Recap

Conservation of linear momentum

$$\int_{V(t)} \rho \frac{D\vec{v}}{Dt}\, dV = \int_{V(t)} \rho \vec{g}\, dV + \int_{S(t)} \vec{n} \cdot \underline{\underline{\sigma}}\, dS \quad \text{(macroscopic balance)}$$

$$\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} + \vec{\nabla} \cdot \underline{\underline{\sigma}} \quad \text{(microscopic balance)}$$

Cauchy Stress Tensor: $\underline{\underline{\sigma}} = (\sigma_{ij}) = -P \underline{\underline{\delta}} + \underline{\underline{\tau}}$

normal stress

viscous shear tensor

σ_{xy} stress on x-plane along y-direction

plane upon stress acts

direction of stress component

Cauchy Momentum Eq

$\underline{\underline{\tau}} = \underline{(\tau_{ij})}$ shear stress tensor

$$\boxed{\rho \frac{D\vec{v}}{Dt} = \underbrace{\rho \vec{g}}_{\text{grav.}} - \underbrace{\vec{\nabla} P}_{\text{pressure}} + \underbrace{\vec{\nabla} \cdot \underline{\underline{\tau}}}_{\text{viscous stress}}}$$

Eq. 6.3-10

see Tables 6-1/2/3

for $\vec{v} = 0$ (static fluid): $\vec{\nabla} P = \rho \vec{g}$ static pressure

for $\vec{v} \neq 0$: define Dynamic Pressure: $\vec{\nabla} \mathcal{P} \equiv \vec{\nabla} P - \rho \vec{g}$

$\Rightarrow$ Cauchy eq: $\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} \mathcal{P} + \vec{\nabla} \cdot \underline{\underline{\tau}}$

The pressure P can be expressed by

- the thermodynamic pressure, defined locally by the Equation of State (EOS): $P = P(\rho, T)$

Ex: ideal gas: $P = \frac{NkT}{V} = \frac{nRT}{V} = \frac{RT}{V_m}$

V_m .. molar volume

VdW fluids: $\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$

a, b. subst. specific constants