ISSUES TO ADDRESS...

- **Stress** and **strain**: What are they and why are they used instead of load and deformation?

- **Elastic** behavior: When loads are small, how much deformation occurs? What materials deform least?

- **Plastic** behavior: At what point do dislocations cause permanent deformation? What materials are most resistant to permanent deformation?

- **Toughness** and **ductility**: What are they and how do we measure them?
Chapter 6

Elastic deformation

1. Initial
2. Small load
3. Unload

Elastic means reversible!

Linear-elastic
Non-Linear-elastic
Plastic deformation (metals)

1. Initial
2. Small load
   - Bonds stretch
   - Planes shear
3. Unload
   - Planes still sheared

Plastic means permanent!

\[ \delta_{\text{elastic + plastic}} \]

\[ \delta_{\text{plastic}} \]

\[ \delta \]

\[ F \]

Linear elastic

Chapter 6-3
Engineering stress

- **Tensile stress, $\sigma$:**

$$\sigma = \frac{F_t}{A_0}$$

original area before loading

- **Shear stress, $\tau$:**

$$\tau = \frac{F_s}{A_0}$$

Stress has units: N/m$^2$ or lb/in$^2$
Common states of stress

• **Simple** tension: cable

\[ \sigma = \frac{F}{A_0} \]

\( A_0 \) = cross sectional Area (when unloaded)

\[ F \rightarrow \sigma \rightarrow F \]

• **Simple** shear: drive shaft

\[ \tau = \frac{F_s}{A_o} \]

\( \tau \) = \( M/A_cR \) here.

\( F_s \rightarrow \tau \rightarrow A_o \)

\( \tau \)

Ski lift (photo courtesy P.M. Anderson)
Other common stress states (1)

- **Simple** compression:

\[ \sigma = \frac{F}{A_o} \]

Note: compressive structure member \((\sigma < 0\) here).
Other common stress states (2)

- **Bi-axial tension:**
  \[ \sigma_\theta > 0 \quad \sigma_Z > 0 \]

- **Hydrostatic compression:**
  \[ \sigma_h < 0 \]

*Pressurized tank (photo courtesy P.M. Anderson)*

*Fish under water (photo courtesy P.M. Anderson)*
Engineering strain

- **Tensile strain:**
  \[ \varepsilon = \frac{\delta}{L_0} \]

- **Lateral strain:**
  \[ \varepsilon_L = -\frac{\delta_L}{W_0} \]

- **Shear strain:**
  \[ \gamma = \tan \theta \]

Strain is always dimensionless.
Stress-strain testing

• Typical tensile specimen

- Other types of tests:
  -- compression: brittle materials (e.g., concrete)
  -- torsion: cylindrical tubes, shafts.

- Typical tensile test machine

  load cell

  extensometer

  specimen

  moving cross head

  gauge length = (portion of sample with reduced cross section)

Adapted from Fig. 6.2, *Callister 6e.*

Adapted from Fig. 6.3, *Callister 6e.*


Chapter 6- 9
Linear elastic properties

- Modulus of Elasticity, $E$: (also known as Young's modulus)
- Hooke's Law: $\sigma = E \varepsilon$
- Poisson's ratio, $\nu$:
  
  $$\nu = -\frac{\varepsilon_L}{\varepsilon}$$

  - metals: $\nu \sim 0.33$
  - ceramics: $\sim 0.25$
  - polymers: $\sim 0.40$

Units:
- $E$: [GPa] or [psi]
- $\nu$: dimensionless
Other elastic properties

- Elastic Shear modulus, $G$:
  \[ \tau = G \gamma \]

- Elastic Bulk modulus, $K$:
  \[ P = -K \frac{\Delta V}{V_0} \]

- Special relations for isotropic materials:
  \[ G = \frac{E}{2(1 + \nu)} \quad K = \frac{E}{3(1 - 2\nu)} \]

Simple torsion test
Pressure test: Init. vol $= V_0$. Vol chg. $= \Delta V$
Based on data in Table B2, *Callister 6e.*
Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.
Useful linear elastic relations

- Simple tension:
  \[ \delta = \frac{FL_0}{EA_0} \quad \delta_L = -\nu \frac{Fw_0}{EA_0} \]

- Simple torsion:
  \[ \alpha = \frac{2ML_0}{\pi r_0^4 G} \]

- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.
Plastic (permanent) deformation
(at lower temperatures, $T < T_{\text{melt}}/3$)

- Simple tension test:
  - Tensile stress, $\sigma$
  - Elastic initially
  - Elastic + Plastic at larger stress
  - Permanent (plastic) deformation after load is removed
  - Plastic strain, $\varepsilon_p$
  - Engineering strain, $\varepsilon$
Yield strength, $\sigma_y$

- Stress at which *noticeable* plastic deformation has occurred.

When $\varepsilon_p = 0.002$

![Diagram showing tensile stress and engineering strain with yield strength at $\varepsilon_p = 0.002$](image)
Yield strength: comparison

Based on data in Table B4, *Callister 6e.*

- a = annealed
- hr = hot rolled
- ag = aged
- cd = cold drawn
- cw = cold worked
- qt = quenched & tempered
Tensile strength, TS

- Maximum possible engineering stress in tension.

- Metals: occurs when noticeable necking starts.
- Ceramics: occurs when crack propagation starts.
- Polymers: occurs when polymer backbones are aligned and about to break.

Adapted from Fig. 6.11, Callister 6e.
Based on data in Table B4, Callister 6e.

- a = annealed
- hr = hot rolled
- ag = aged
- cd = cold drawn
- cw = cold worked
- qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.
Ductility, %EL

- Plastic tensile strain at failure:
  \[ \%EL = \frac{L_f - L_o}{L_o} \times 100 \]

  \[ \begin{align*}
  \text{Engineering tensile strain, } \varepsilon \\
  \text{Engineering tensile stress, } \sigma
  \end{align*} \]

- Another ductility measure:
  \[ \%AR = \frac{A_o - A_f}{A_o} \times 100 \]

- Note: %AR and %EL are often comparable.
  --Reason: crystal slip does not change material volume.
  --%AR > %EL possible if internal voids form in neck.

Adapted from Fig. 6.13, Callister 6e.
Toughness

• Energy to break a unit volume of material
• Approximate by the area under the stress-strain curve.

Engineering tensile stress, $\sigma$

Engineering tensile strain, $\varepsilon$

smaller toughness (ceramics)
larger toughness (metals, PMCs)
smaller toughness-unreinforced polymers
Resilience

Resilience is the capacity to absorb energy in the elastic region. Modulus of resilience is the total elastic strain energy per unit volume (Area under elastic portion of $\sigma$ vs. $\varepsilon$)

$$U_r = \int_0^{\sigma_y} E \varepsilon d\varepsilon = \int_0^{\sigma_y} E \frac{\sigma}{E} d(\sigma/E) = \frac{1}{E} \int_0^{\sigma_y} \sigma d\sigma$$

$$U_r = \frac{\sigma_y^2}{2E}, \text{ J/m}^3$$

Resilient materials are used in spring applications. They have high yield stress and low modulus of elasticity.
Elastic properties of spring mtl s

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_y$ (ksi)</th>
<th>Prop limit (ksi)</th>
<th>$E \times 10^3$ ksi</th>
<th>Recov. strain %</th>
<th>Normalized Mod. of resil.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-55Ni</td>
<td>30</td>
<td>27</td>
<td>10</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>17-7PH</td>
<td>200</td>
<td>180</td>
<td>29</td>
<td>0.62</td>
<td>2.2</td>
</tr>
<tr>
<td>MP55N-Co</td>
<td>225</td>
<td>203</td>
<td>33.6</td>
<td>0.6</td>
<td>2.4</td>
</tr>
<tr>
<td>5160</td>
<td>250</td>
<td>225</td>
<td>30</td>
<td>0.75</td>
<td>3.3</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>120</td>
<td>108</td>
<td>16.5</td>
<td>0.65</td>
<td>1.4</td>
</tr>
<tr>
<td>Beta C Ann.</td>
<td>120</td>
<td>108</td>
<td>13.0</td>
<td>0.83</td>
<td>1.8</td>
</tr>
<tr>
<td>Beta C + CW</td>
<td>170</td>
<td>160</td>
<td>13.0</td>
<td>1.18</td>
<td>3.6</td>
</tr>
<tr>
<td>Beta C + CW+ Aged</td>
<td>210</td>
<td>185</td>
<td>15.5</td>
<td>1.22</td>
<td>4.6</td>
</tr>
</tbody>
</table>
Hardness

• Resistance to permanently indenting the surface.
• Large hardness means:
  --resistance to local plastic deformation or cracking in compression.
  --better wear properties.

[Diagram of hardness testing with an 10mm sphere (Brinell Hardness)]

- apply known force (1 to 1000g)
- measure size of indent after removing load

Smaller indents mean larger hardness.

most plastics  brasses  easy to machine  file hard  cutting tools  nitrided steels  diamond

increasing hardness

Adapted from Fig. 6.18, *Callister 6e*. (Fig. 6.18 is adapted from G.F. Kinney, *Engineering Properties and Applications of Plastics*, p. 202, John Wiley and Sons, 1957.)
Strain hardening

- An increase in $\sigma_y$ due to plastic deformation.

- Curve fit to the stress-strain response:

  \[
  \sigma_T = C(\varepsilon_T)^n
  \]

  "true" stress (F/A)

  Strain hardening exponent:
  $n=0.15$ (some steels) to $n=0.5$ (some copper)

  "true" strain: $\ln(L/L_o)$
Design or safety factors

• Design uncertainties mean we do not push the limit.
• Factor of safety, \( N \)

\[
\sigma_{\text{working}} = \frac{\sigma_y}{N}
\]

Often \( N \) is between 1.2 and 4

• Ex: Calculate a diameter, \( d \), to ensure that yield does not occur in the 1045 carbon steel rod below when subjected to a load of 200,000N. Use a factor of safety of 5.

1045 plain carbon steel:
\( \sigma_y = 310 \text{MPa} \)
\( \text{TS} = 565 \text{MPa} \)

\[
\frac{220,000 \text{N}}{\pi \left( \frac{d^2}{4} \right)} \cdot 5 = \frac{\sigma_y}{N}
\]

F = 220,000N
Summary

• **Stress** and **strain**: These are size-independent measures of load and displacement, respectively.

• **Elastic** behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).

• **Plastic** behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches $\sigma_y$.

• **Toughness**: The energy needed to break a unit volume of material.

• **Resilience**: Energy absorbed during elastic deformation.

• **Ductility**: The plastic strain at failure.
ANNOUNCEMENTS

Reading: Chapter 6.1-12

Core Problems: Chapter 6, Problems 4, 8, 25, 29, 46

Self-help Problems: Review Example problems 6.2, 6.3, 6.4, 6.4, 6.5