Table 12.1 *Table of Expressions for Elementary Flows*

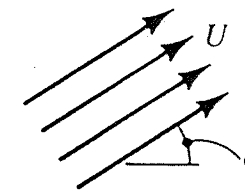
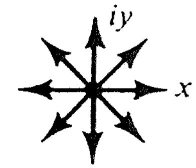
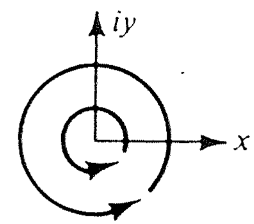
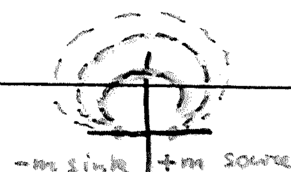
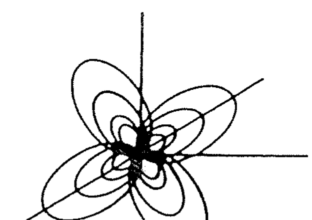
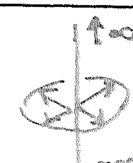
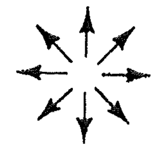
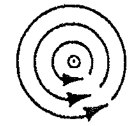
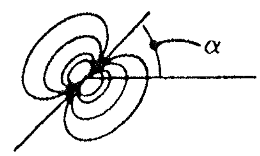
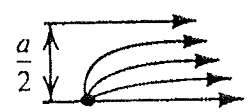
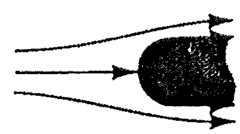
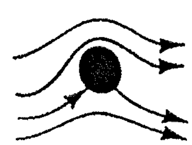
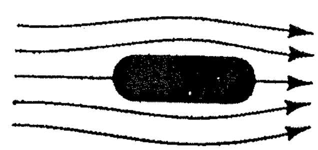
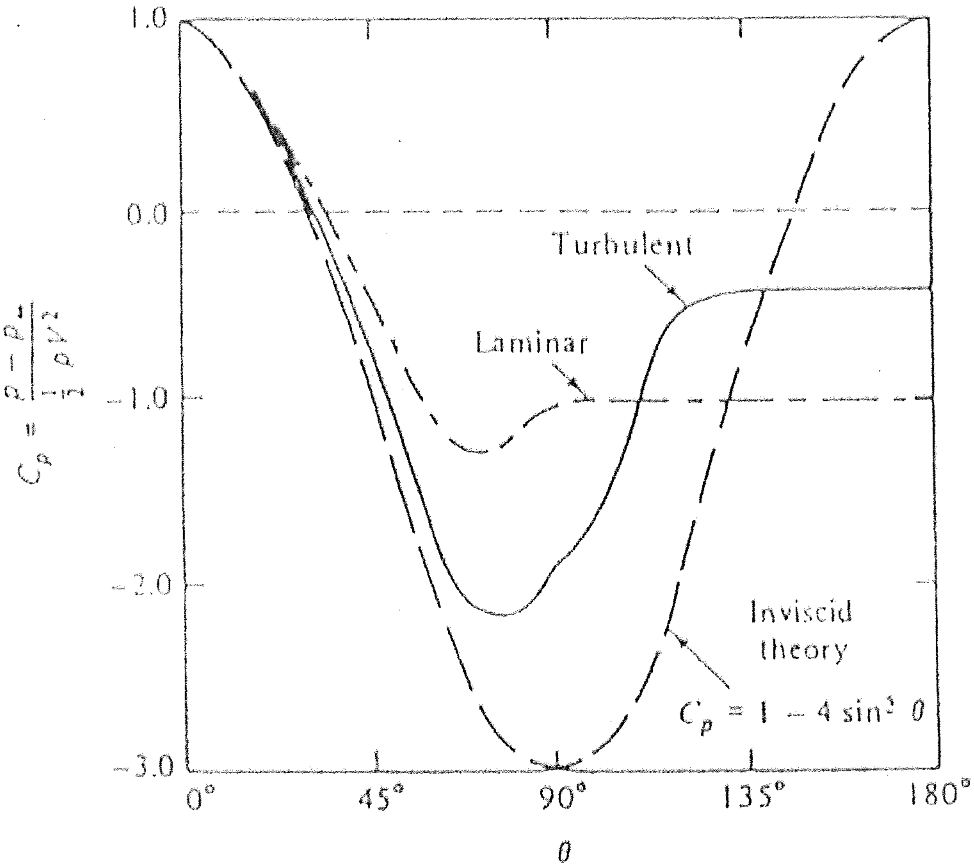
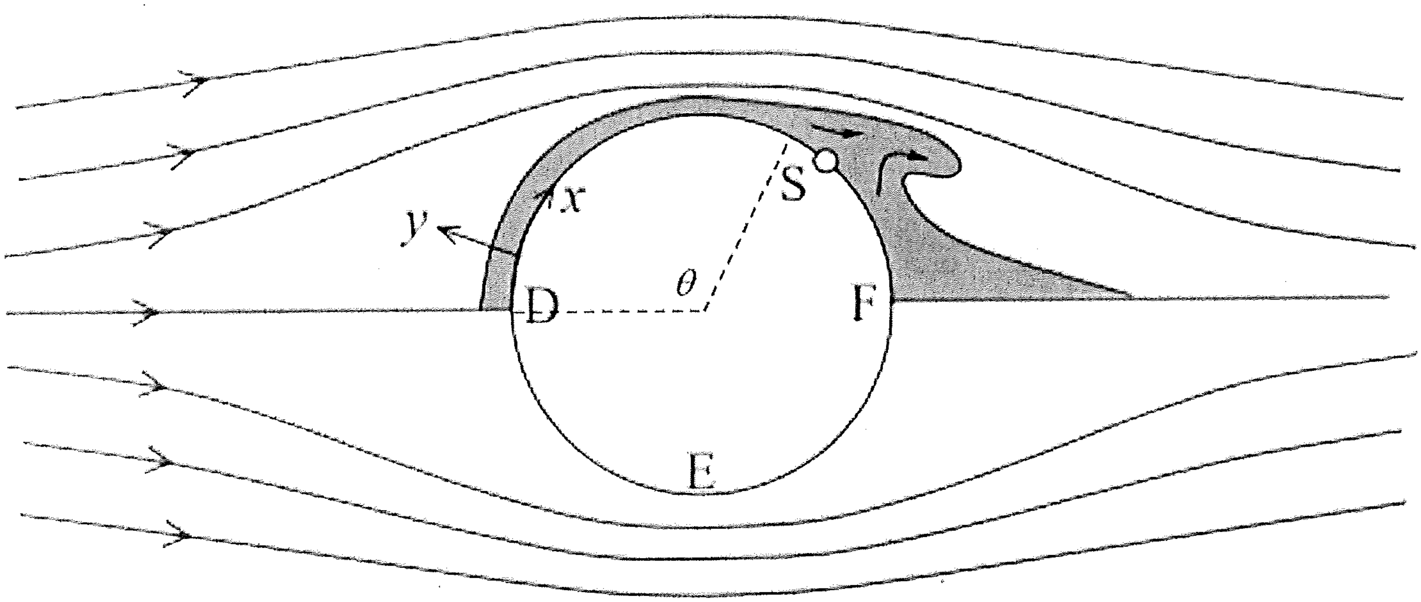
Elementary Flow	$\Omega = \phi + i\psi$ Complex Potential ($z_o = 0$)	velocity potential ϕ: $\phi(x, y)$	$\phi(r, \theta)$	stream function ψ: $\psi(x, y)$	$\psi(r, \theta)$	Streamlines
1. Uniform flow	$Uze^{-i\alpha}$	$U(x \cos \alpha + y \sin \alpha)$	$Ur \cos (\theta - \alpha)$	$U(y \cos \alpha - x \sin \alpha)$	$Ur \sin (\theta - \alpha)$	
2. Source	$\frac{q}{2\pi} \ln z$	$\frac{q}{4\pi} \ln (x^2 + y^2)$	$\frac{q}{2\pi} \ln r$ $m \equiv \frac{q}{2\pi}$ measure of source strength	$\frac{q}{2\pi} \tan^{-1} \left(\frac{y}{x}\right)$	$\frac{q}{2\pi} \theta$	
irrotational 3. Vortex	$\frac{-i\Gamma_\infty}{2\pi} \ln z$	$\frac{\Gamma_\infty}{2\pi} \tan^{-1} \left(\frac{y}{x}\right)$	$\frac{\Gamma_\infty}{2\pi} \theta$ $\frac{\Gamma_\infty}{2\pi}$ vortex strength	$-\frac{\Gamma_\infty}{4\pi} \ln (x^2 + y^2)$ Γ_∞ potential circulation	$-\frac{\Gamma_\infty}{2\pi} \ln r$	
4. Doublet	$\frac{qae^{i\alpha}}{\pi z}$	$\frac{qax \cos \alpha}{\pi(x^2 + y^2)}$	$\frac{qa \cos (\alpha - \theta)}{\pi r}$ 2a ... distance between sink and source $s \equiv \frac{qa}{\pi}$ doublet strength	$-\frac{qay \sin \alpha}{\pi(x^2 + y^2)}$	 $\frac{qa \sin (\alpha - \theta)}{\pi r}$	
5. Line Source			$m \ln r$	$m \tan^{-1}\left(\frac{y}{x}\right)$ emits flow uniformly in all radial directions	$m \theta$	

Table 12.2 Various Complex Potentials and Corresponding Flows

Complex Potential Ω $\Omega = \phi + i\psi$	**Configuration**	**Flow Geometry**
$Uze^{-i\alpha}$	Uniform flow in the direction α	
$\frac{q}{2\pi} \ln (z - z_o)$	Source located at z_o	
$\frac{-i\Gamma_\infty}{2\pi} \ln (z - z_o)$	Vortex located at z_o	
$\frac{qae^{i\alpha}}{\pi(z - z_o)}$	Doublet located at z_o	
Az^n	Flow in a corner of angle π/n	
$\frac{q}{2\pi} \ln \sinh \pi z/a$	Source at the center of a channel	
$Uz + \frac{q}{2\pi} \ln z$	Flow about a half body	
$U\left(z + \frac{b^2}{z}\right) + i\frac{\Gamma_\infty}{2\pi} \ln z$	Flow about a cylinder with circulation	
$Uz + \frac{q}{2\pi} \ln \frac{z + a}{z - a}$	Flow about a Rankine oval	
$\frac{i\Gamma_\infty}{2\pi} \ln \frac{z + a}{z - a}$	Line vortex near a wall	

Crossflow around Cylinder





Boundary Layer Separation at θ
Laminar BL: $\theta \approx 80$ deg. (smooth cylinder surface)
Turbulent BL: $\theta = 107.7$ deg. (smooth cylinder surface)