

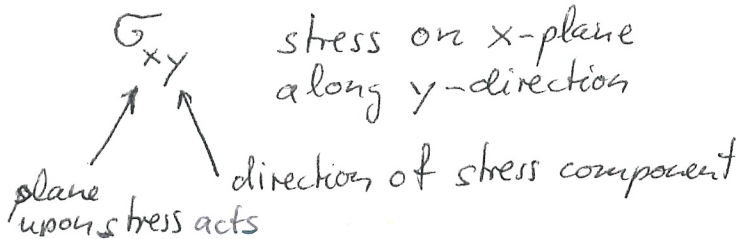
Recap

Conservation of linear momentum

$$\int_{V(t)} \rho \frac{D\vec{v}}{Dt} dV = \int_{V(t)} \rho \vec{g} dV + \int_{S(t)} \vec{n} \cdot \underline{\underline{\underline{\sigma}}} dS \quad (\text{macroscopic balance})$$

$$\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} + \vec{\nabla} \cdot \underline{\underline{\underline{\sigma}}} \quad (\text{microscopic balance})$$

Cauchy Stress Tensor: $\underline{\underline{\underline{\sigma}}} = (\sigma_{ij}) = -P\underline{\underline{\underline{\delta}}} + \underline{\underline{\underline{\tau}}}$



↑ normal stress
↑ viscous shear tensor

Cauchy Momentum Eq $\underline{\underline{\underline{\tau}}} = (\tau_{ij})$ shear stress tensor

$$\rho \frac{D\vec{v}}{Dt} = \underbrace{\rho \vec{g}}_{\text{grav.}} - \underbrace{\vec{\nabla} P}_{\text{pressure}} + \underbrace{\vec{\nabla} \cdot \underline{\underline{\underline{\tau}}}}_{\text{viscous stress}}$$

Eq. 6.3-10
see Tables
6-1/2/3

for $\vec{v}' = 0$ (static fluid): $\vec{\nabla} P = \rho \vec{g}$ static pressure

for $\vec{v}' \neq 0$: define Dynamic Pressure: $\vec{\nabla} \mathcal{P} \equiv \vec{\nabla} P - \rho \vec{g}$

$$\Rightarrow \text{Cauchy eq: } \rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} \mathcal{P} + \vec{\nabla} \cdot \underline{\underline{\underline{\tau}}}$$

The pressure P can be expressed by

- the thermodynamic pressure, defined locally by the Equation of State (EOS): $P = P(\rho, T)$

Ex: ideal gas: $P = \frac{NkT}{V} = \frac{nRT}{V} = \frac{RT}{V_m}$
 V_m : molar volume

Van der Waals fluids: $\left(P + \frac{a}{V_m^2} \right) (V_m - b) = RT$

a, b : subst. specific constants