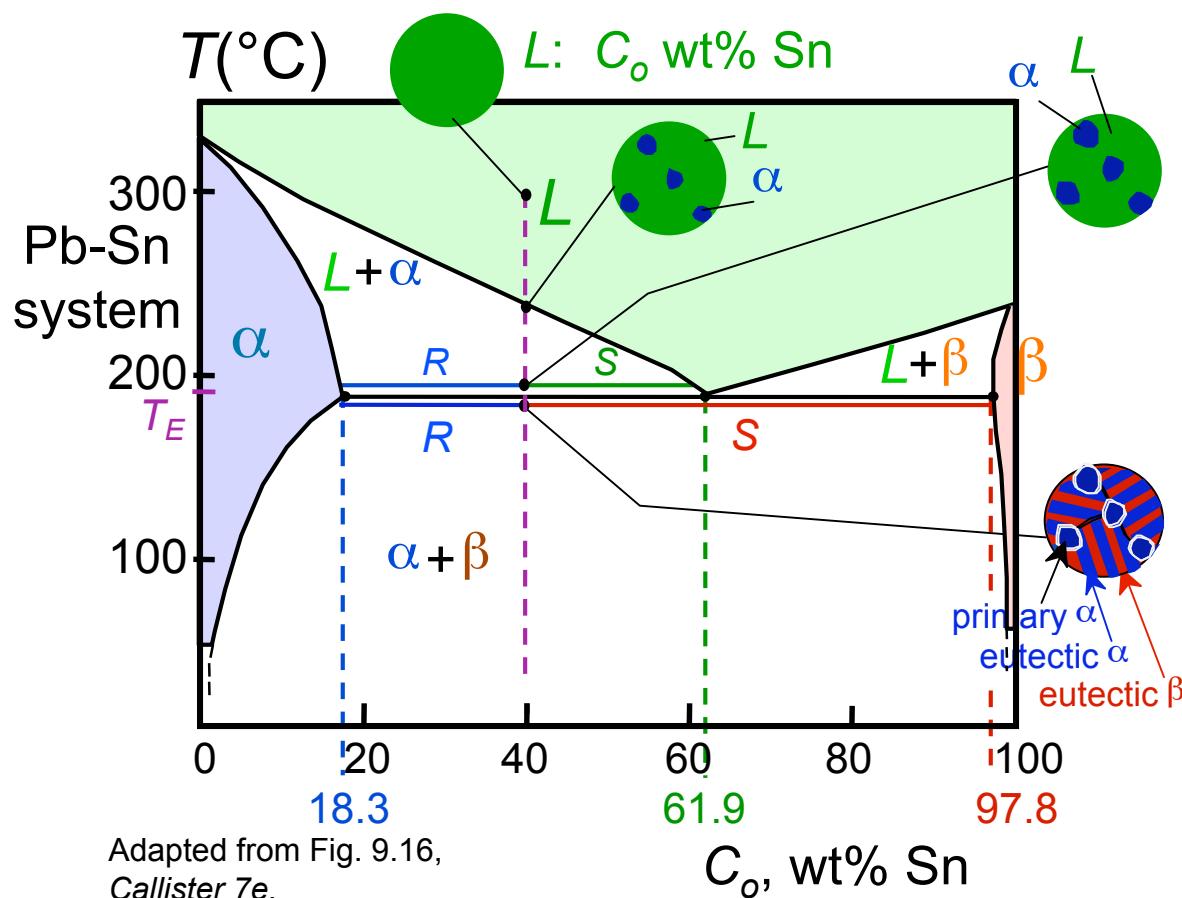
Micrograph of Pb-Sn eutectic microstructure



160  $\mu\text{m}$   
Adapted from Fig. 9.14, Callister 7e.

# Microstructures in eutectic systems

- $18.3 \text{ wt\% Sn} < C_o < 61.9 \text{ wt\% Sn}$
- Result:  $\alpha$  crystals and a eutectic microstructure



- Just above  $T_E$ :

$$C_\alpha = 18.3 \text{ wt\% Sn}$$

$$C_L = 61.9 \text{ wt\% Sn}$$

$$W_\alpha = \frac{S}{R + S} = 50 \text{ wt\%}$$

$$W_L = (1 - W_\alpha) = 50 \text{ wt\%}$$

- Just below  $T_E$ :

$$C_\alpha = 18.3 \text{ wt\% Sn}$$

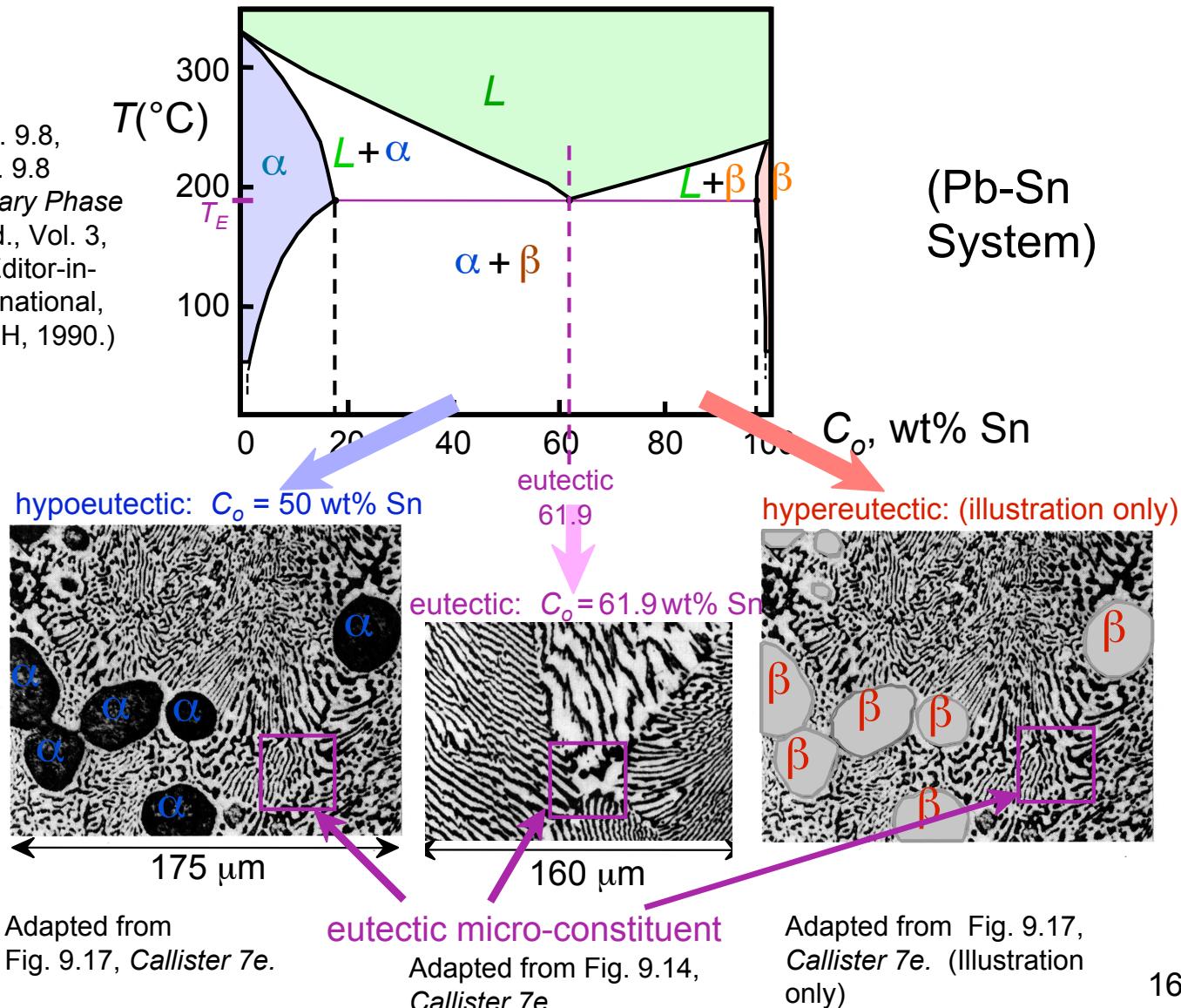
$$C_\beta = 97.8 \text{ wt\% Sn}$$

$$W_\alpha = \frac{S}{R + S} = 73 \text{ wt\%}$$

$$W_\beta = 27 \text{ wt\%}$$

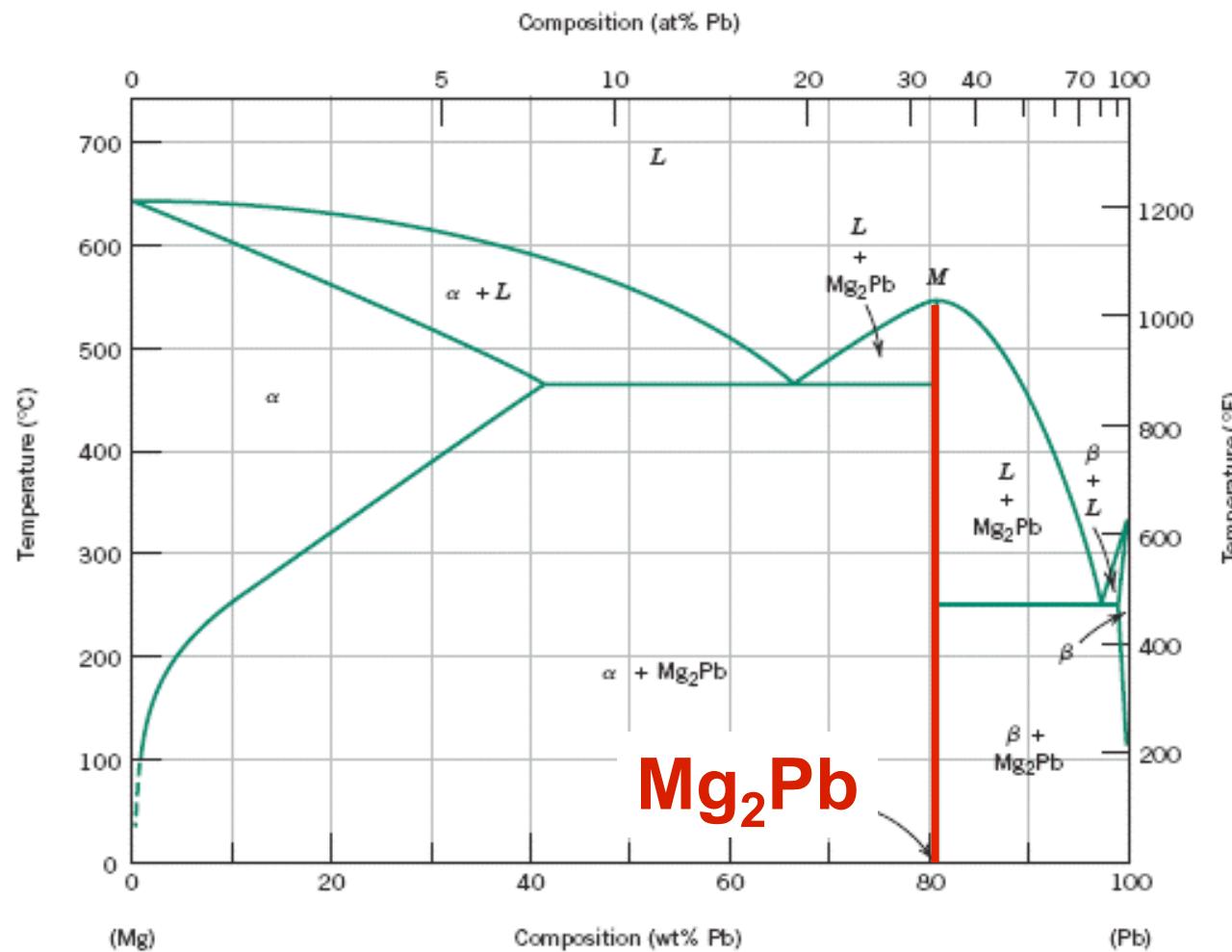
# Hypo<sup>e</sup>utectic & hyper<sup>e</sup>utectic

Adapted from Fig. 9.8,  
Callister 7e. (Fig. 9.8  
adapted from *Binary Phase  
Diagrams*, 2nd ed., Vol. 3,  
T.B. Massalski (Editor-in-  
Chief), ASM International,  
Materials Park, OH, 1990.)



(Figs. 9.14 and 9.17  
from *Metals  
Handbook*, 9th ed.,  
Vol. 9,  
*Metallography and  
Microstructures*,  
American Society for  
Metals, Materials  
Park, OH, 1985.)

# Intermetallic compounds



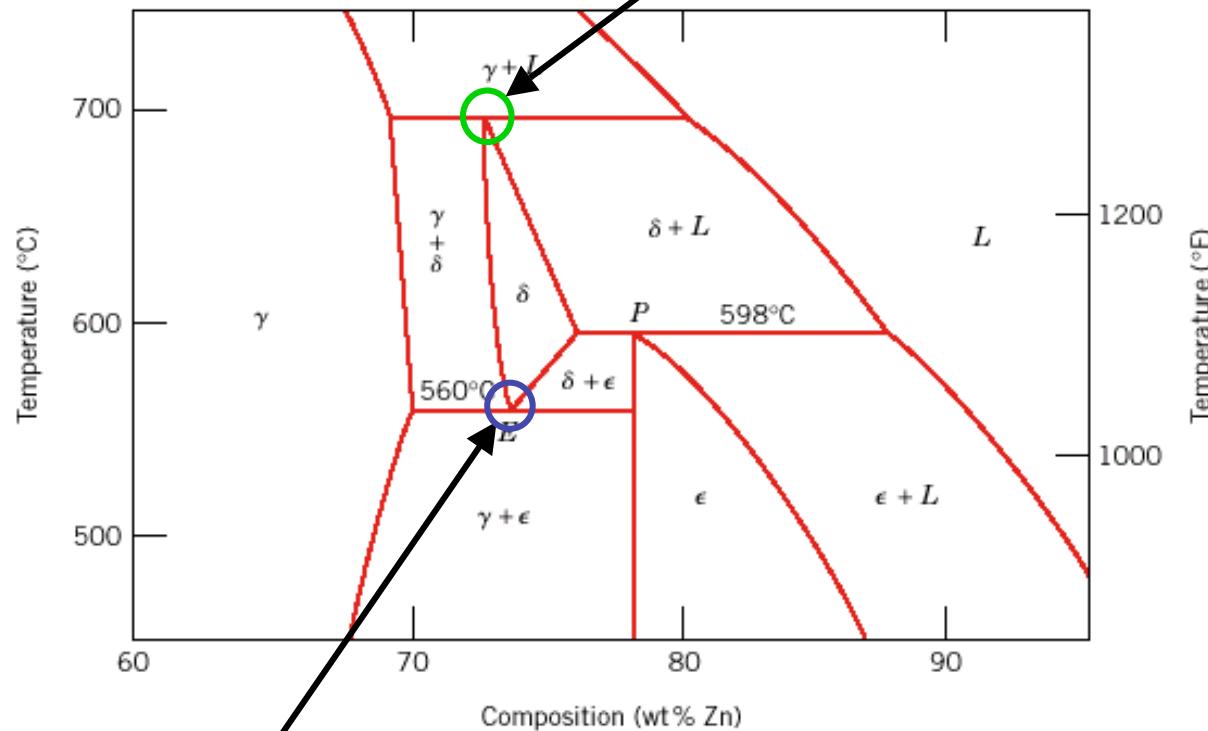
Adapted from  
Fig. 9.20, Callister 7e.

Note: intermetallic compound forms a line - not an area - because stoichiometry (i.e. composition) is exact.

# Peritectic & eutectoid

- Cu-Zn Phase diagram

Peritectic transition  $\gamma + L \rightleftharpoons \delta$



Eutectoid transition  $\delta \rightleftharpoons \gamma + \epsilon$

Adapted from  
Fig. 9.21, Callister 7e.

# Fe-C phase diagram

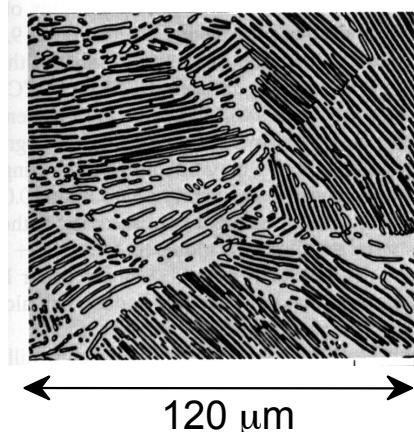
- 2 important points

-Eutectic (A):

$$L \Rightarrow \gamma + \text{Fe}_3\text{C}$$

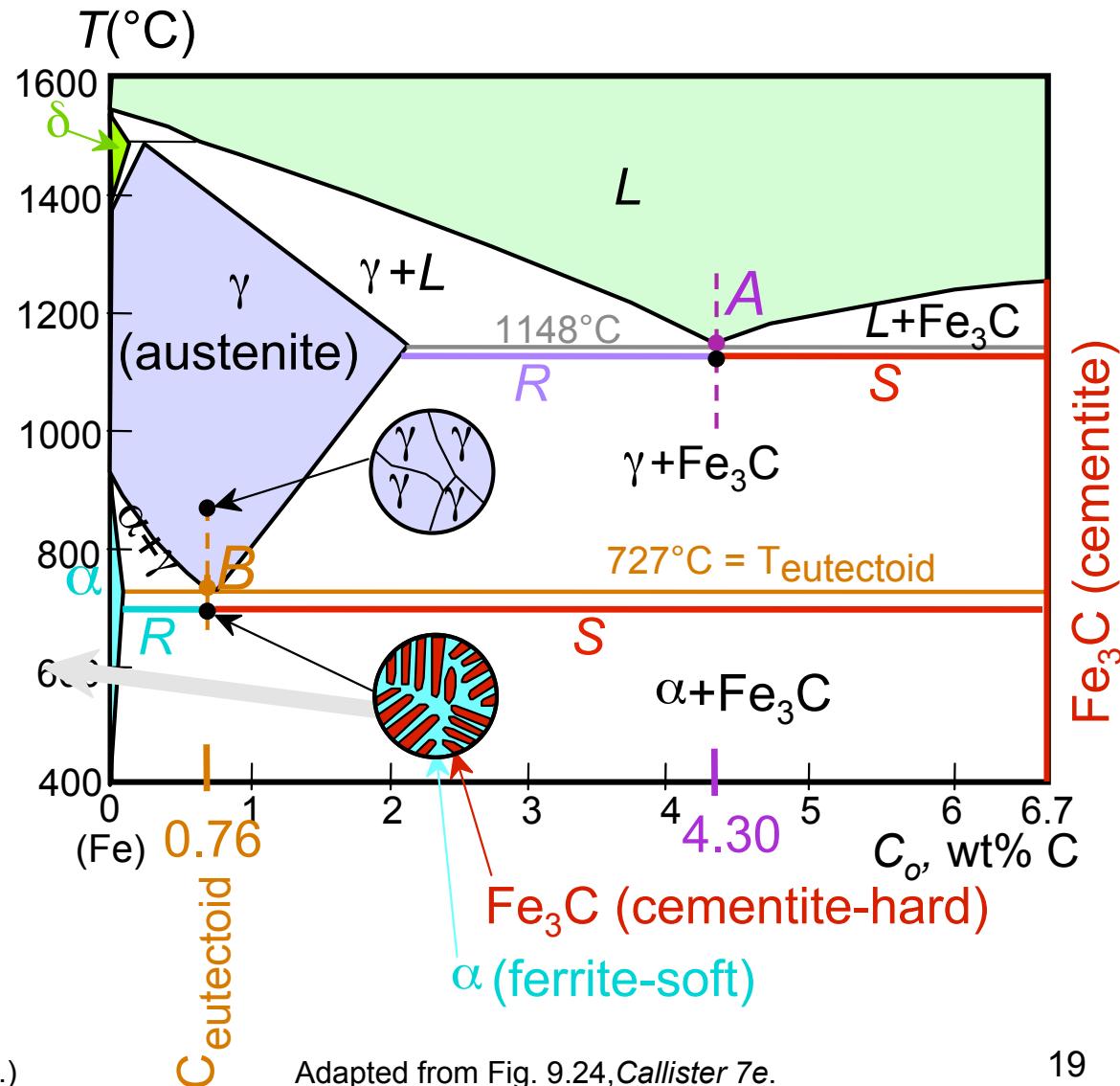
-Eutectoid (B):

$$\gamma \Rightarrow \alpha + \text{Fe}_3\text{C}$$



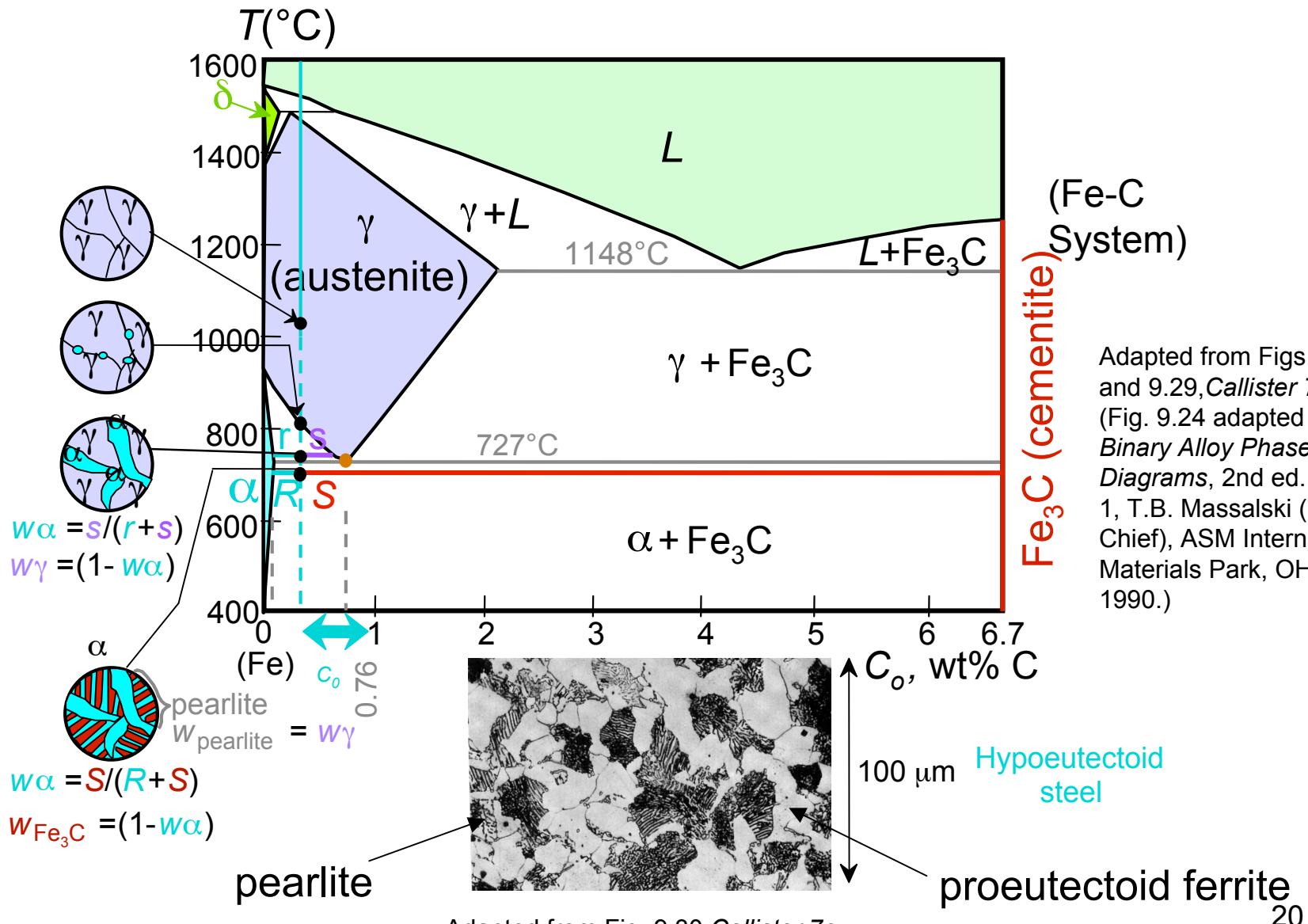
Result: Pearlite =  
alternating layers of  
 $\alpha$  and  $\text{Fe}_3\text{C}$  phases

(Adapted from Fig. 9.27, Callister 7e.)



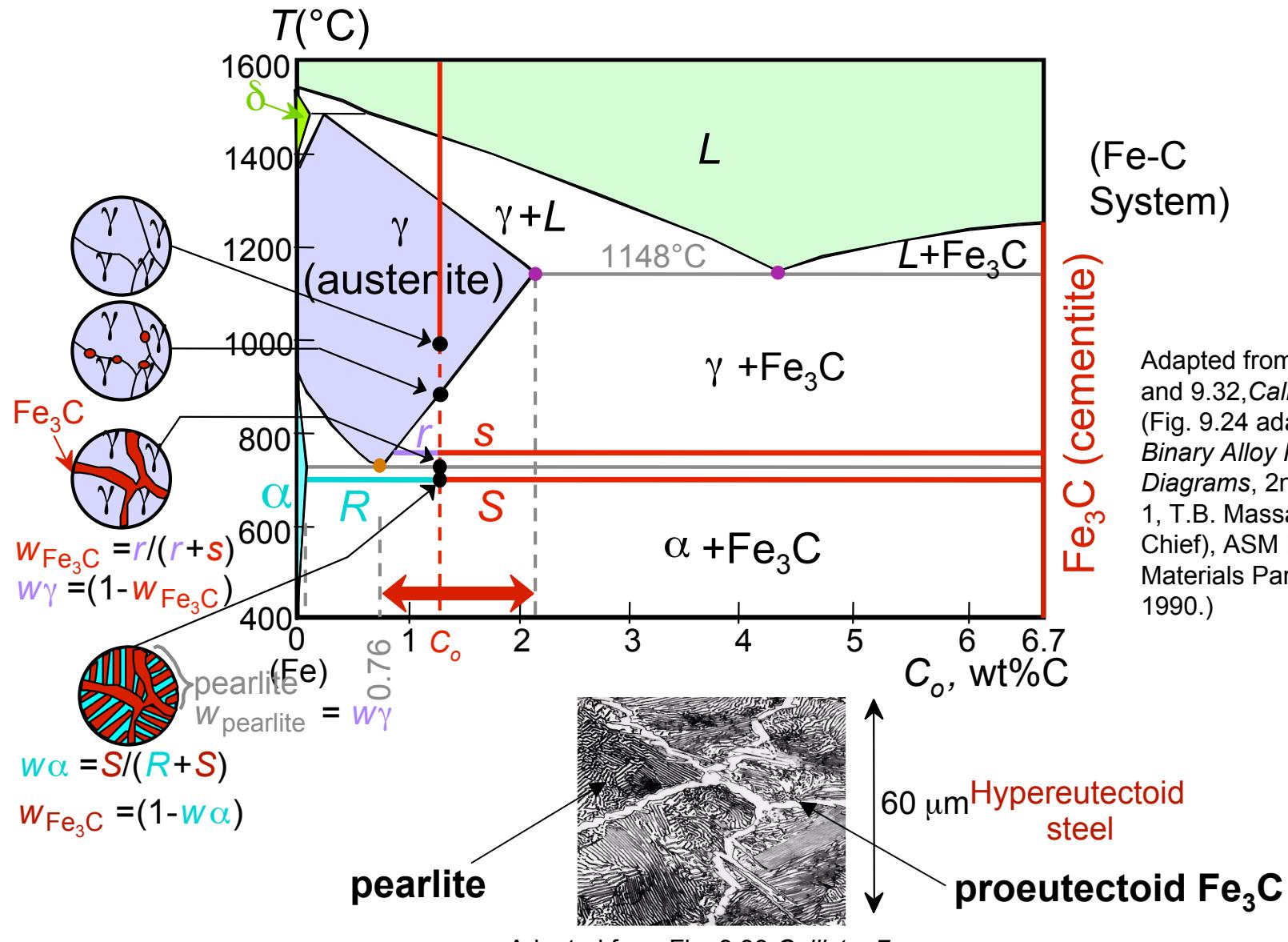
Adapted from Fig. 9.24, Callister 7e.

# Hypo<sup>e</sup>tectoid steel



Adapted from Fig. 9.30, Callister 7e.

# Hypereutectoid steel



Adapted from Figs. 9.24 and 9.32, Callister 7e.  
 (Fig. 9.24 adapted from *Binary Alloy Phase Diagrams*, 2nd ed., Vol. 1, T.B. Massalski (Ed.-in-Chief), ASM International, Materials Park, OH, 1990.)

Adapted from Fig. 9.33, Callister 7e.