Recap : Bi-Directional Flow

Introduced streamfunction $\psi(x,y)$ or $\psi(r,\theta)$

Defined: (rect. coord.) $\boxed{v_x = \frac{\partial \psi}{\partial y}}$ and $\boxed{v_y = -\frac{\partial \psi}{\partial x}}$

(cyl. coord) $\boxed{v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}}$ and $\boxed{v_\theta = -\frac{\partial \psi}{\partial r}}$

More equations see Table 6-12 in Deen

e.g. Navier Stokes in terms of ψ:

$$\frac{\partial}{\partial t}\left(\nabla^2 \psi\right) - \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x,y)} = \nu \nabla^4 \psi$$

For irrot. flow: $\vec{w} = 0$

$\boxed{\nabla^2 \psi = 0}$ Laplace eq. applicable to steady and unsteady planar flow

Setting the streamfunction constant defines a streamline

$\boxed{\psi = c}$ streamline

Important: Streamlines cannot cross as $\vec{v}$ is always tangent to streamlines.