Study Problems : MSE 170 Final

1. Polyethylene contains equal numbers of molecules with 100 mers, 200 mers, 300 mers, 400 mers, and 500 mers, respectively. What is the 'number-average' molecular weight ? and the 'mass-average' molecular weight ? (Ans. : M_w 10,300 g/mole)

2. Calculate the 'number-average' molecular weight of a copolymer consisting of random mix of isobutylene and isoprene *repeat units*. Assume that the isoprene-fraction is 0.75 and the degree of polymerization (DP) is 1500. (Ans.: M_n 97,675 g/mole)

3. The relaxation time (τ) for a polymer is given as 45 days and the modulus elasticity (E) is 70 MPa, under isothermal conditions at 100°C. The polymer is compressed 5% (ε_o = 0.05) and held at the specified temperature. Determine the initial stress (σ_o). Assuming that the polymer undergoes viscoelastic relaxation at 100°C, calculate the stress $\sigma(t)$ after a period of (a) 1 day (b) 30 days (c) 365 days ? (Ans.: 3.5 MPa ; a. 3.4 MPa ; c. 1000 Pa)

4. List the *mer* structure for the following : (a) Butadiene (b) *cis*-Isoprene (c) Chloroprene.

5. The vulcanization of polyisoprene is accomplished with sulfur atoms which provide the cross-linking between molecules. If the vulcanized polyisoprene showed 32 wt% of S, how many sulfur cross-links exist per isoprene *mer* ? (Ans.: 1 S per *mer*)

6. Ten kg of polychloroprene is vulcanized with 0.72 kg of sulfur. Assuming one sulfur-atom participates in each bond, determine the fraction of the sulfur-cross-linked bonds at the specified sulfur concentration. (Ans.: 0.20)

7. A structural member 250 mm long must be able to support a load of 44,400 N without experiencing any plastic deformation. Using the following data for aluminum, brass, and steel determine the alloy that will meet the requirements at the lowest weight:

Alloy	Yield strength, MPa	Density, g/cm^3
Aluminum	275	2.7
Brass	415	8.5
Steel	860	7.9

8. Suppose that a steel of eutectoid composition is cooled from 760°C to 550°C in less than 0.5 sec and held at this temperature (Fig. 10.6). How much time is required for the austenite-to-pearlite reaction to reach 50% completion ? to reach 100% completion ?

9. The kinetics of phase-transformation in a system obey the Avrami equation: fraction transformed, $y = 1 - \exp(-kt^{1.7})$. It was noted that in 100 sec duration, 'y' reached 0.5; calculate the length of time required to obtain a transformation of $y = 0.99$.

10. What is the driving force for the formation of spheroidite in iron-carbon alloys ?

Problem 8

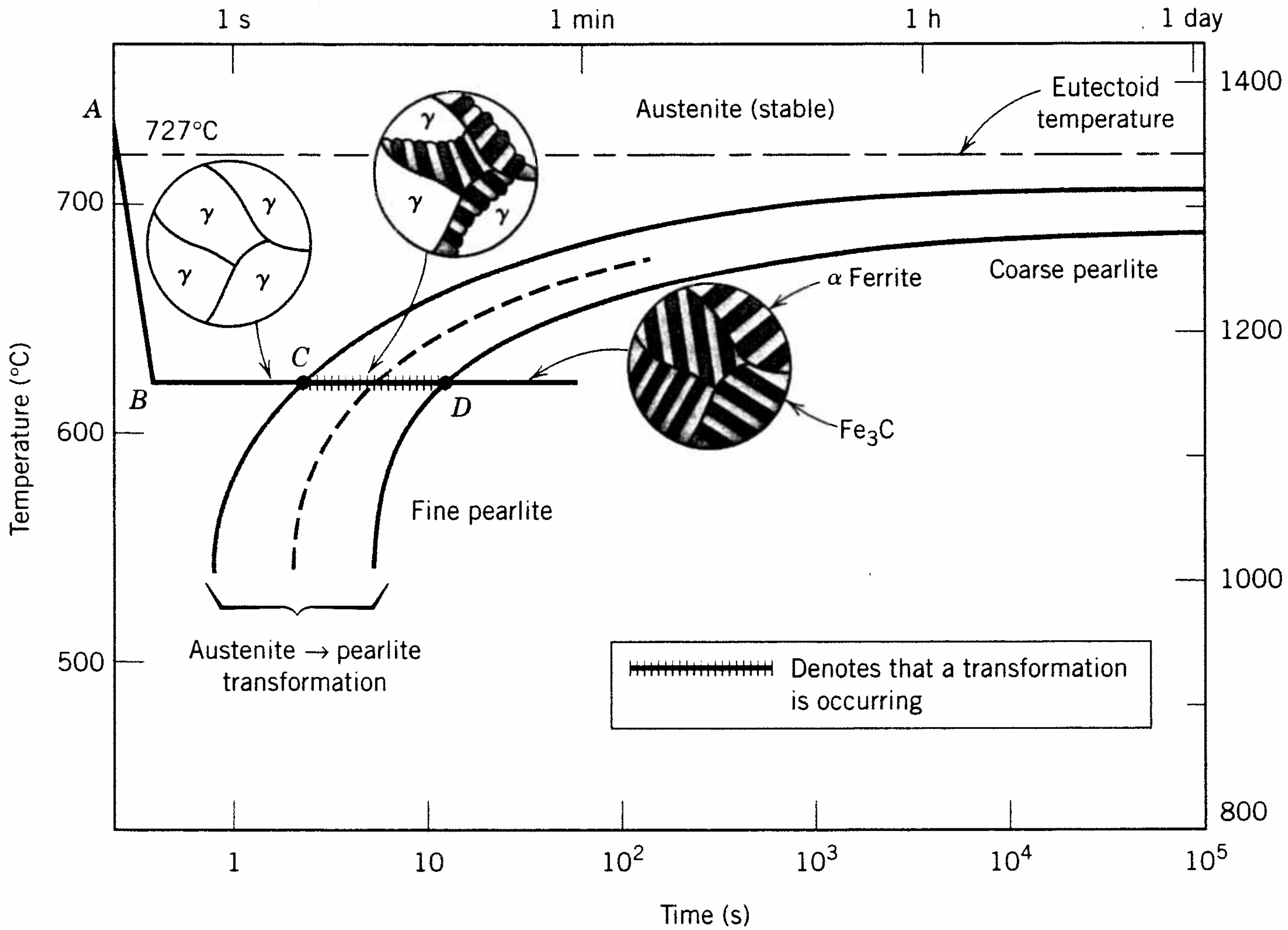


Figure 10.14 Isothermal transformation diagram for a eutectoid iron–carbon alloy, with superimposed isothermal heat treatment curve (*ABCD*). Microstructures before, during, and after the austenite-to-pearlite transformation are shown. [Adapted from H. Boyer (Editor), *Atlas of Isothermal Transformation and Cooling Transformation Diagrams,* American Society for Metals, 1977, p. 28.]

11(a). Recrystallization is a thermally-activated process with the rate **r** $=$ A.exp (- Q/RT) where **r** is expressed as reciprocal of time t_R; the latter is the time required to obtain fully-recrystallized microstructure. It was found that a 75% cold-worked aluminum alloy needs for complete recrystallization (i) $t_R = 100$ hours at 256°C and (ii) $t_R = 10$ hours at 283°C. Find the activation energy Q in kJ/mole and use the result to determine the temperature at which the recrystallization would be completed in a duration (t_R) of 2.5 hours.
11(b). During an investigation of age-hardening in aluminum-based alloys, it was found that maximum hardness could be achieved by aging for 10 hr at 600°K or 280 hr at 500°K. How long would it take to reach maximum hardness for the same alloy at 530°K ?

12. The fracture-toughness is given by : $K_{IC} = Y.\sigma_f [\pi a]^{1/2}$ with $Y \approx 1$. The yield strength of a special steel is 1400 MPa and its fracture-toughness is $K_{IC} = 150$ MPa.(m)$^{1/2}$. Find the size (length) of the inner-crack that will propagate and produce failure at an applied stress (σ_f) of 600 MPa. What would be the critical crack-length if it occurs on surface ?

13. Calculate the breaking strength (σ_f) for MgO(s) when a surface-crack 1 mm long is present. Use $K_{IC} = 3$ MPa.(m)$^{1/2}$ and the equation given in the previous problem.

14. A 1-mm rod drawn from plate glass ($\alpha_L = 9\text{x}10^{-6}\ {}^{o}K^{-1}$) is heated to 200°C to remove any residual stresses. Now the rod is cooled to 20°C at fixed-length. Calculate the stress (σ) that develops in the rod. The modulus of elasticity E = 70 GPa for the plate glass.

15. A fiberglass contains 50 vol% E-glass (54 w/o SiO_2, 17 w/o CaO, 5 w/o MgO, 15 w/o $Al_2O_3 + Fe_2O_3$, 8 w/o B_2O_3 and < 1 w/o each of K_2O and Na_2O) in an epoxy matrix. Find (a) the w/o of E-glass fibers in the composite and (b) the density. For the two components of this composite, it is given that $\rho_E = 2.54$ g/cm^3 and $\rho_{epy} = 1.1$ g/cm^3, respectively.

16. Calculate the elastic modulus, $E_{com} = v_{epy}E_{epy} + v_C E_C$ for a carbon-fiber reinforced epoxy matrix composite and compare the result with the experimental value of 221 GPa. Assume that the volume-fractions are 0.33 for epoxy and 0.67 for carbon; furthermore, the corresponding elastic moduli are: $E_{epy} = 6.9$ GPa and $E_C = 360$ GPa. This equation for E_{com} predicts the so-called *isostrain modulus* of the composite.

17. An intrinsic semiconductor (probably Ge) has a conductivity of 390 ohm^{-1}.m^{-1} at 5°C and 1010 ohm^{-1}.m^{-1} at 25°C, respectively. (a) How large is the band-gapenergy (E_g) ? (b) What is the conductivity at 15°C ? The Boltzmann constant k = 8.617x10^{-5} eV/°K. It is well to note that, to a good approximation, $\sigma = \sigma_o.\exp[-E_g/(2kT)]$, ohm^{-1}.m^{-1}.

18(a). Calculate the induction (**B**) and magnetization (**M**) of a paramagnetic material with $\mu_r = 1.001$ under an applied field strength (**H**) of 5x10^5 amperes/m. Note that $\mu_r = \mu/\mu_o$.
18(b). The ceramic magnetic material *nickel ferrite* has eight [$NiFe_2O_4$] molecules per unit cell, which is cubic with lattice parameter a = 0.834-nm. Assuming that all the unit cells have the same magnetic orientation, calculate the saturation magnetization. It is to be noted that each Ni^{2+} ion has two Bohr magnetons. In the foregoing, μ and μ_r denote the permeability and the relative permeability of the solid where $\mu_r = \mu/\mu_o$; the permeability in vacuum, $\mu_o = 4\pi\text{x}10^{-7}$ Henry/meter.

Study Problems: Final

1. Ethylene 'mer' : C_2H_4 ; mol. wt. 28.054 g/mole
The polyethylene has polymer molecules having five different molecular weights.

Type	Number	X_N, fraction	mol. wt.	X_w, fraction	X_w(mol. wt)
1	E *	0.200	2805.4	0.0667	187
2	E	0.200	5610.8	0.1333	748
3	E	0.200	8416.2	0.2000	1683
4	E	0.200	11221.6	0.2667	2993
5	E	0.200	14027	0.3333	4676
total:	5E	1.000	—	1.000	10,287

* Equal number.

number average mol. wt. $= M_N$

$M_N = 0.2(2805.4 + 5610.8 + \cdots + 14027) = \underline{8416}$

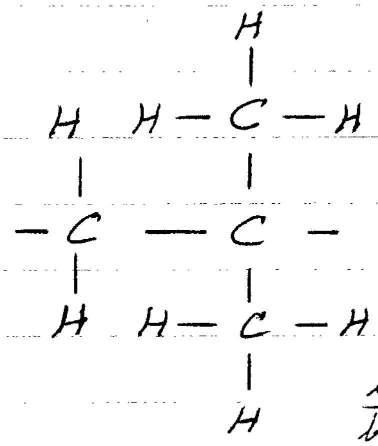
mass-average mol. wt. $= M_w$

$M_w = 0.0667(2805.4) + \cdots + 0.3333(14027)$

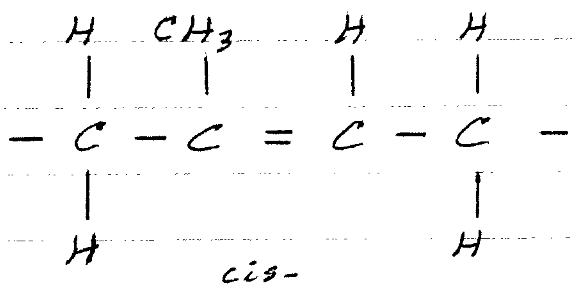
or $M_w = \underline{10,287}$; $DP = \frac{10287}{28.054} = \underline{367}$

A smaller DP is M_N/mer mol. wt. ; it is $\underline{300}$ only.

2.



iso-butylene



cis-isoprene

mol. wt. isobutylene mer ($-C_4H_8-$) $= 56.108 \frac{g}{mole}$

mol. wt. isoprene ($-C_4H_5CH_3-$) mer $= 68.119$ g/mole

Co-polymer (random mix):

mole-fraction of isoprene $= 0.75$

mol. wt. of co-polymer 'mer' $= 0.75(68.119) + 0.25(56.108)$

$= 65.12$ g/mole

'number-average' mol. wt. $= 1500 \times DP$

$= \underline{97{,}675 \text{ g/mole}}$

3. relaxation time (τ) $= 45$ days

modulus of elasticity (E) $= 70$ MPa

At 100°C :

compressive strain, $\varepsilon_0 = 0.05$

Initial condition :

compressive stress, $\sigma_0 = E\varepsilon_0 = \underline{3.5 \text{ MPa}}$

Due to viscoelastic relaxation, at any time t, with temperature at 100°C,

$$\frac{d\sigma}{\sigma} = -\frac{dt}{\tau}$$

Integrating with $\sigma = \sigma_0$ at $t = 0$,

$$\sigma(t) = \sigma_0 \cdot \exp(-t/\tau)$$

(a) At $t = 1$ day :

$\sigma = 3.5 \exp(-1/45) = \underline{3.423 \text{ MPa}}$

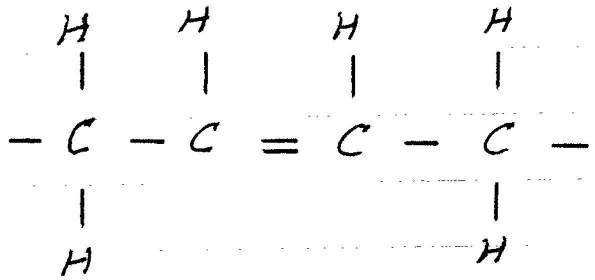
(b) At $t = 30$ days :

$\sigma = 3.5 \exp(-2/3) = \underline{1.797 \text{ MPa}}$

(c) $\sigma = 3.5 \exp(-365/45) = \underline{1050 \text{ Pa}}$: 365 days

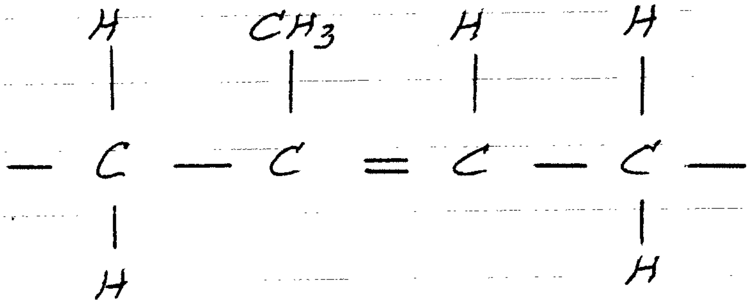
4.

Butadiene: mol. wt. 54.092



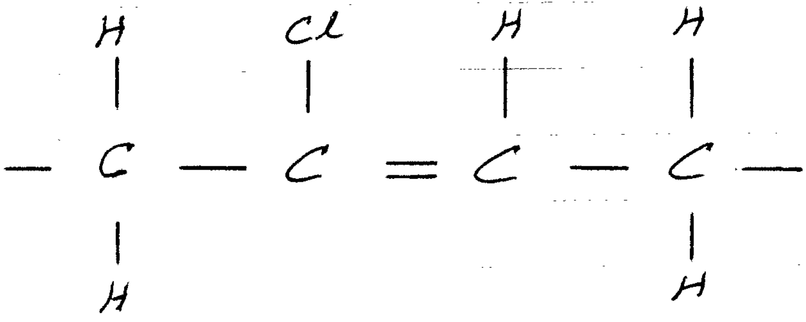
$-\mathrm{CH_2}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH_2}-$

cis-Isoprene: mol. wt. 68.119

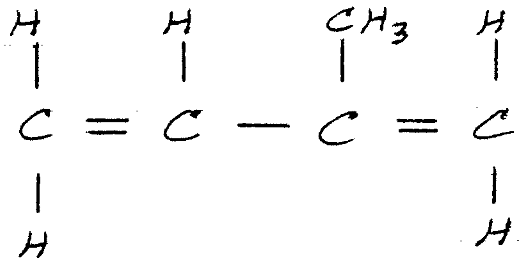


$-\mathrm{CH_2}-\mathrm{C(CH_3)}=\mathrm{CH}-\mathrm{CH_2}-$

Chloroprene: mol. wt 88.537



$-\mathrm{CH_2}-\mathrm{C(Cl)}=\mathrm{CH}-\mathrm{CH_2}-$

5. 

$$\begin{array}{ccccccc} H & & H & & CH_3 & & H \\ | & & | & & | & & | \\ C & = & C & - & C & = & C \\ | & & & & & & | \\ H & & & & & & H \end{array}$$

<u>isoprene monomer</u>

mol. wt. $68.113 \frac{g}{mole}$

$\approx$ <u>68 g/mole</u>

Consider 400 g of vulcanized polyisoprene:

mass of isoprene $= 400(0.68) = 272$ g

isoprene 'mers' $= \frac{272}{68} = \underline{4}$

mass of sulfur $= 400(0.32) = 128$ g

mol. wt. of S $= 32$ g/mole

sulfur atoms (to cross-link) $= \frac{128}{32} = \underline{4}$

There are 4 sulfur cross-links per 4 mers; hence, <u>one S per mer.</u>

6. polychloroprene $(-C_4H_5Cl-) = 10$ kg

mol. wt. of a 'mer' $= 88.537$ g/mole

number of chloroprene mers $= \frac{10^4}{88.537} = \underline{113}$

amount of S $= 0.72$ kg

number of S atoms $= \frac{0.72 \times 10^3}{32} = \underline{22.5}$

For a fully-cross-linked polymer, there would be 113 sulfur atoms (1 S per mer); at 22.5 S-atoms per 113 chloroprene mers,

fraction cross-linked $= \frac{22.5}{113} = \underline{0.20}$

7. In general, for the i-th material,

area of cross-section $= A_i = \frac{F}{\sigma_i} = \frac{\text{Load}}{\sigma_i}$

where σ_i = yield strength of the ith alloy.

At a specified length $L = 0.25\,m$, the respective weights can be found using

$$w_i = (\text{area})\,\text{length}\,(\text{density}) = 0.25 A_i \rho_i$$

where ρ_i is the density of the i-th alloy.

Load, $F = 44,400\,N$

Thus

$$w_{Al} = 0.25 F \frac{2.7}{275} = 2.455 \times 10^{-3} F \equiv \underline{1.07}$$

$$w_{Br} = 0.25 F \frac{8.5}{415} = 5.121 \times 10^{-3} F \equiv \underline{2.23}$$

$$w_{St} = 0.25 F \frac{7.9}{860} = 2.300 \times 10^{-3} F \equiv \underline{1.0}$$

Thus, steel offers the lowest weight with aluminum a close second. One can show the respective data are as follows:

A_i m^2	diameter m	wt., w_i in kg	Alloy i
0.000161	0.01432	0.109	Al
0.000107	0.01167	0.2275	Brass
0.00005163	0.008108	0.102	Steel

8. <u>Fig. 10.14 (W.D. Callister, Jr.):</u>

Austenite → pearlite transformation:

i) begins at $t \approx 2.4$ sec

ii) 50% completed at <u>$t \approx 5.4$ sec</u>

iii) 100% completed at <u>$t \approx 12.8$ sec</u>

Problem 8

5.a

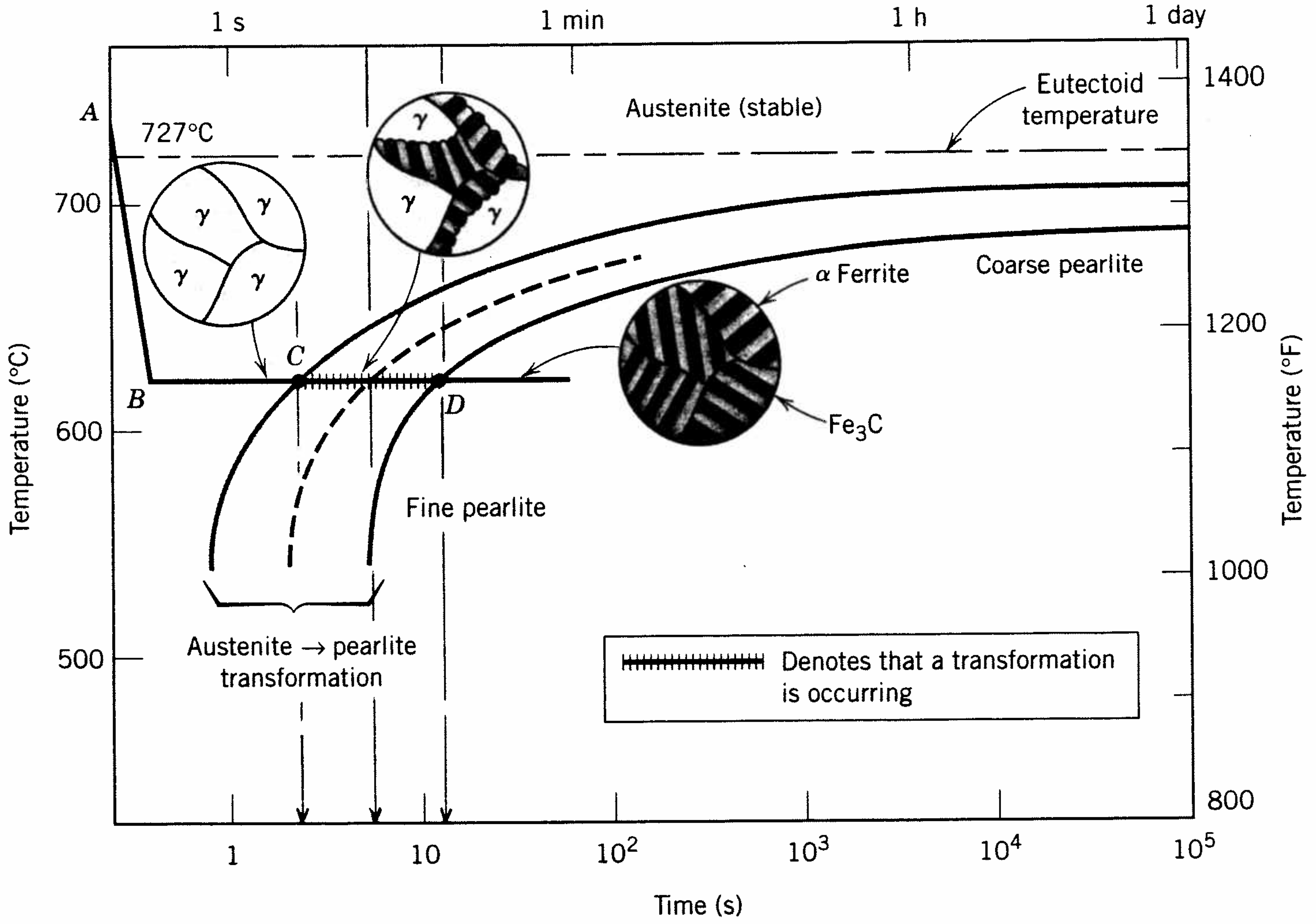


Figure 10.14 Isothermal transformation diagram for a eutectoid iron–carbon alloy, with superimposed isothermal heat treatment curve (*ABCD*). Microstructures before, during, and after the austenite-to-pearlite transformation are shown. [Adapted from H. Boyer (Editor), *Atlas of Isothermal Transformation and Cooling Transformation Diagrams*, American Society for Metals, 1977, p. 28.]

9. Phase-transformation kinetics can be suitably represented by the Avrami equation:

$$y = 1 - \exp(-kt^{1.7}) \qquad (1)$$

Rearranging terms and taking logarithms,

$$-\ln(1-y) = kt^{1.7} \qquad (2)$$

At $t = 100$ sec, $y = 0.5$:

$$k = 0.6932/(100)^{1.7} = 2.76 \times 10^{-4}\ \text{sec}^{-1.7}$$

For $y = 0.99$, $t = t_{0.99}$:

$$4.6052 = 2.76 \times 10^{-4}(t_{0.99})^{1.7}$$

$$t_{0.99} = 304.6\ \text{sec.}$$

10. The lamellar pearlite microstructure consisting of α-Ferrite and Fe_3C undergoes a marked change when subjected to prolonged heat treatment (for 18-24 hrs) at about 700°C: the elongated Fe_3C phase becomes spherical-shaped Fe_3C distributed in the α-ferrite matrix. The driving force for this change is decrease in the interfacial free energy: for a given volume, spherical-shape offers the lowest surface area. The much larger interfacial area of the lamellar pearlite (layers of α-ferrite/Fe_3C) gives away to the smaller area of Fe_3C/ferrite phase in the spheroidite. This is more ductile and less hard as compared to the fine pearlite.

11(a). Recrystallization-rate, $r = \frac{1}{t_R}$ (1)

$$\ln r = -\ln t_R = \frac{-Q}{R \cdot T} + \ln A \qquad (2)$$

At 256°C or 529°K:

$$-\ln 100 = \ln A - \frac{Q}{529 R} \qquad (3)$$

At 283°C or 556°K:

$$-\ln 10 = \ln A - \frac{Q}{556 R} \qquad (4)$$

Subtracting

$$\ln \frac{10}{100} = \frac{Q}{R} \cdot \frac{529 - 556}{(556)(529)}$$

$$Q = (R \ln 0.1)\, 556 (529) / (-27)$$

(i) $Q = 208{,}550$ J/mole

From Eqn (4), we find

$$\ln A = -\ln 10 + \frac{208550}{8.3143 \times 556}$$

(ii) $A = 3.915 \times 10^{18}$ hr^{-1}

At $T = T_3$, $t_R = 2.5$ hrs.

$$-\ln 2.5 = \ln (3.915 \times 10^{18}) - \frac{208550}{8.3143\, T_3}$$

(iii) $T_3 = \frac{208550}{8.3143 \times 43.7276} = \underline{573.6°K}$

11(b). Since age-hardening is a thermally-activated process,

$$\ln \left(\frac{1}{t_{ag}}\right) = -\ln t_{ag} = \ln A_{ag} - \frac{Q_{ag}}{8.3143 \cdot T}$$

Substituting

$$-\ln 10 = \ln A_{ag} - \frac{Q_{ag}}{8.3143 \times 600}$$

$$-\ln 280 = \ln A_{ag} - \frac{Q_{ag}}{8.3143 \times 500}$$

$$Q_{ag} = \frac{8.3143 \cdot \ln 28}{(600 - 500)} (600 \times 500) = \underline{83{,}115 \text{ J/mole}}$$

$A_{ag} = 1.721 \times 10^6 \text{ hr}^{-1}$

<u>At $T = 530°K$:</u>

$\ln \frac{1}{t_{ag}} = \ln 1.721 \times 10^6 - \frac{83115}{8.3143 \times 530}$

$-\ln t_{ag} = -4.5032$; $\underline{t_{ag} = 90.3 \text{ hrs}}$

12. Yield strength of special steel $= 1400$ MPa

$150 \text{ MPa}\cdot\text{m}^{1/2} = (1) 1400 (\pi a)^{1/2}$

$a = 0.003654$ m or 3.654 mm

length of inner crack $= 2a = \underline{7.308 \text{ mm}}$

surface crack-length, $a = \underline{3.654 \text{ mm}}$

13. For MgO(s), fracture toughness $= 3 \text{ MPa}\cdot\text{m}^{1/2}$

Surface crack-length, $a = 1$ mm or 10^{-3} m

$3 \text{ MPa}\cdot\text{m}^{1/2} = (1)\ \sigma_f\ (\pi \times 10^{-3} \text{m})^{1/2}$

breaking strength, $\sigma_f = \underline{53.52 \text{ MPa}}$

14. For the 1-mm rod drawn from plate glass,

α_l, linear coeff. thermal exp. $= \frac{\Delta L}{L_o} \cdot \Delta T$

$\Delta T = 180°K$

Strain, $\epsilon = \frac{\Delta L}{L_o} = 9 \times 10^{-6} \times 180 = 0.00162$

Elastic modulus, $E = \frac{\sigma}{\epsilon} = 70 \times 10^3$ MPa

Stress, $\sigma = E\epsilon = \underline{113.4 \text{ MPa}}$

As the rod-length is a fixed L_o, σ is tensile.

15. Consider 1 cm^3 of composite: $v_E = 0.5\,cm^3$; $v_{epy} = 0.5\,cm^3$

mass of E-glass $= 0.5 \times 2.54 = 1.27\,g$

mass of epoxy $= 0.5 \times 1.10 = 0.55\,g$

total $= 1.82\,g$

w/o E $= 1.27 \times 100)/1.82 = 69.8\,\%$

w/o epoxy $= 0.55 \times 100)/1.82 = 30.2\%$

ρ_{com}, density of the composite $= \underline{1.82\,g/cm^3}$

16. $E_{com} = 0.33(6.9) + 0.67(360) = \underline{243.5\,GPa}$

E_{com}(expt.) $= 221$ GPa; difference $= 10.2\%$

Thus, the upper limit (isostrain) modulus E_{com} shows <u>good agreement</u> with the experimental value.

17. For an intrinsic semiconductor,

$$\ln \sigma = \ln \sigma_0 - \frac{E_g}{2kT}$$

At $T = 278°K$:

$$\ln 390 = \ln \sigma_0 - \frac{E_g}{2k(278)}$$

At $T = 298°K$:

$$\ln 1010 = \ln \sigma_0 - \frac{E_g}{2k(298)}$$

Subtracting and solving,

$$\frac{E_g}{2k} = \ln\left(\frac{1010}{390}\right)\frac{298 \times 278}{(298-278)} = 3942\,;\ \underline{E_g = 0.679\,eV}$$

$$\ln \sigma_0 = \ln 390 + \frac{3942}{278}\ ;\ \sigma_0 = 5.60 \times 10^8\,\Omega^{-1}.m^{-1}$$

At $T = 288°K$:

$$\sigma_{288} = \sigma_0 \cdot e^{-13.686} = \underline{637.5\,\Omega^{-1}.m^{-1}}$$