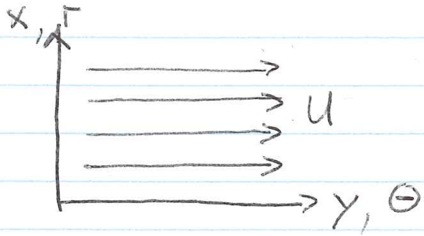
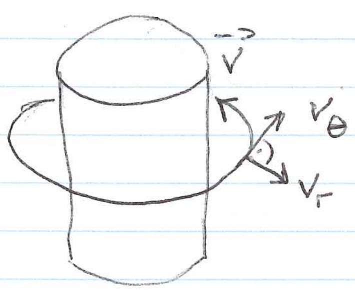
Planar Flow : $v_z = 0$



$w_z = -\nabla^2 \psi$ rectangular (x, y)

$w_z = -\nabla^2 \psi$ cylindrical (r, Θ)



Navier - Stokes :

$$\boxed{\begin{aligned} \frac{D}{Dt} w_z &= \nu \nabla^2 w_z \; ; \; \text{rect.} \, (x, y) \\ \frac{D}{Dt} w_z &= \nu \nabla^2 w_z \; ; \; \text{cyl.} \, (r, \theta) \end{aligned}}$$

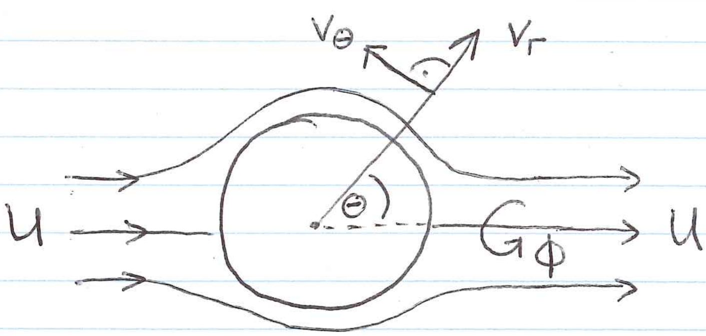
Axisymmetric Flow :

for cylinder /or sphere

$$E^2 \equiv \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

$w_\Theta = -\frac{1}{r} E^2 \psi$; cyl. (z, r)

$w_\phi = -\frac{1}{r \sin\Theta} E^2 \psi$; sph (r, Θ)



Navier-Stokes

$$\boxed{\frac{D}{Dt} w_\Theta = \frac{w_\Theta v_r}{r} + \nu\left(\nabla^2 w_\Theta - \frac{w_\Theta}{r^2}\right)} \; ; \; \text{cyl.} \, (z, r)$$

Eq. 6-8-8c

$$\boxed{\frac{D w_\phi}{Dt} = \frac{w_\phi}{r \sin\Theta}\left(v_r \sin\Theta + v_\Theta \cos\Theta\right) + \nu\left(\nabla^2 w_\phi - \frac{w_\phi}{r^2 \sin^2\Theta}\right)} \; ; \; \text{sph.} \, (r, \Theta)$$

Eq. 6.8-8d