

# Diffusion

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**Diffusion** - Mass transport by atomic motion

## Mechanisms

- Gases & Liquids – random (Brownian) motion
- Solids – vacancy diffusion or interstitial diffusion

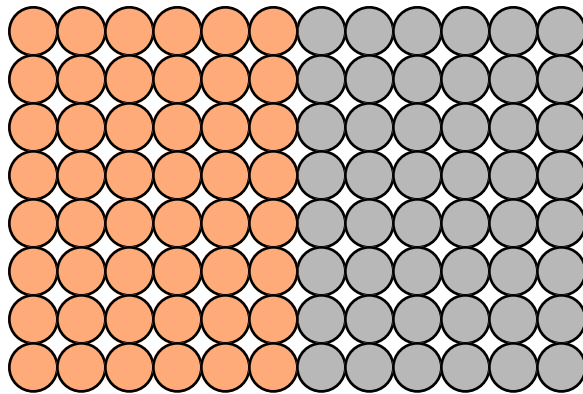
**Interdiffusion**: In an alloy, atoms tend to migrate from regions of high conc. to regions of low conc.

**Self-diffusion**: In an elemental solid, atoms also migrate.

# Interdiffusion

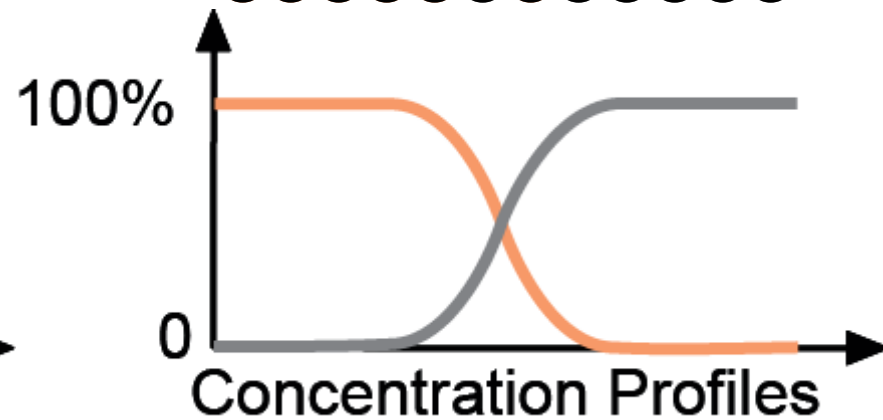
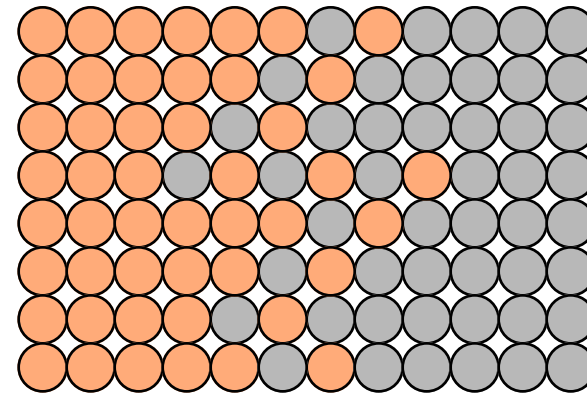
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Initially



Adapted from  
Figs. 5.1 and  
5.2, *Callister*  
7e.

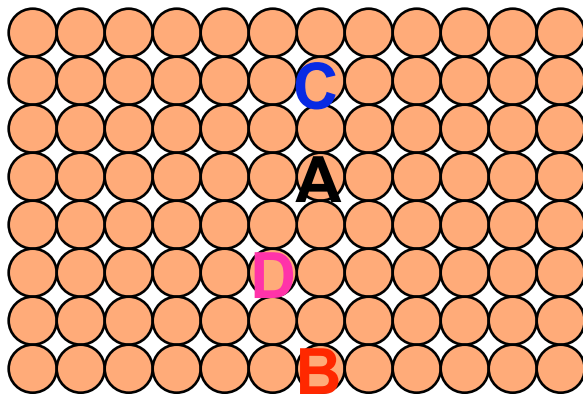
After some time



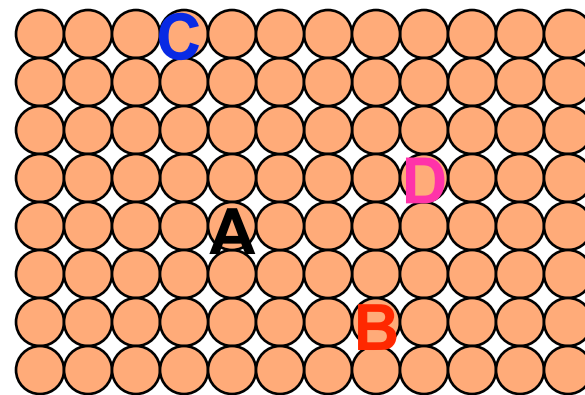
# Self-diffusion

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Label some atoms



After some time



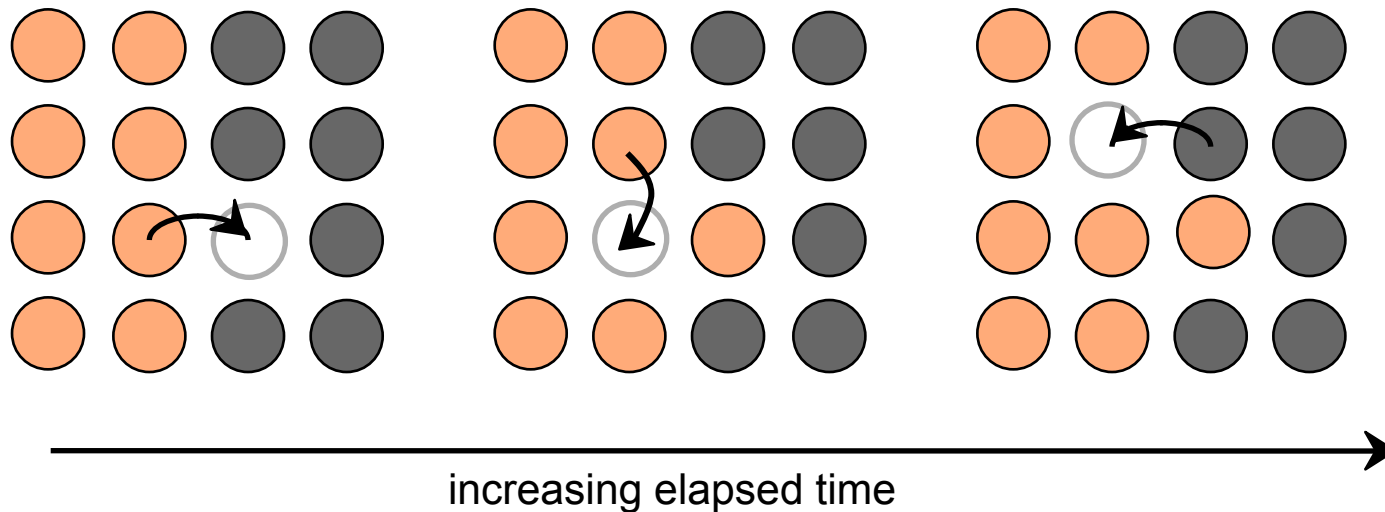
# Diffusion mechanisms

## Conditions:

- There must be an empty site available
- Atoms must have the energy to make the jump

## Vacancy Diffusion:

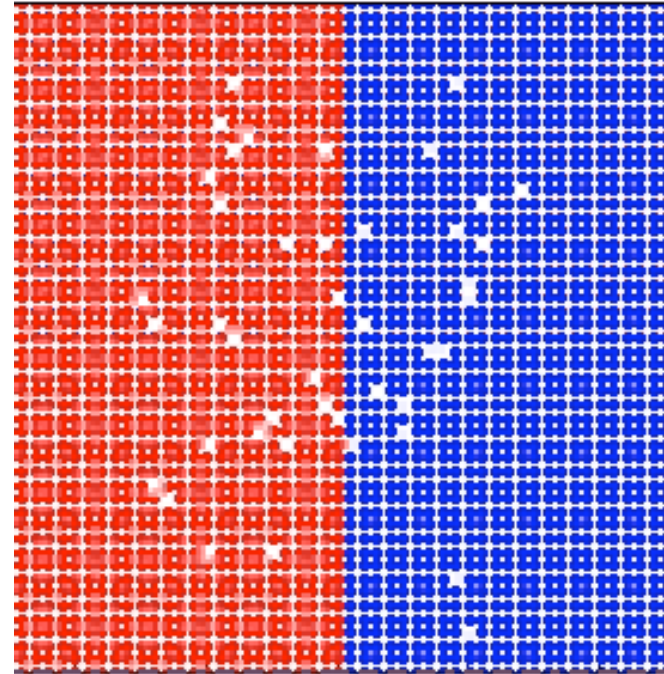
- atoms exchange with vacancies
- applies to substitutional impurities atoms
- rate depends on:
  - number of vacancies
  - activation energy to exchange.



# Diffusion simulation

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- Simulation of interdiffusion across an interface:
- Rate of substitutional diffusion depends on:
  - vacancy concentration
  - frequency of jumping.



(Courtesy P.M. Anderson)

# Diffusion flux

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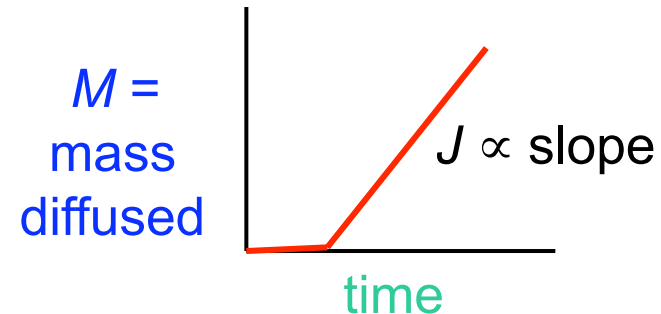
How do we quantify the amount or rate of diffusion?

$$J \equiv \text{Flux} \equiv \frac{\text{moles (or mass) diffusing}}{(\text{surface area})(\text{time})} = \frac{\text{mol}}{\text{cm}^2\text{s}} \text{ or } \frac{\text{kg}}{\text{m}^2\text{s}}$$

Measured empirically

- Make thin film (membrane) of known surface area
- Impose concentration gradient
- Measure how fast atoms or molecules diffuse through the membrane

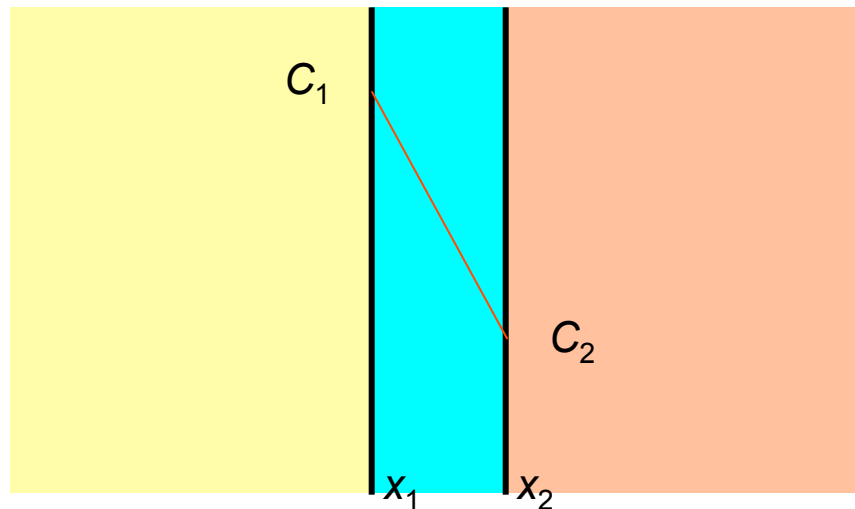
$$J = \frac{M}{At} = \frac{l}{A} \frac{dM}{dt}$$



# Steady-state diffusion

*Rate of diffusion independent of time*

Flux proportional to concentration gradient =  $\frac{dC}{dx}$



Fick's first law of diffusion

$$J = -D \frac{dC}{dx}$$

if linear  $\frac{dC}{dx} \cong \frac{\Delta C}{\Delta x} = \frac{C_2 - C_1}{x_2 - x_1}$

$D$   $\equiv$  diffusion coefficient

# Diffusion and temperature

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- Diffusion coefficient increases with increasing  $T$ .

$$D = D_o \exp \left( - \frac{Q_d}{R T} \right)$$

$D$  = diffusion coefficient [ $\text{m}^2/\text{s}$ ]

$D_o$  = pre-exponential [ $\text{m}^2/\text{s}$ ]

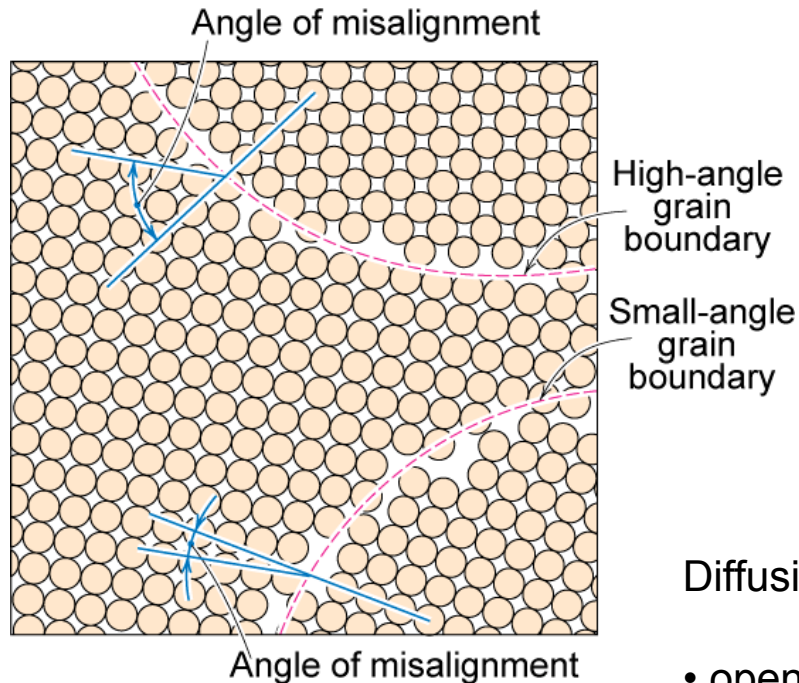
$Q_d$  = activation energy [ $\text{J/mol}$  or  $\text{eV/atom}$ ]

$R$  = gas constant [ $8.314 \text{ J/mol-K}$ ]

$T$  = absolute temperature [ $\text{K}$ ]



# Diffusion paths



Diffusion **FASTER** for...

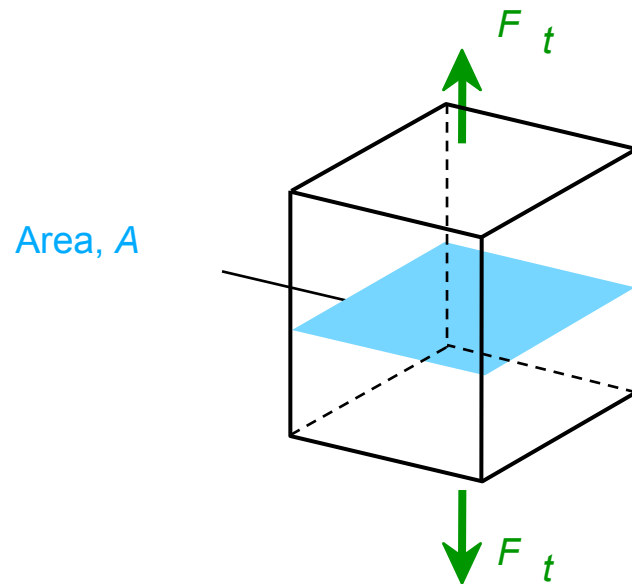
- open crystal structures
- materials w/secondary bonding
- smaller diffusing atoms
- lower density materials

Diffusion **SLOWER** for...

- close-packed structures
- materials w/covalent bonding
- larger diffusing atoms
- higher density materials

# Engineering stress

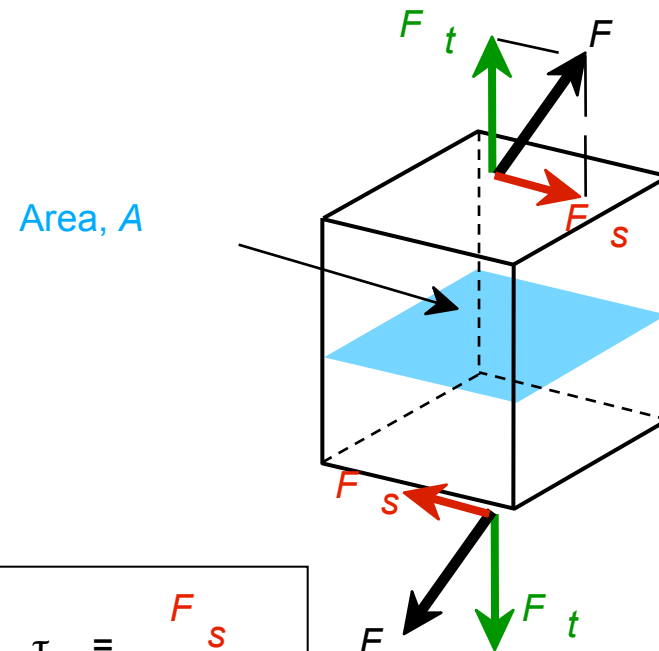
- Tensile stress,  $\sigma$ :



$$\sigma = \frac{F_t}{A_o} = \frac{\text{lb}_f}{\text{in}^2} \text{ or } \frac{\text{N}}{\text{m}^2}$$

original area  
before loading

- Shear stress,  $\tau$ :



$$\tau = \frac{F_s}{A_o}$$

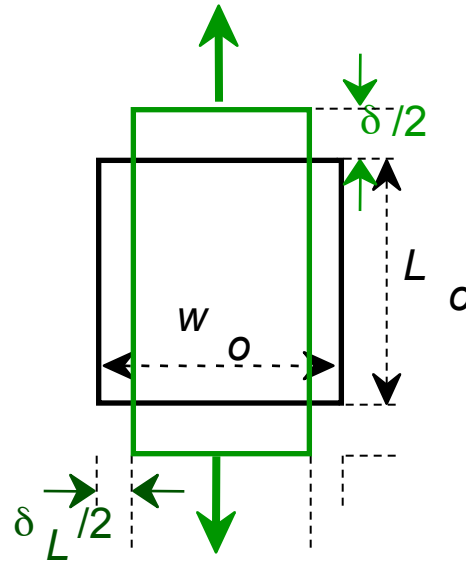
# Engineering strain

- **Tensile strain:**

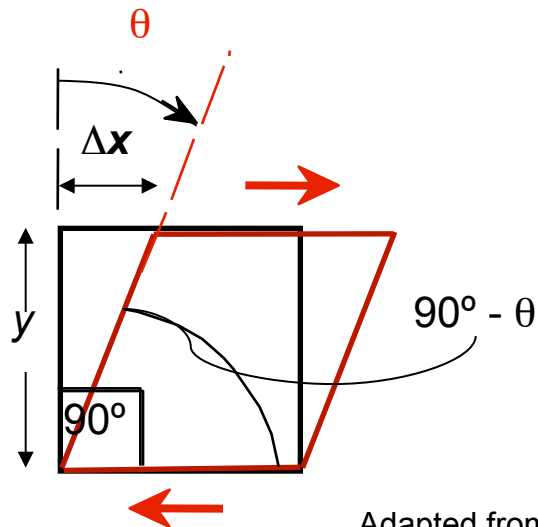
$$\varepsilon = \frac{\delta}{L_o}$$

- **Lateral strain:**

$$\varepsilon_L = \frac{-\delta_L}{w_o}$$



- **Shear strain:**



$$\gamma = \Delta x / y = \tan \theta$$

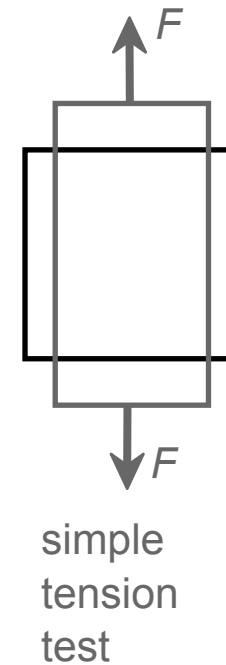
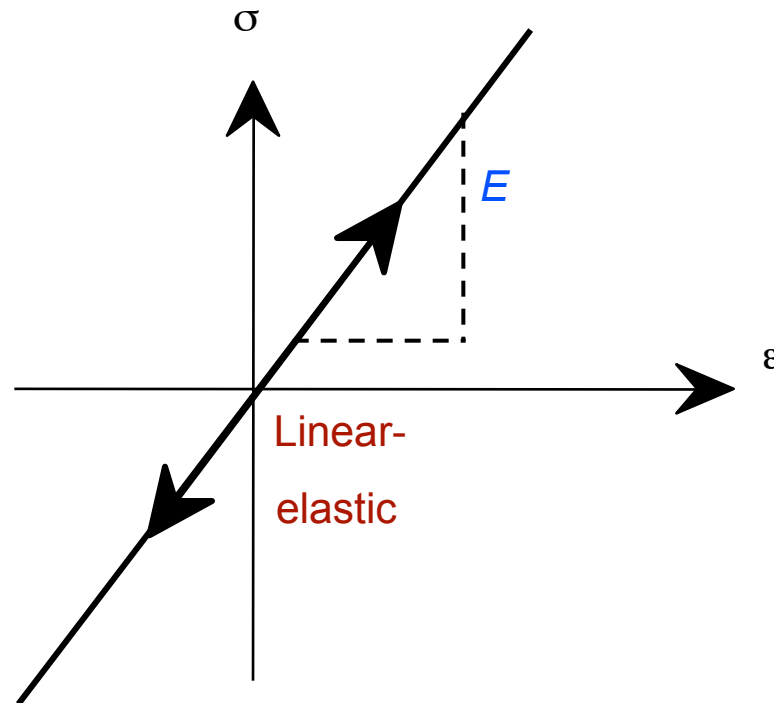
Adapted from Fig. 6.1 (a) and (c), *Callister 7e*.

# Engineering strain

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- **Modulus of Elasticity,  $E$ :**  
(also known as Young's modulus)
- **Hooke's Law:**

$$\sigma = E \varepsilon$$



# Elastic properties of materials

- **Poisson's ratio,  $\nu$ :**

$$\nu = - \frac{\epsilon_L}{\epsilon}$$

metals:  $\nu \sim 0.33$

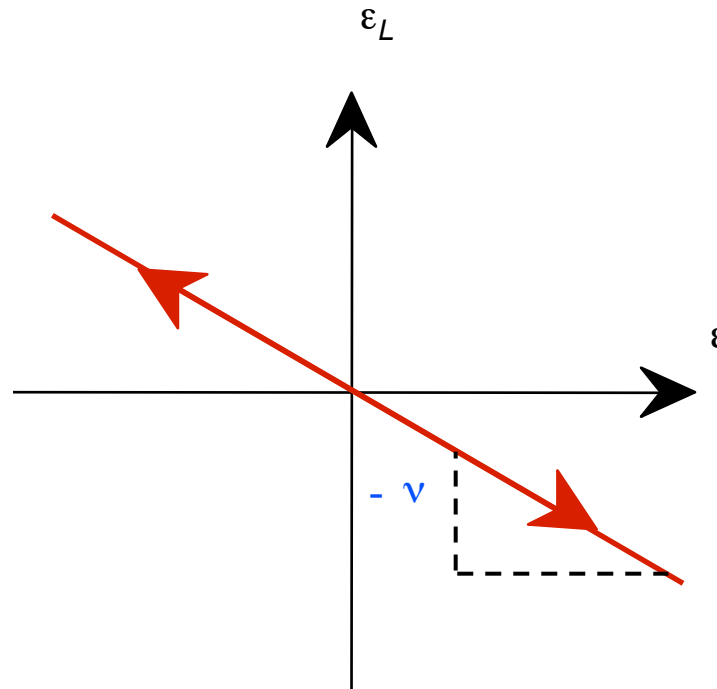
ceramics:  $\nu \sim 0.25$

polymers:  $\nu \sim 0.40$

Units:

$E$ : [GPa] or [psi]

$\nu$ : dimensionless



$-\nu > 0.50$  density increases

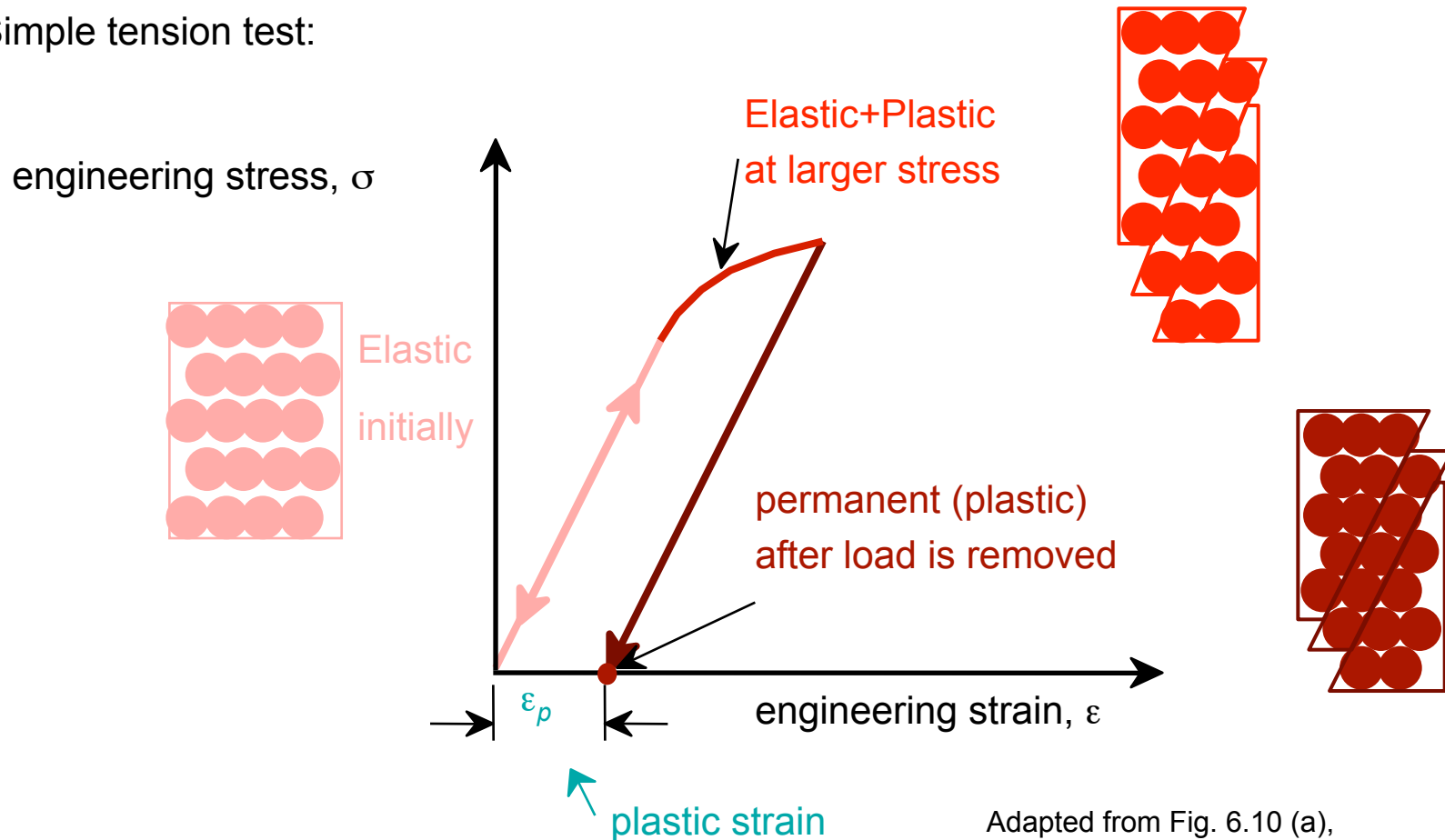
$-\nu < 0.50$  density decreases  
(voids form)

–For *isotropic* materials

$$E = 2G(1+\nu)$$

# Plastic deformation

- Simple tension test:

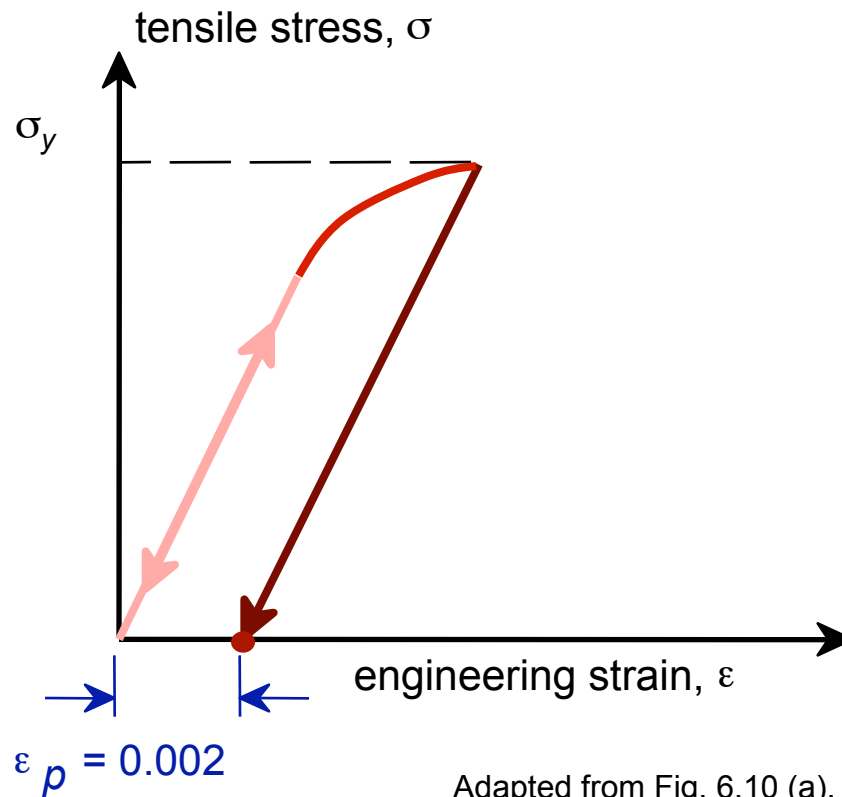


Adapted from Fig. 6.10 (a),  
Callister 7e.

# Yield strength, $\sigma_y$

- Stress at which *noticeable* plastic deformation has occurred.

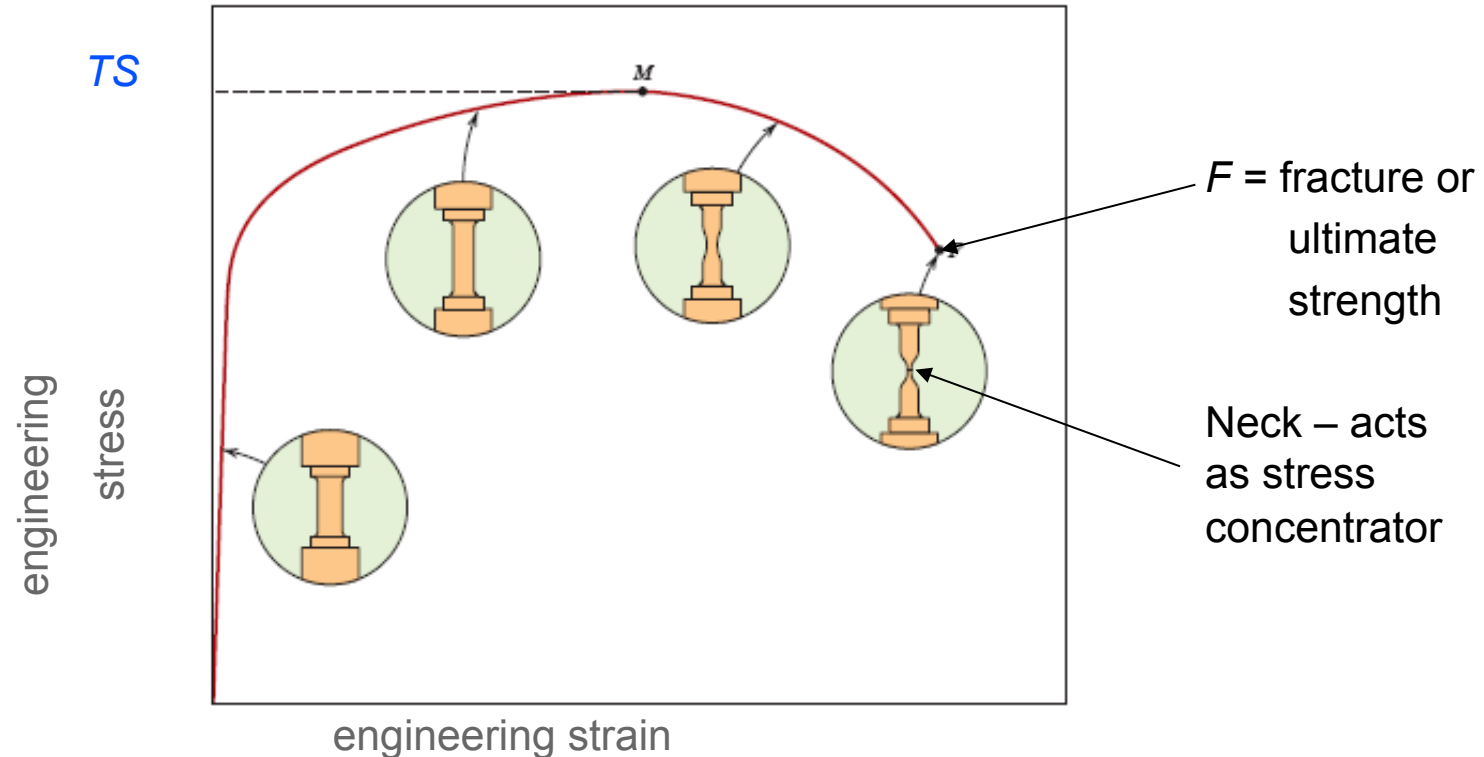
when  $\epsilon_p = 0.002$



Adapted from Fig. 6.10 (a),  
*Callister 7e*.

# Tensile strength, TS

- Maximum stress on engineering stress-strain curve.

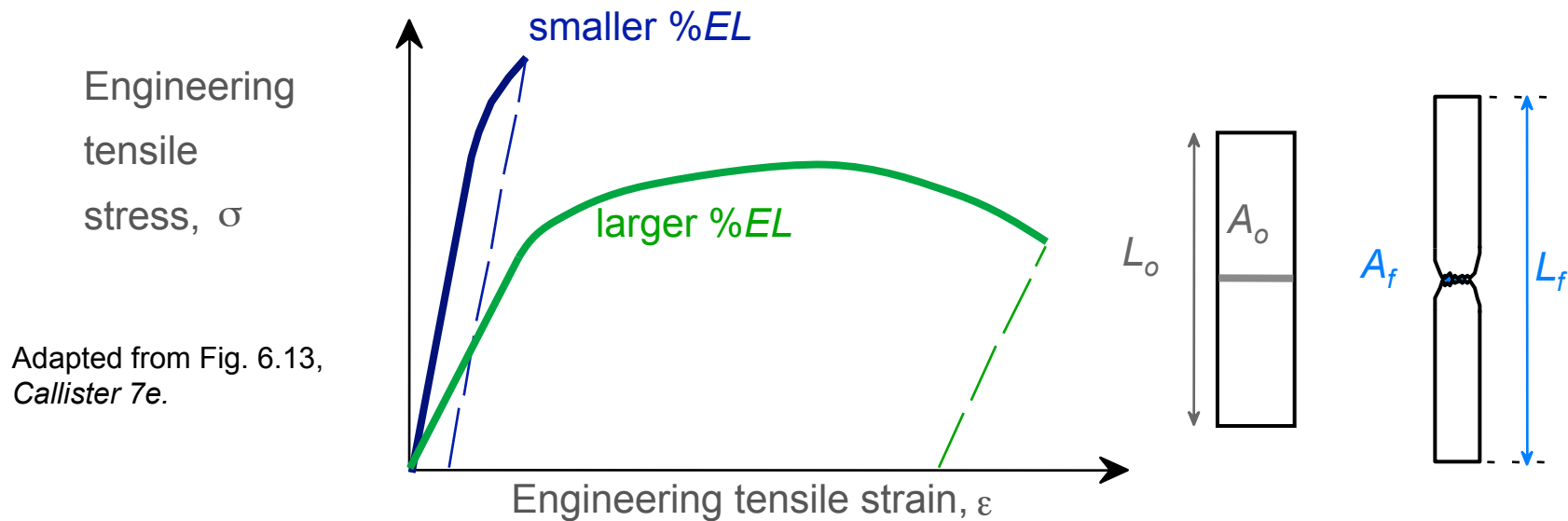


- Metals:** occurs when noticeable necking starts.
- Polymers:** occurs when polymer backbone chains are aligned and about to break.



# Ductility

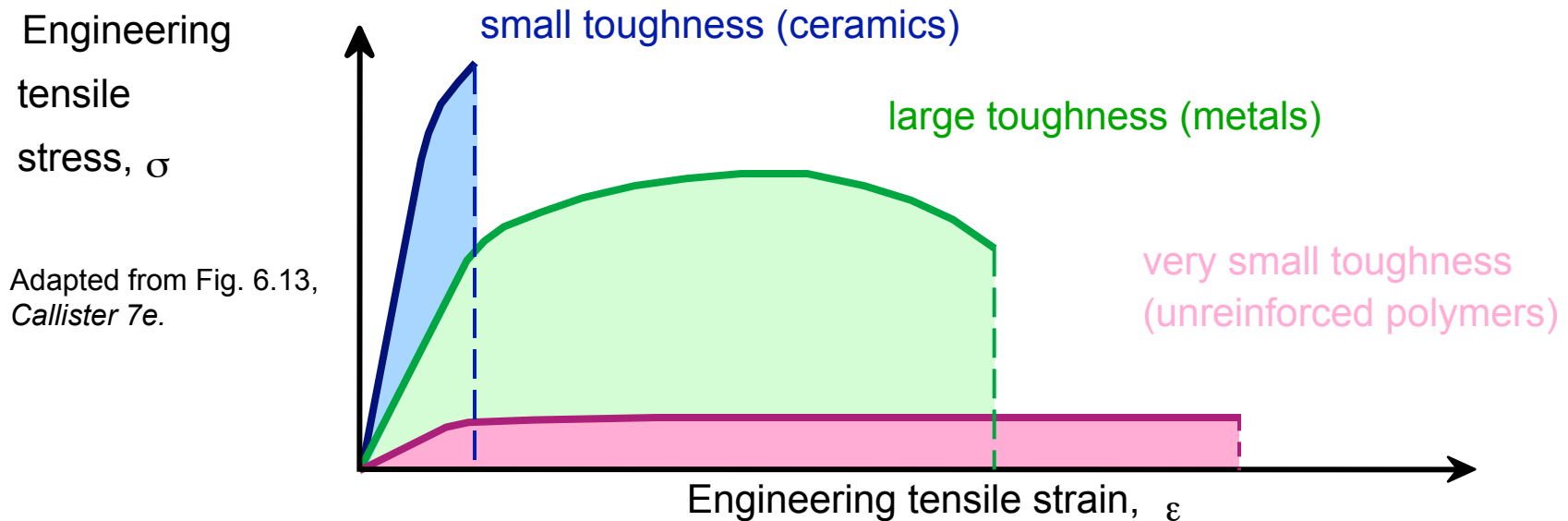
- Plastic tensile strain at failure: 
$$\% EL = \frac{L_f - L_o}{L_o} \times 100$$



- Another ductility measure: 
$$\% RA = \frac{A_o - A_f}{A_o} \times 100$$

# Toughness

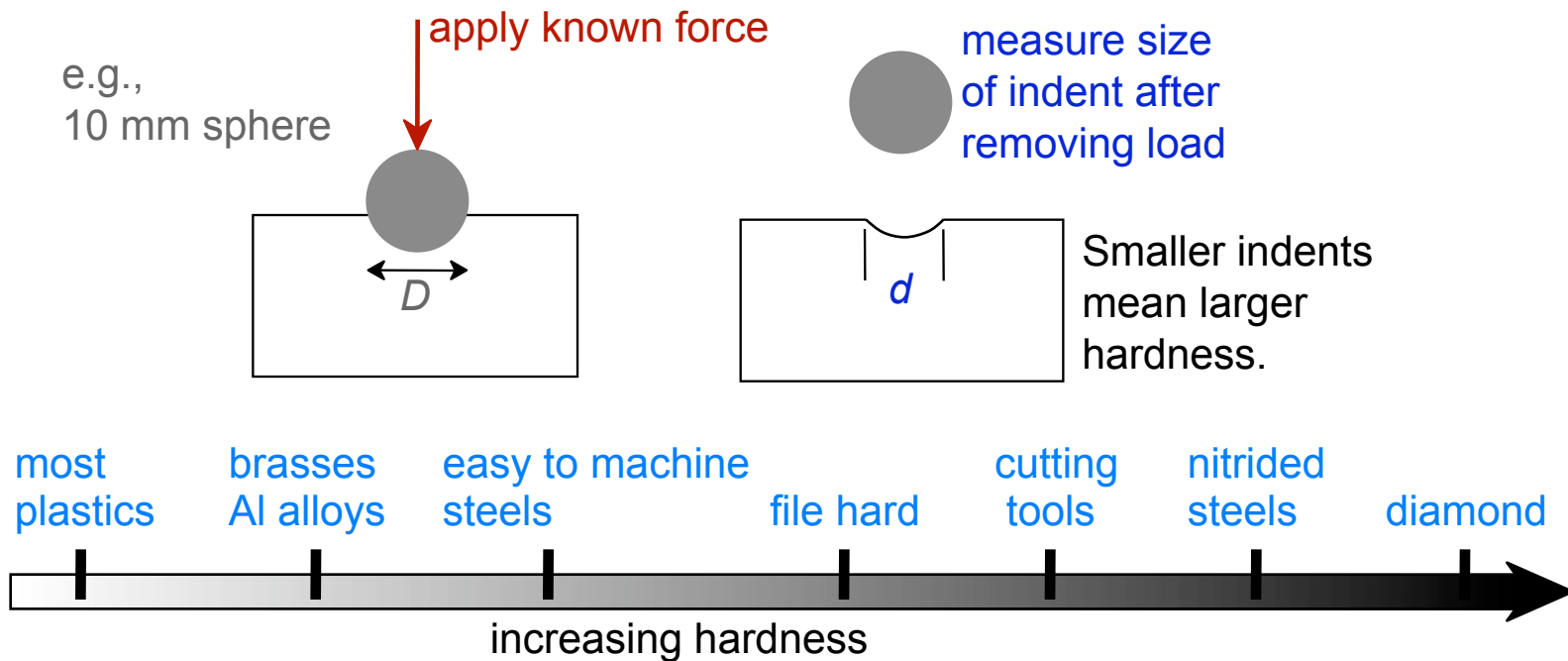
- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.



Brittle fracture: elastic energy  
Ductile fracture: elastic + plastic energy

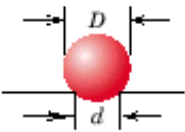
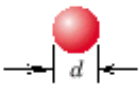
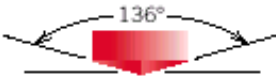
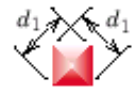
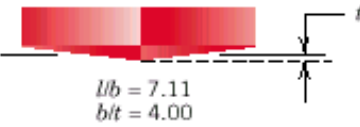
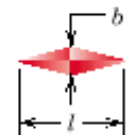
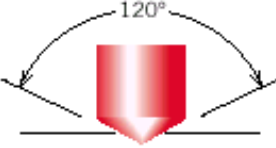



# Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  - resistance to plastic deformation or cracking in compression.
  - better wear properties.



# Hardness Measurement

Table 6.5 Hardness Testing Techniques

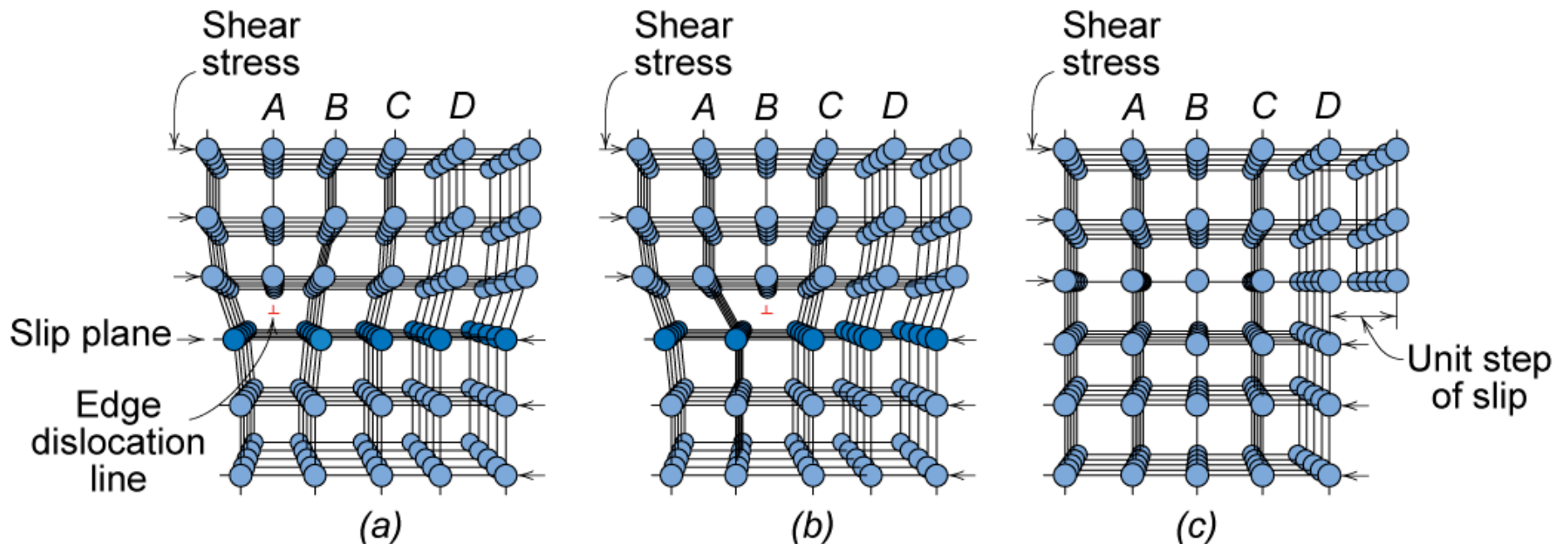
Test	Indenter	Shape of Indentation		Load	Formula for Hardness Number <sup>a</sup>
		Side View	Top View		
Brinell	10-mm sphere of steel or tungsten carbide			$P$	$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			$P$	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			$P$	$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	<div> <div>Diamond cone</div> <div><math>\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}</math> in. diameter steel spheres</div> </div>	 	 	<div>60 kg</div> <div>100 kg</div> <div>150 kg</div> <div>15 kg</div> <div>30 kg</div> <div>45 kg</div>	<div>Rockwell</div> <div>Superficial Rockwell</div>

<sup>a</sup> For the hardness formulas given,  $P$  (the applied load) is in kg, while  $D$ ,  $d$ ,  $d_1$ , and  $l$  are all in mm.

**Source:** Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

# Dislocation and plastic deformation

- Cubic & hexagonal metals - plastic deformation by **plastic shear or slip** where one plane of atoms slides over adjacent plane by defect motion (dislocations).

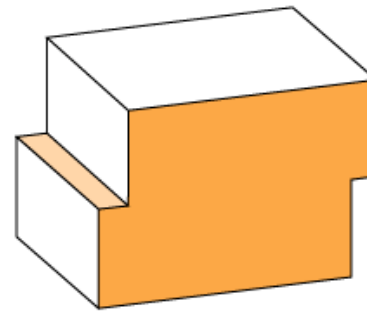
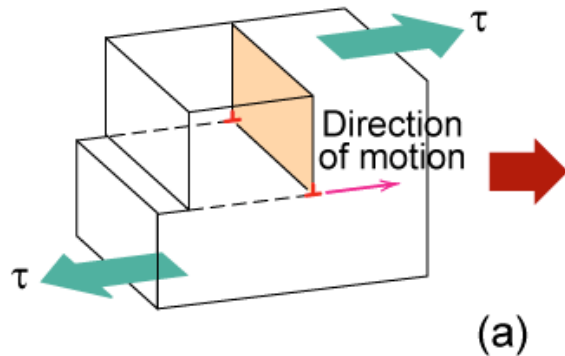


- If dislocations don't move, deformation doesn't occur!

Adapted from Fig. 7.1,  
*Callister 7e*.

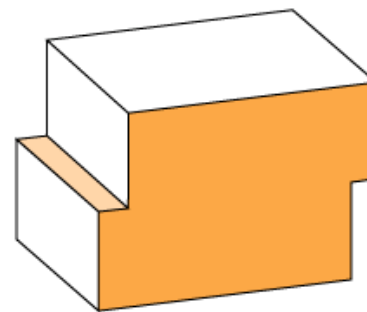
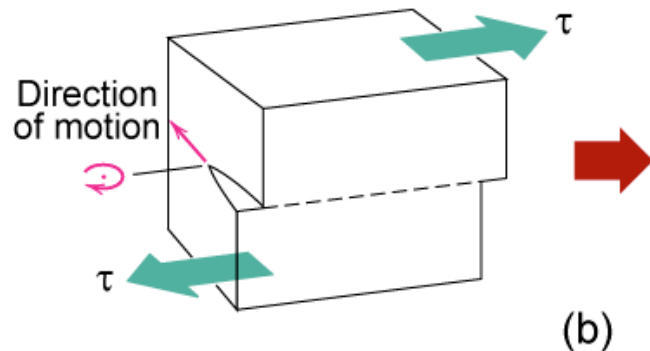
# Dislocation motion

- Dislocation moves along **slip plane** in **slip direction** perpendicular to dislocation line
- Edge dislocations** move **parallel** to the applied force, **screw dislocations** move **perpendicular** to the applied force



**Edge dislocation**

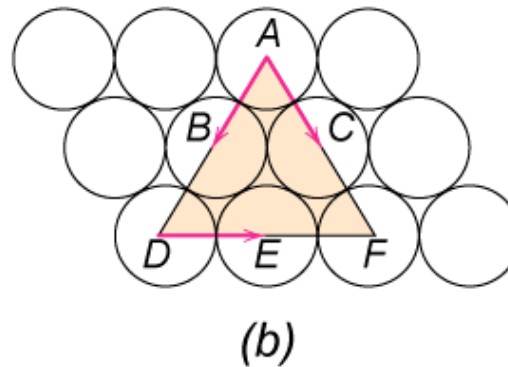
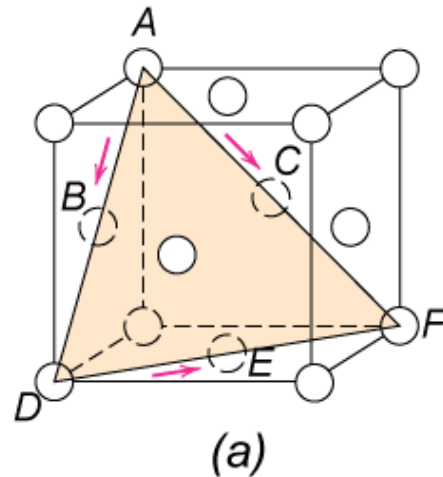
Adapted from Fig. 7.2,  
*Callister 7e.*



**Screw dislocation**

# Deformation mechanisms

- Slip System
  - Slip plane - plane allowing easiest slippage
  - Slip direction - direction of movement

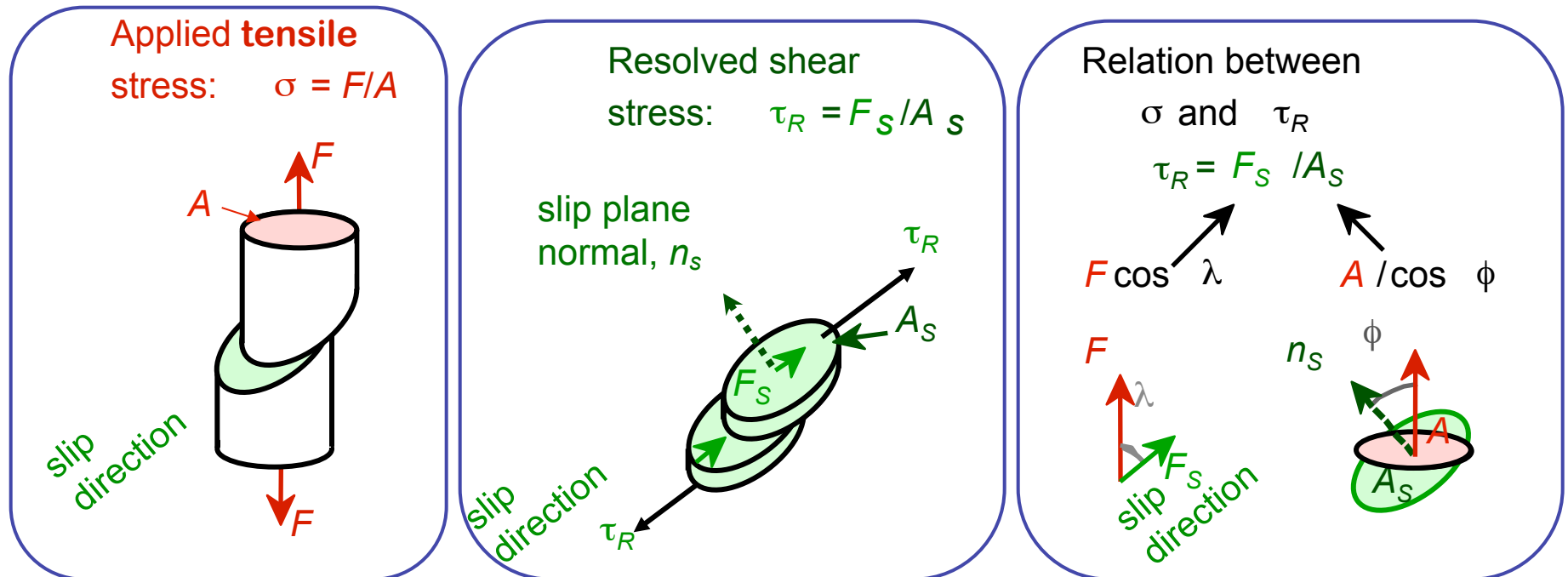


Adapted from Fig. 7.6, Callister 7e.

- FCC Slip occurs on  $\{111\}$  planes (close-packed) in  $\langle 110 \rangle$  directions (close-packed)
  - $\Rightarrow$  total of 12 slip systems in FCC

# Slip in single crystals

- Crystals slip due to a **resolved shear stress**,  $\tau_R$ .
- Applied tension can produce such a stress.



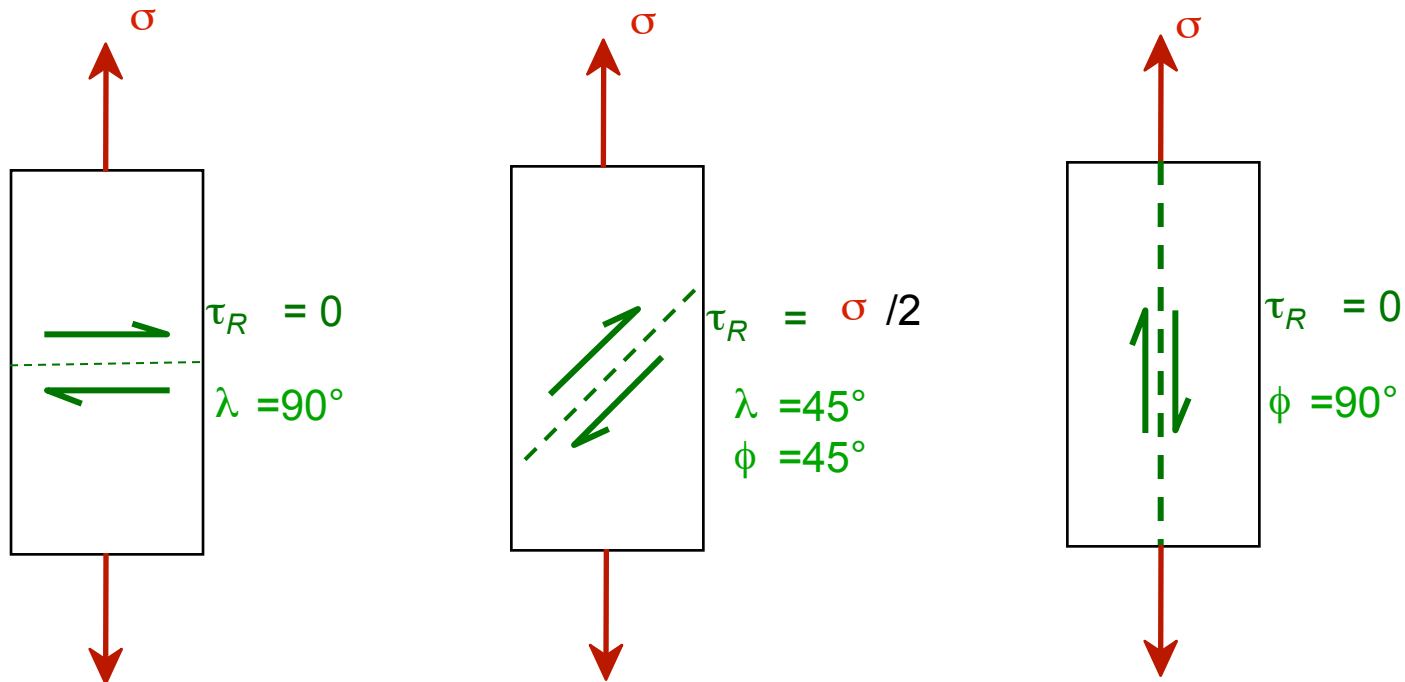
$$\tau_R = \sigma \cos \lambda \cos \phi$$



# Critical resolved shear stress

- Condition for dislocation motion:  $\tau_R > \tau_{CRSS}$
- Crystal orientation can make it easy or hard to move dislocation

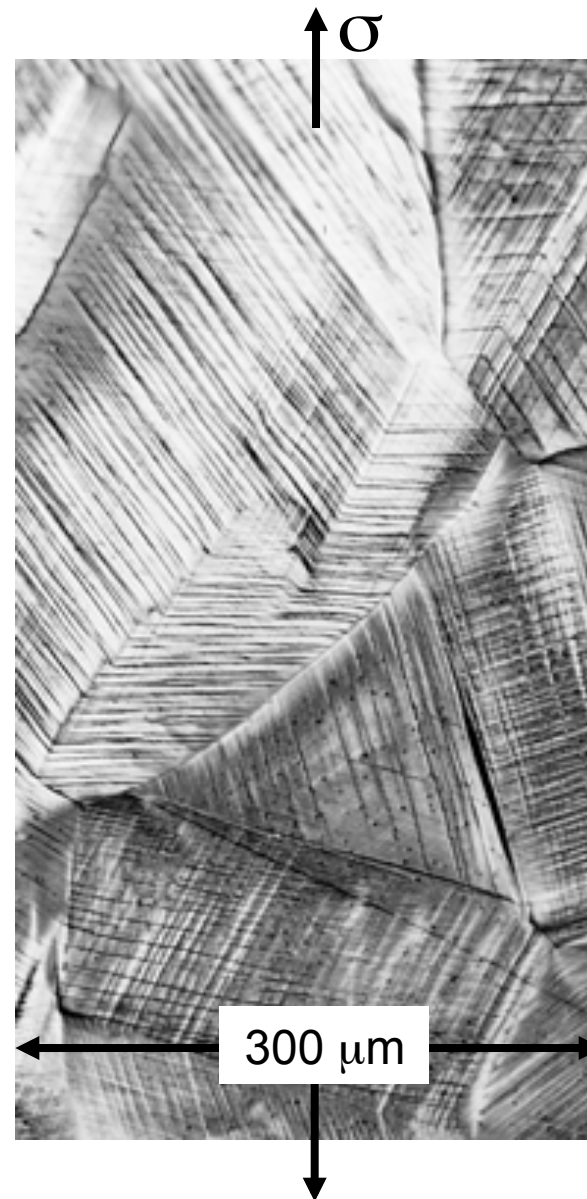
$$\tau_R = \sigma \cos \lambda \cos \phi$$



$\tau$  maximum at  $\lambda = \phi = 45^\circ$

# Slip motion in polycrystals

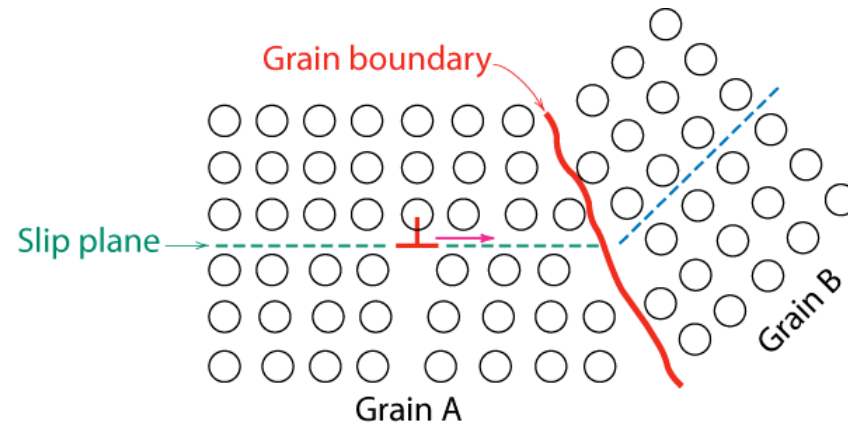
- Stronger - grain boundaries pin deformations
- Slip planes & directions ( $\lambda$ ,  $\phi$ ) change from one crystal to another.
- $\tau_R$  will vary from one crystal to another.
- The crystal with the largest  $\tau_R$  yields first.
- Other (less favorably oriented) crystals yield later.



Adapted from Fig. 7.10, *Callister 7e*. (Fig. 7.10 is courtesy of C. Brady, National Bureau of Standards [now the National Institute of Standards and Technology, Gaithersburg, MD].)

# Strategies for strengthening: grain size reduction

- Grain boundaries are barriers to slip.
- Barrier "strength" increases with increasing angle of misorientation.
- Smaller grain size: more barriers to slip.

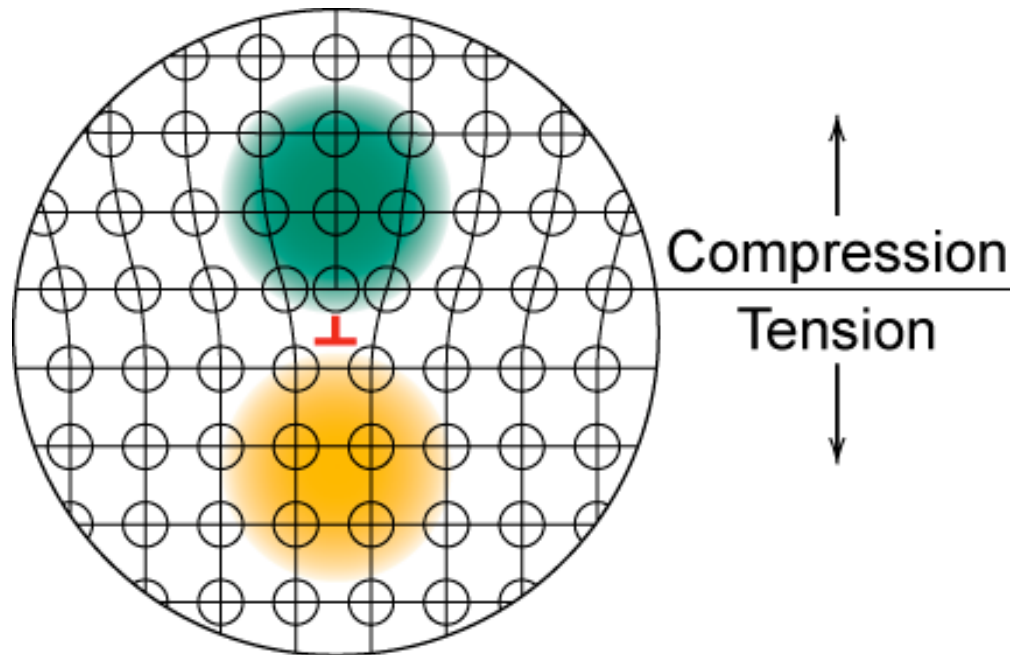


Adapted from Fig. 7.14, *Callister 7e*.  
(Fig. 7.14 is from *A Textbook of Materials Technology*, by Van Vlack, Pearson Education, Inc., Upper Saddle River, NJ.)

- Hall-Petch Equation:  $\sigma_{yield} = \sigma_o + k_y d^{-1/2}$

## Strategies for strengthening: solid solutions

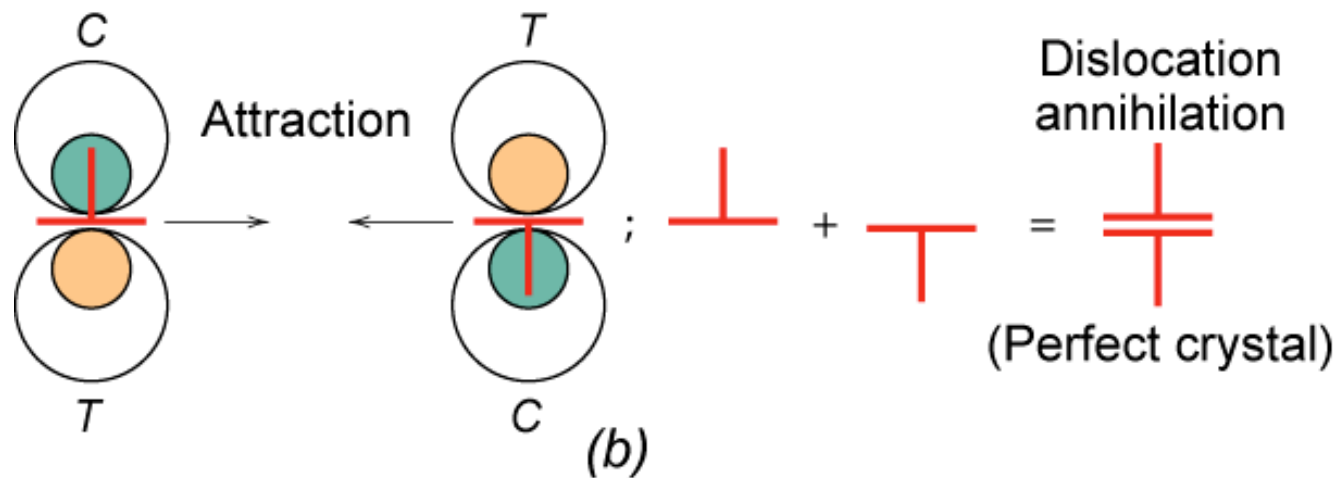
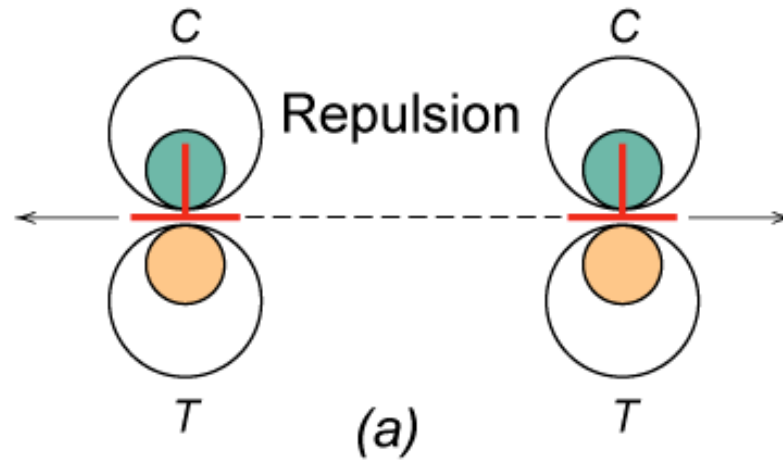
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Adapted from Fig. 7.4,  
*Callister 7e*.

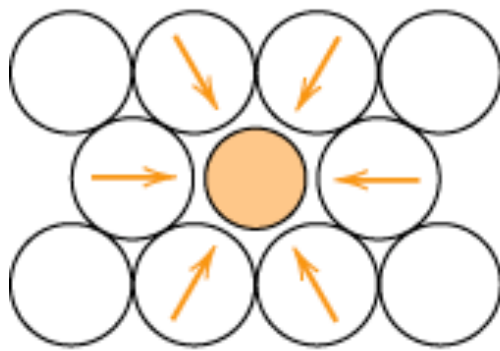
# Effects of stress at dislocations

Adapted from Fig.  
7.5, Callister 7e.

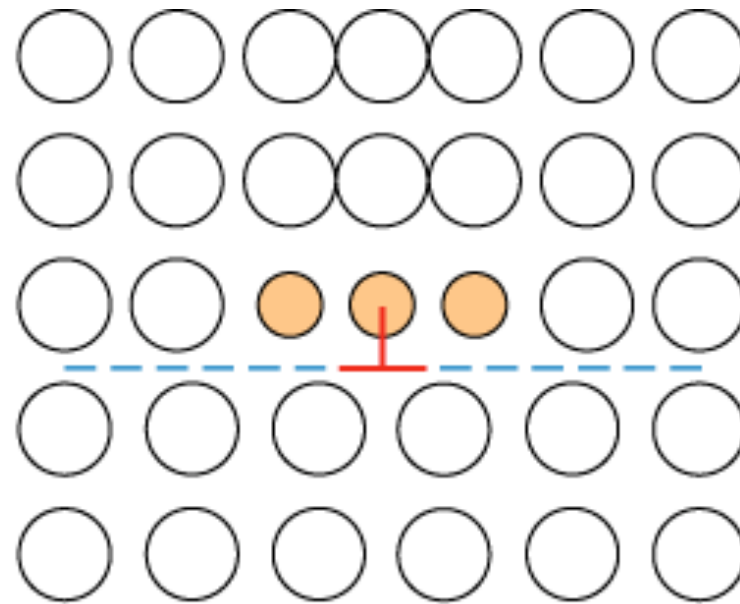


# Strengthening by alloying

- small impurities tend to concentrate at dislocations
- reduce mobility of dislocation  $\therefore$  increase strength



(a)

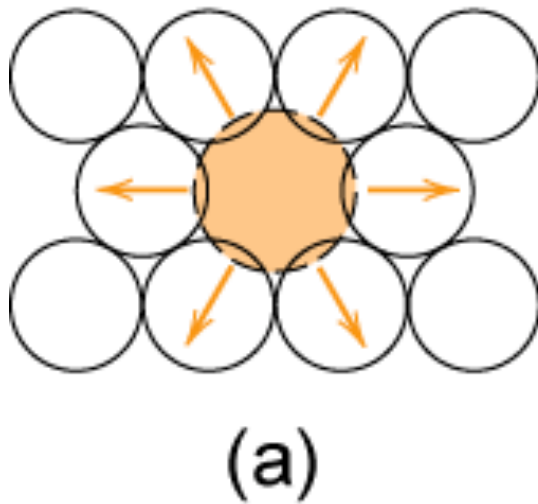


(b)

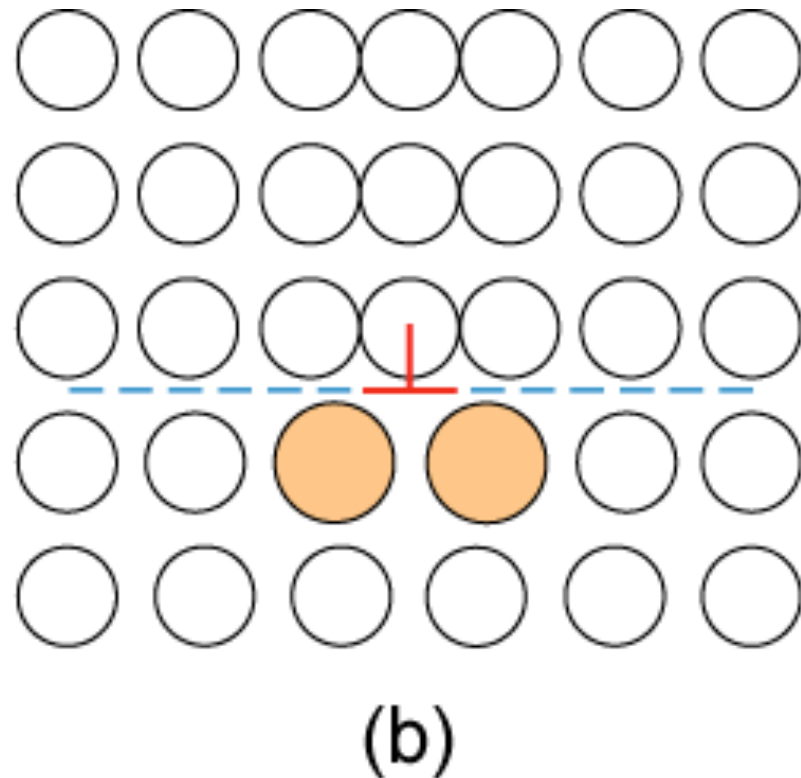
Adapted from Fig.  
7.17, *Callister 7e*.

# Strengthening by alloying

- large impurities concentrate at dislocations on low density side

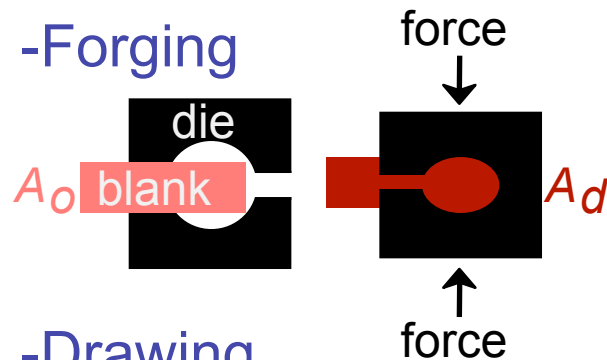


Adapted from Fig.  
7.18, *Callister 7e*.

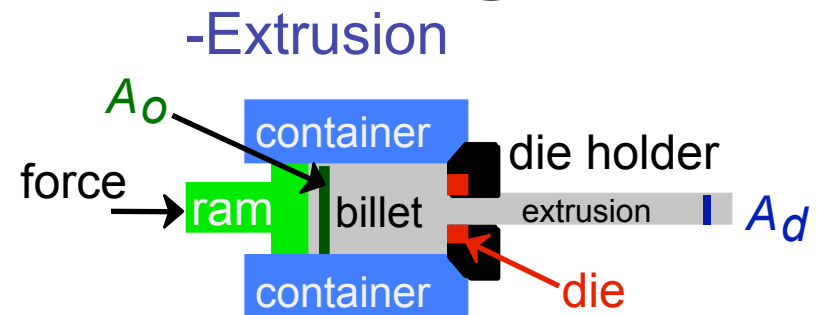
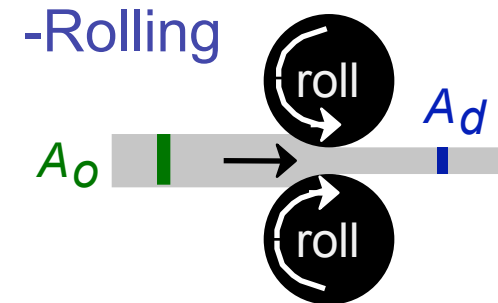
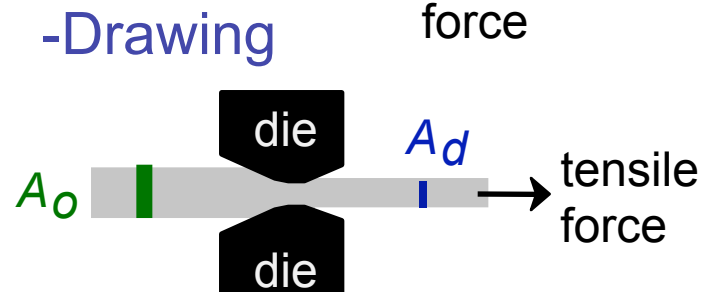


# Strategies for strengthening: Cold work (%CW)

- Room temperature deformation.
- Common forming operations change the cross sectional area:



Adapted from Fig. 11.8, Callister 7e.



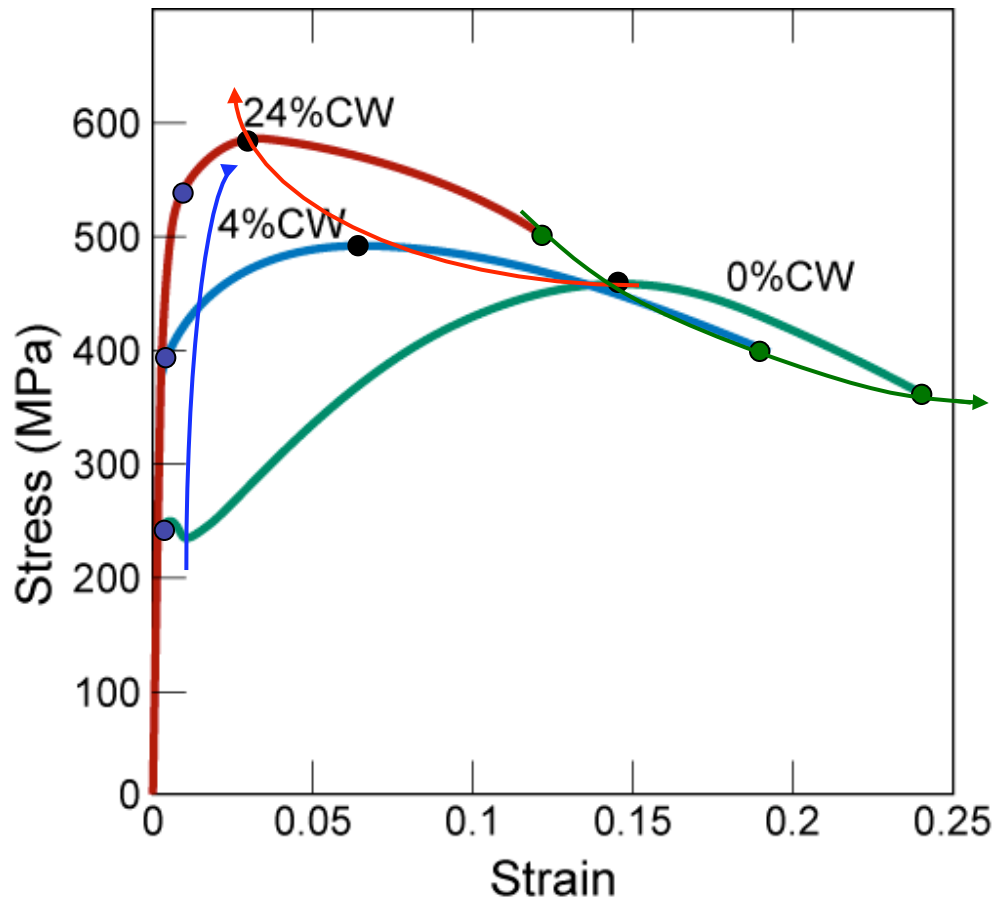
$$\%CW = \frac{A_o - A_d}{A_o} \times 100$$



# Impact of cold work

As cold work is increased

- Yield strength ( $\sigma_y$ ) increases.
- Tensile strength (TS) increases.
- Ductility (%EL or %AR) decreases.



Adapted from Fig. 7.20,  
Callister 7e.