Chapter 6:
Mechanical Properties II

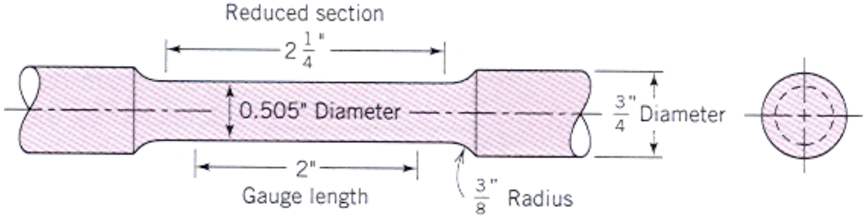
Outline

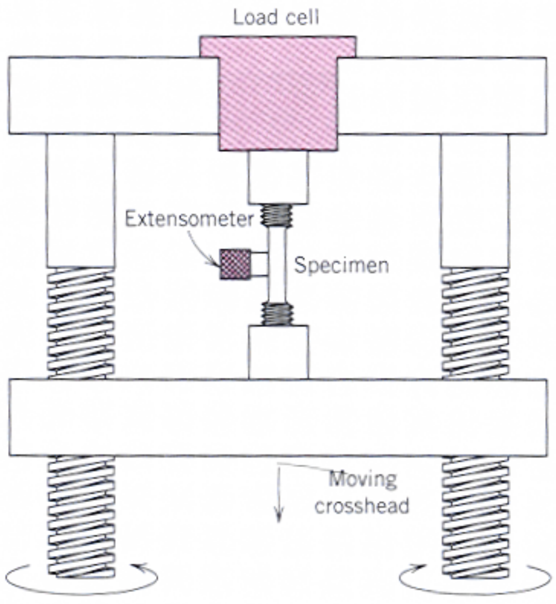
- Elastic recovery during plastic deformation
- Compressive, shear, and torsional deformation
- Hardness
- Variability of material properties
- Design/safety factors

Concepts of stress and strain

- **Tension tests**
  - **engineering stress** $$\sigma = \frac{F}{A_0}$$
  - **engineering strain**

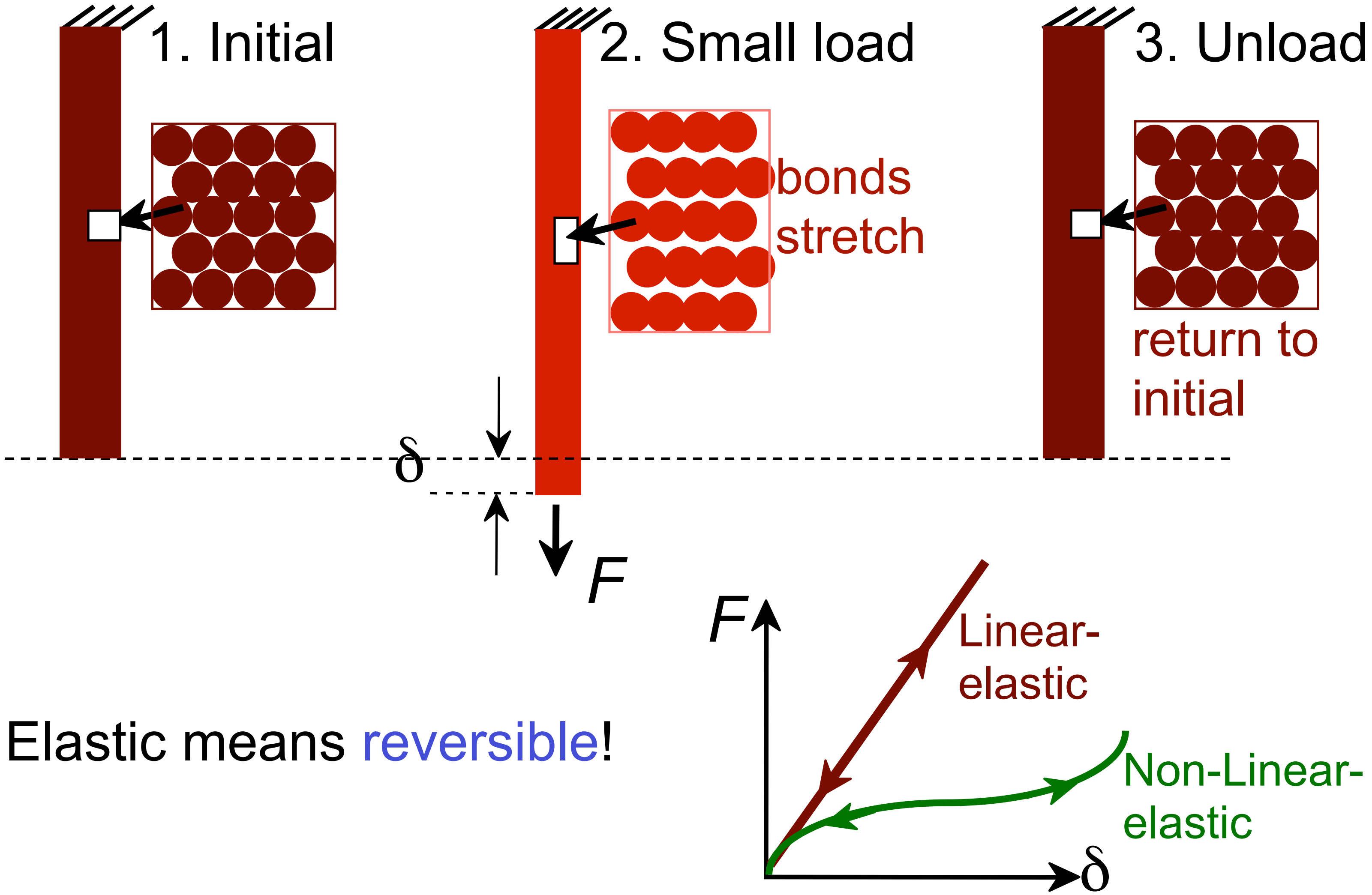
$$\varepsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}$$





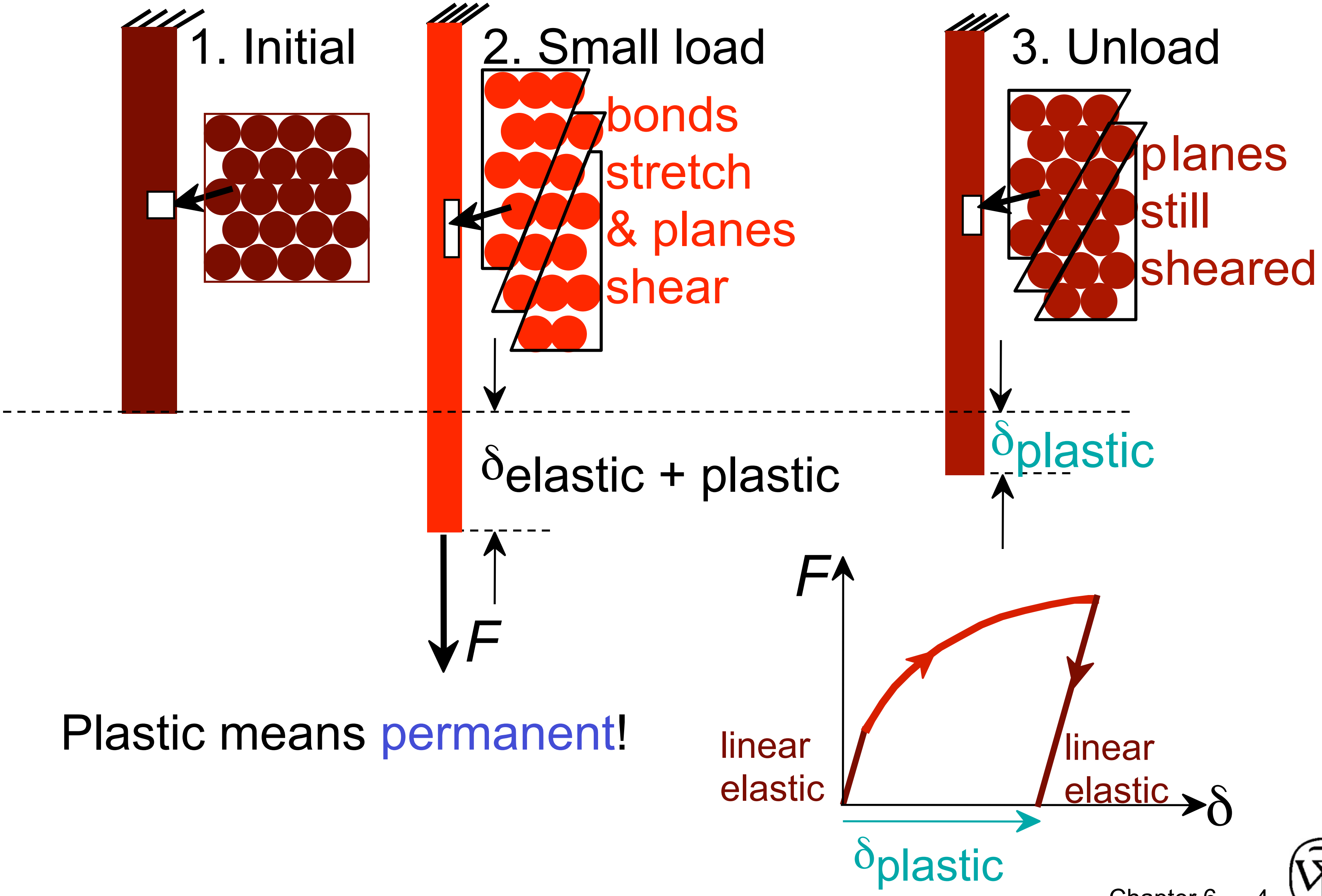
- **Compression tests**

Elastic Deformation



Elastic means reversible!

Plastic Deformation (Metals)



1. Initial

2. Small load

bonds
stretch
& planes
shear

$\delta_{\text{elastic + plastic}}$

F

3. Unload

planes
still
sheared

δ_{plastic}

F

linear
elastic

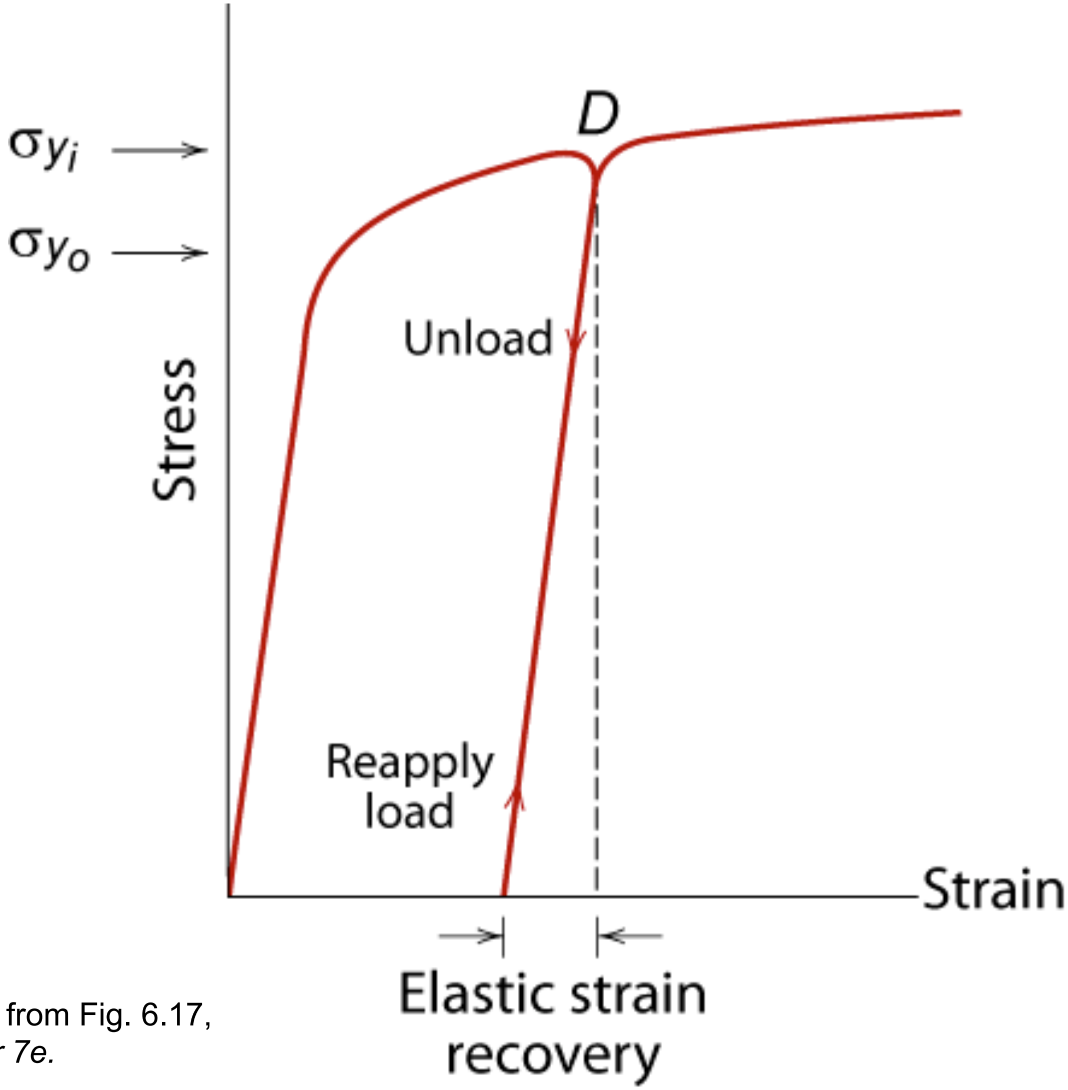
linear
elastic

δ

δ_{plastic}

Plastic means permanent!

Elastic Strain Recovery

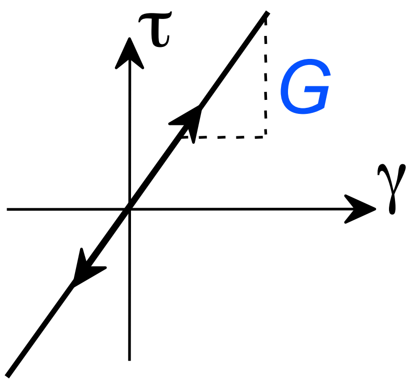


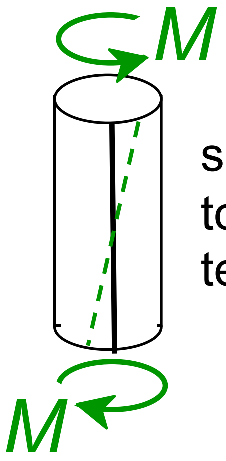
Adapted from Fig. 6.17, *Callister 7e.*

Other Elastic Properties

- **Elastic Shear modulus, *G*:**

$$\tau = G\,\gamma$$

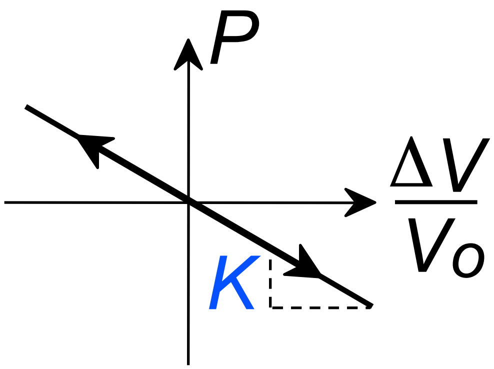


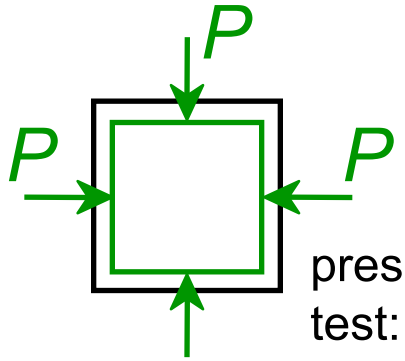


simple torsion test

- **Elastic Bulk modulus, K:**

$$P = -K\frac{\Delta V}{V_o}$$





pressure test: Init. vol $= V_o$. Vol chg. $= \Delta V$

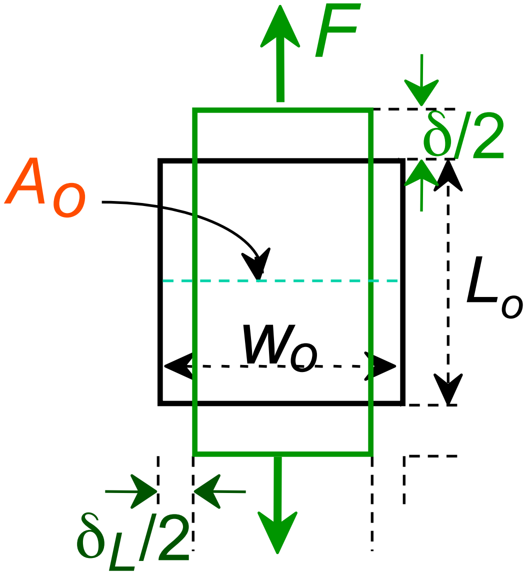
- **Special relations for isotropic materials:**

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Useful Linear Elastic Relationships

- **Simple tension:**

$$\delta = \frac{FL_o}{EA_o} \qquad \delta_L = -\nu \frac{Fw_o}{EA_o}$$

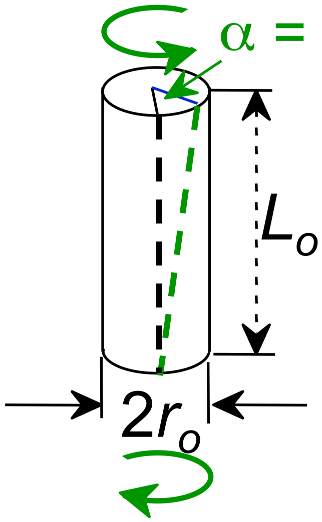


- **Simple torsion:**

$$\alpha = \frac{2ML_o}{\pi r_o^4 G}$$

M = moment

α = angle of twist



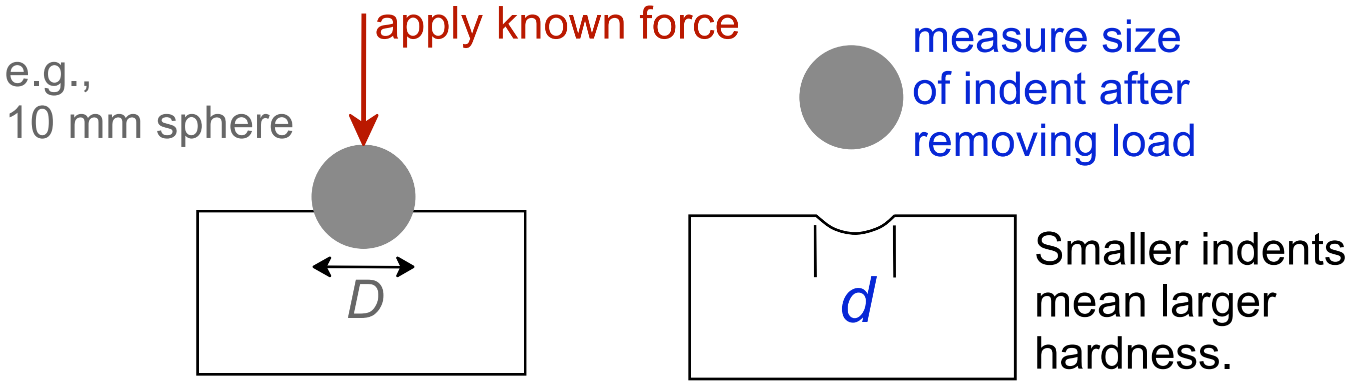
- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.

Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:

--resistance to plastic deformation or cracking in compression.

--better wear properties.



apply known force

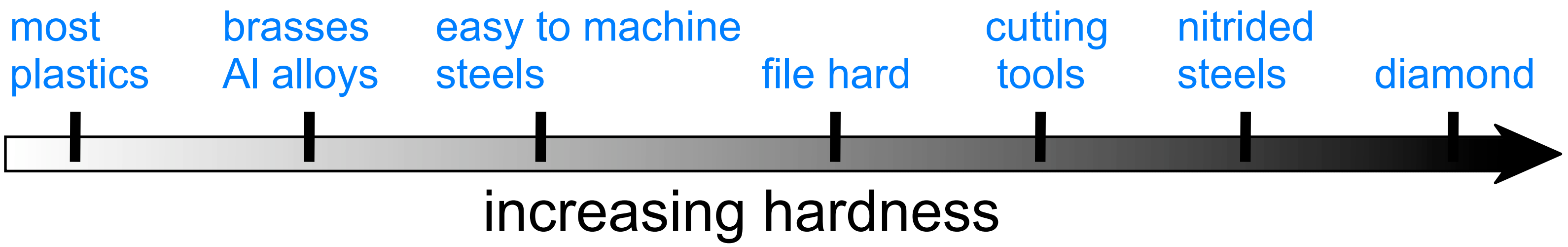
e.g.,
10 mm sphere

D

measure size
of indent after
removing load

d

Smaller indents
mean larger
hardness.



most plastics

brasses Al alloys

easy to machine steels

file hard

cutting tools

nitrided steels

diamond

increasing hardness

Hardness: Measurement

- Rockwell
  - No major sample damage
  - Each scale runs to 130 but only useful in range 20-100.
  - Minor load 10 kg
  - Major load 60 (A), 100 (B) & 150 (C) kg
    - A = diamond, B = 1/16 in. ball, C = diamond

- HB = Brinell Hardness
  - *TS* (psia) = 500 x HB
  - *TS* (MPa) = 3.45 x HB

Hardness: Measurement

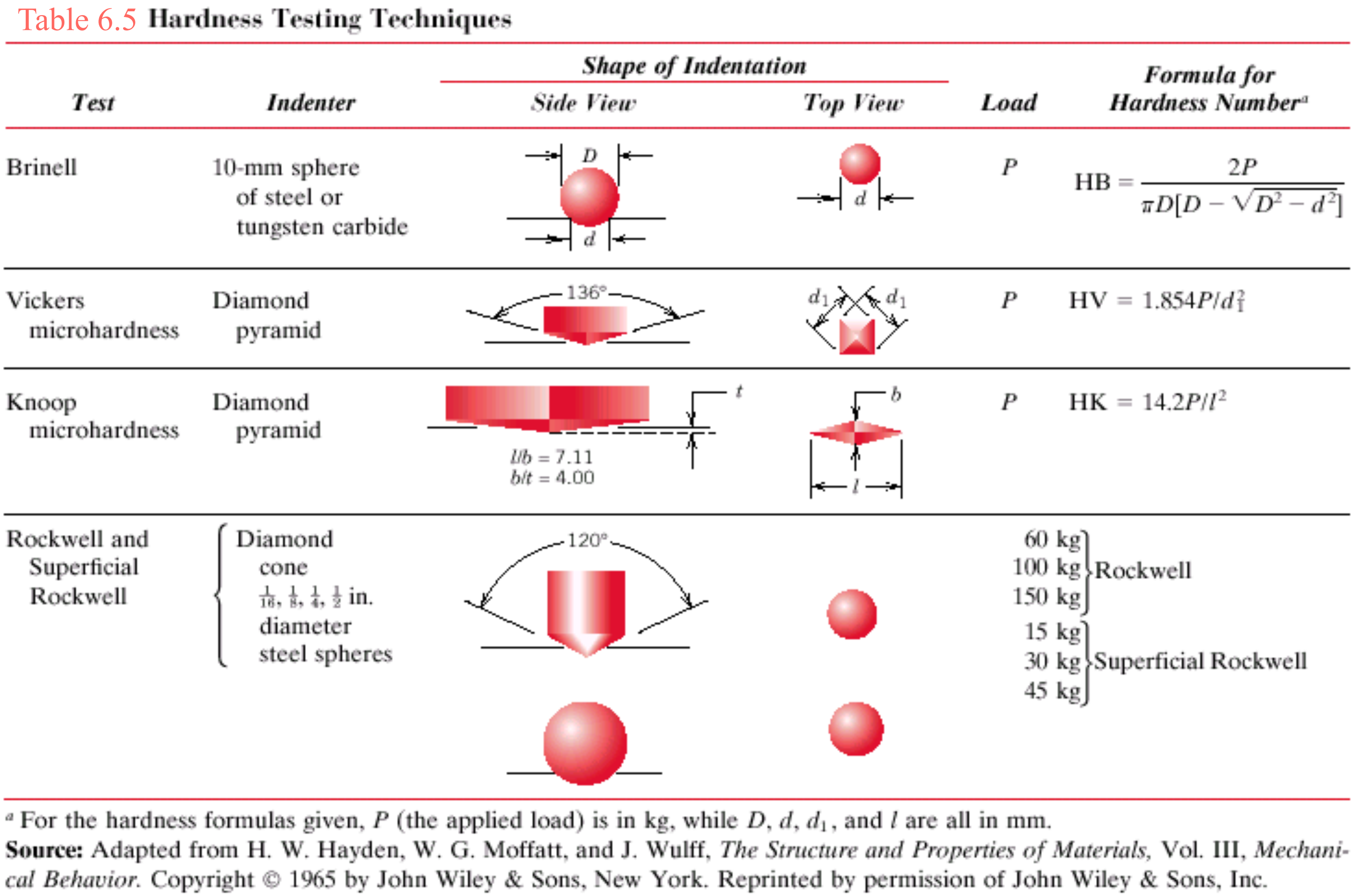


Table 6.5 **Hardness Testing Techniques**

Test	*Indenter*	*Shape of Indentation* Side View	*Shape of Indentation* Top View	*Load*	*Formula for Hardness Number*[a]
Brinell	10-mm sphere of steel or tungsten carbide	D d	d	P	$\mathrm{HB} = \dfrac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid	136°	d_1 d_1	P	$\mathrm{HV} = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid	t $l/b = 7.11$ $b/t = 4.00$	b l	P	$\mathrm{HK} = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres	120°		60 kg 100 kg 150 kg	Rockwell
				15 kg 30 kg 45 kg	Superficial Rockwell

[a] For the hardness formulas given, P (the applied load) is in kg, while D, d, d_1, and l are all in mm.

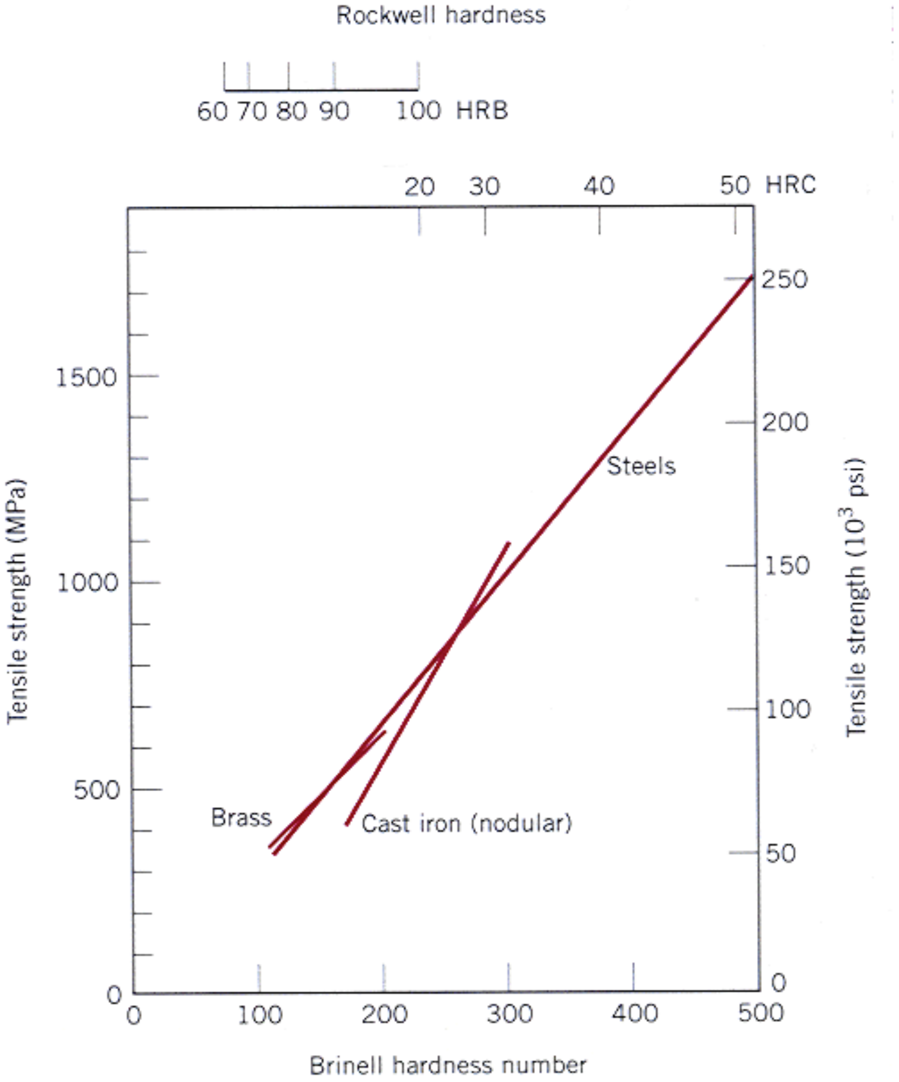
Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials,* Vol. III, *Mechanical Behavior.* Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

Correlation between hardness and tensile strength

- **Relations between hardness and tensile strength for steel, brass, and cast iron.**
- **For most steels:**

TS (MPa)=3.45xHB

TS (psi)=500xHB



Example

- **Estimate the Brinell and Rockwell hardnesses for brass and carbon steel**

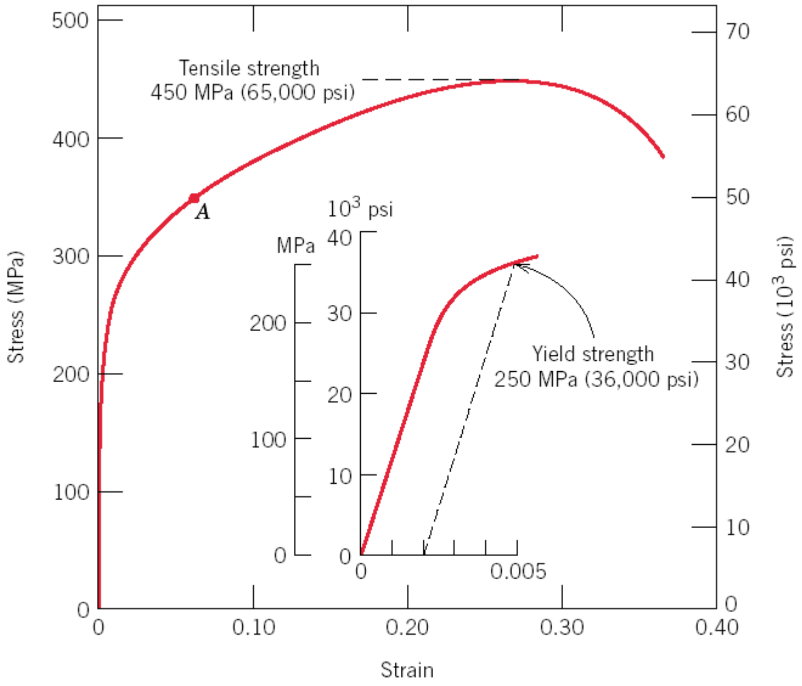


FIGURE 6.12 The stress–strain behavior for the brass specimen

Variability in Material Properties

- Elastic modulus is material property
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics
  - Mean

$$\overline{x} = \frac{\overset{n}{\Sigma} x_n}{n}$$

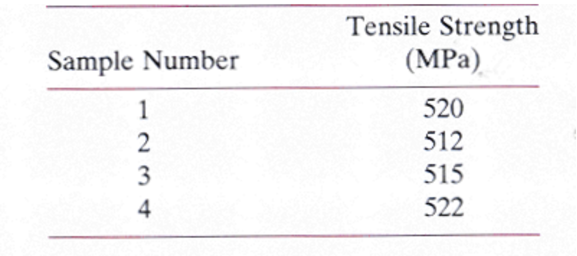
  - Standard Deviation

$$s = \left[\frac{\overset{n}{\Sigma} (x_i - \overline{x})^2}{n - 1} \right]^{\frac{1}{2}}$$

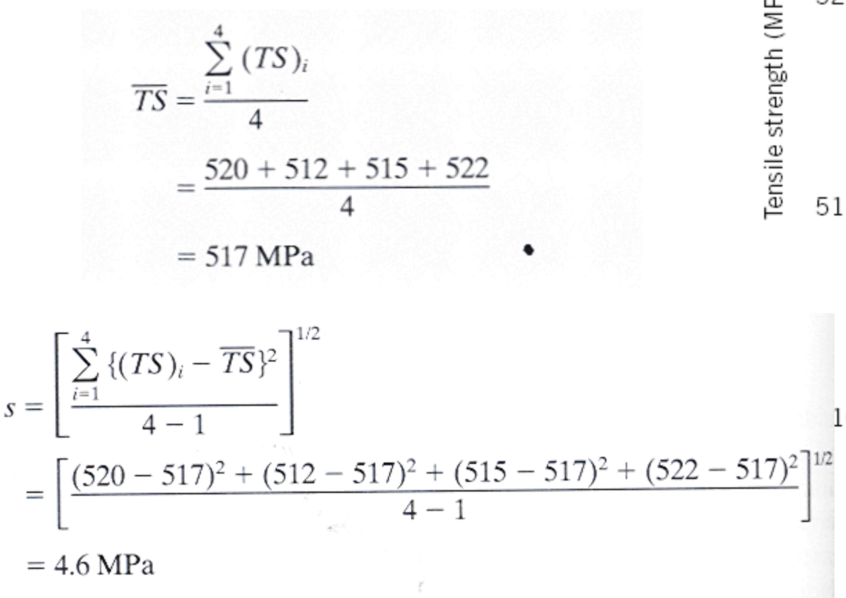
where *n* is the number of data points

Example

- **Determine the average and standard deviation of tensile strength**



Sample Number	Tensile Strength (MPa)
1	520
2	512
3	515
4	522



$$\overline{TS} = \frac{\sum_{i=1}^{4} (TS)_i}{4}$$

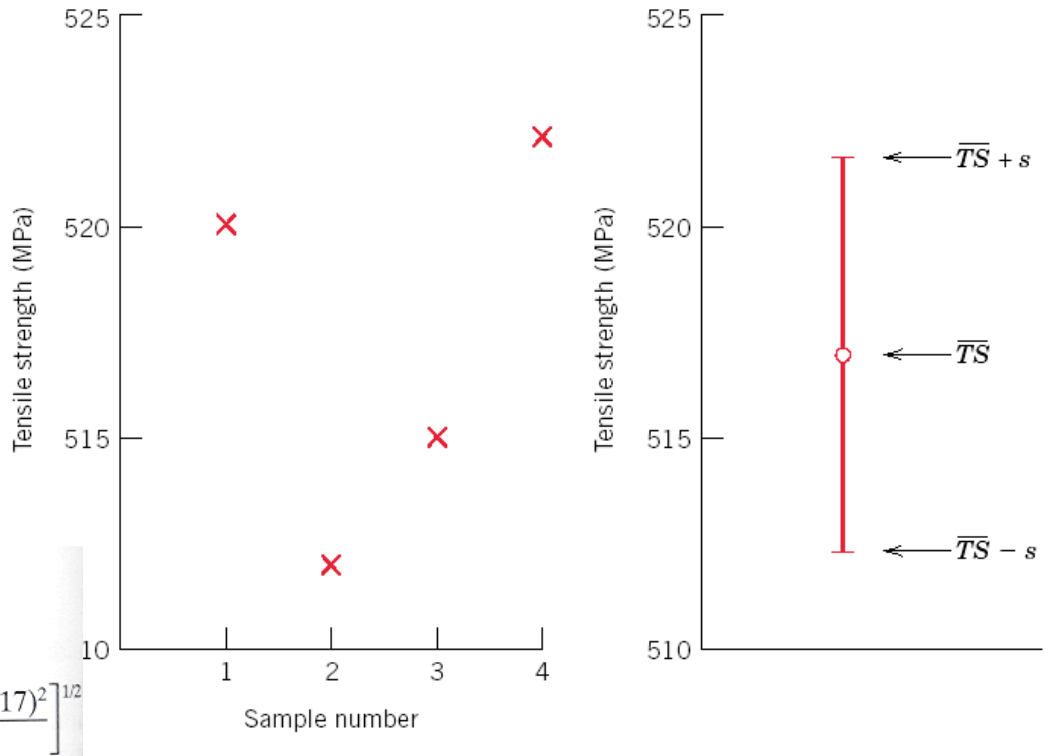
$$= \frac{520 + 512 + 515 + 522}{4}$$

$$= 517 \text{ MPa}$$

$$s = \left[\frac{\sum_{i=1}^{4} \{(TS)_i - \overline{TS}\}^2}{4 - 1} \right]^{1/2}$$

$$= \left[\frac{(520 - 517)^2 + (512 - 517)^2 + (515 - 517)^2 + (522 - 517)^2}{4 - 1} \right]^{1/2}$$

$$= 4.6 \text{ MPa}$$



(a)

(b)

Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, *N*

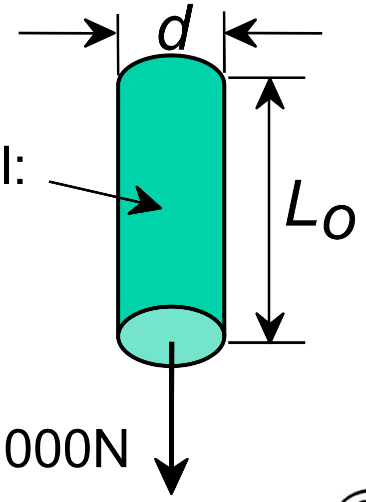
$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often *N* is between
1.2 and 4

- Example: Calculate a diameter, *d*, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$$\underbrace{\sigma_{working}}_{\frac{220{,}000N}{\pi\left(d^2/4\right)}} = \frac{\sigma_y}{\underbrace{N}_{5}}$$

1045 plain
carbon steel:
σ_y = 310 MPa
TS = 565 MPa



F = 220,000N

$$\boxed{d = 0.067 \text{ m} = 6.7 \text{ cm}}$$

Summary

- Stress and strain: These are size-independent measures of load and displacement, respectively.
- Elastic behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).
- Plastic behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches σ_y.
- Toughness: The energy needed to break a unit volume of material.
- Ductility: The plastic strain at failure.
- Hardness: Resistance to permanently indenting the surface.
- Safety: Design uncertainties mean we do not push the limit