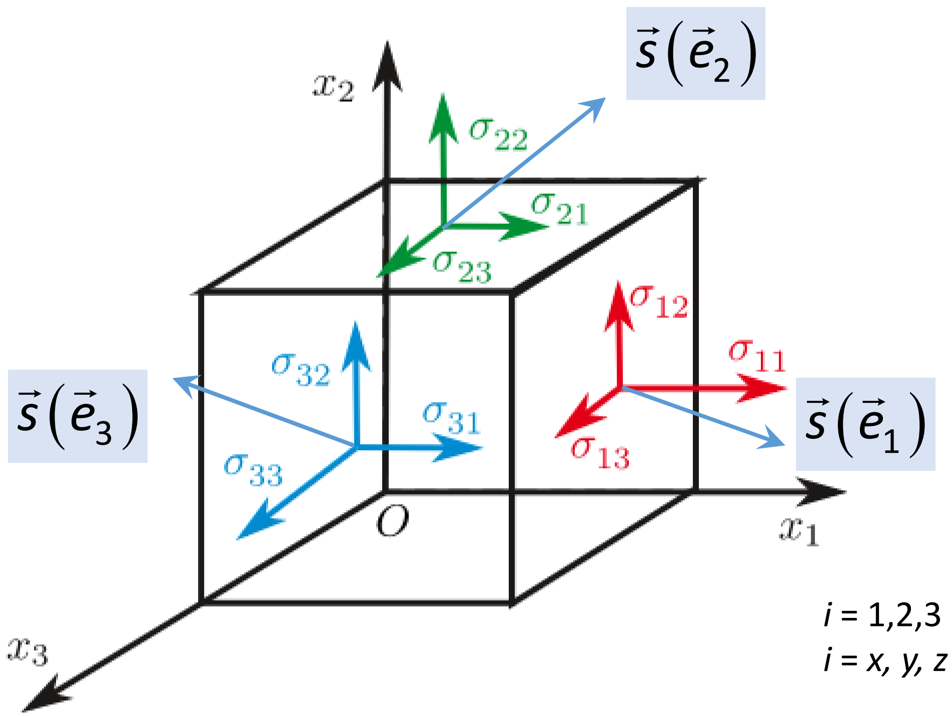
Cauchy Stress Tensor and Stress Vectors



$\vec{s}(\vec{n})$ ... stress vector

$\vec{s}(\vec{e}_i)$ ... basic stress vector

σ_{ij} ... (Cauchy) stress element

$\underline{\underline{\sigma}} = [\sigma_{ij}]$ ... stress tensor

i = 1,2,3 General basis (coordinate system)

i = x, y, z Special basis (Cartesian coord. system)

$$\vec{s}(\vec{n}) = \vec{n} \cdot \sum_i \vec{e}_i \vec{s}(\vec{e}_i) = \vec{n} \cdot \sum_i \vec{e}_i \sum_j \sigma_{ij} \vec{e}_j = \vec{n} \cdot \sum_i \sum_j \sigma_{ij} \vec{e}_i \vec{e}_j = \vec{n} \cdot \underline{\underline{\sigma}}$$

E.g.: $i, j = \{x, y, z\}$

basic stress vectors

$i{=}x$, $i{=}y$, $i{=}z$

$\vec{s}(\vec{e}_x)$ $\vec{s}(\vec{e}_y)$ $\vec{s}(\vec{e}_z)$ (columns)

$$\vec{s}(\vec{e}_i) = \begin{pmatrix} \sigma_{xi} \\ \sigma_{yi} \\ \sigma_{zi} \end{pmatrix} \text{ rows or columns in } \underline{\underline{\sigma}} = \left(\vec{s}(\vec{e}_x), \vec{s}(\vec{e}_y), \vec{s}(\vec{e}_z)\right) = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \begin{matrix} \leftarrow \vec{s}(\vec{e}_x) \\ \leftarrow \vec{s}(\vec{e}_y) \\ \leftarrow \vec{s}(\vec{e}_z) \end{matrix}$$