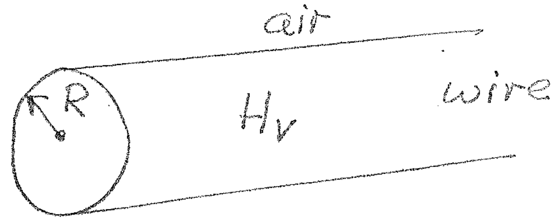
EXAMPLE 3.2-5 Electrically Heated Wire As a heat transfer example, consider the steady-state temperature in a cylindrical wire of radius R that is heated by passage of an electric current and cooled by convective heat transfer to the surrounding air. The local heating rate, H_V, is assumed to be constant, which is equivalent to assuming a uniform current density and electrical resistance. The thermal conductivity and heat transfer coefficient are also assumed to be constant. It is desired to determine the temperature profile in the wire, $T(r)$, where r is the cylindrical radial coordinate. This problem illustrates the use of a convection boundary condition and introduces another important dimensionless group, the Biot number.

$$\boxed{\rho C_p \frac{DT}{Dt} = k \nabla^2 T + H_V}$$



Note: $T = T(r)$ Use cyl. coord.

Table 2-3: $\Rightarrow \nabla^2 = \left\{ \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \right\}$

$$\frac{DT}{Dt} = \left\{ \underbrace{\cancel{\frac{\partial}{\partial t}}}_{\text{steady state}}^{\to 0} + \cancel{\vec{v}} \cdot \vec{\nabla} \right\}$$

$\vec{v} = 0$ (no bulk flow)

$\Rightarrow$ Energy Balance: $\boxed{\frac{1}{r}\frac{d}{dr}\left(r \frac{dT}{dr}\right) = -\frac{H_V}{k}}$

2nd order diff. eq $\Rightarrow$ 2 boundary conditions

Boundary Conditions (BC):

BC1: convective BC at $r = R$: $\underbrace{-k\nabla T}_{\text{wire}}\Big|_{r=R} = \underbrace{h(T_2 - T_\infty)}_{\text{air}}$

BC1 $\Rightarrow \left.\frac{dT}{dr}\right|_{r=R} = -\frac{h}{k}(T - T_\infty)$

BC2: Symmetry Point at $r = 0$, i.e., heat flux $q = -k \left.\frac{dT}{dr}\right|_{r=0} = 0 \Rightarrow$ BC2 $\left.\frac{dT}{dr}\right|_{r=0} = 0$.

Integration of Energy Balance yields with BCs:

Temperature profile in wire: $\boxed{T(r) - T_\infty = \frac{H_V R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right] + \frac{H_V R}{2h}}$