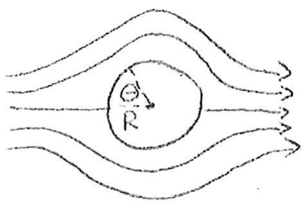
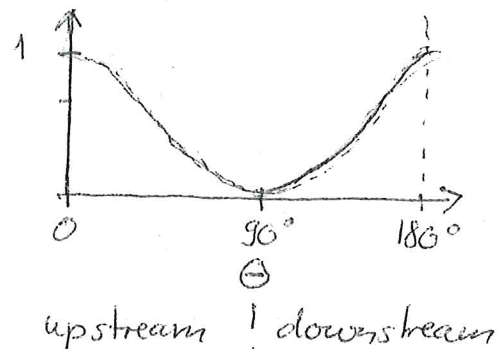
Potential Flow around Cylinder (cross flow)

Pressure coeff. $C_p = \dfrac{\Delta P}{\frac{1}{2}\rho U^2} = \left[1 - 4\sin^2\Theta\right]$





$\Delta P = p - p_\infty$

p ... mean surface pressure

p_∞ stream static pressure

Deen Eq. 9.3-16

$\Delta P \equiv \mathcal{P}(R, \Theta) = \dfrac{F_{\text{Pressure drag}}}{RL}$

(1) The pressure drag is given by:

$$C_{D,p} = \int_0^{\pi} C_p \cos\Theta \, d\Theta$$

For the idealized potential flow situation:

$$C_{D,p} = \int_0^{\pi} \left[1 - 4\sin^2\Theta\right] \cos\Theta \, d\Theta$$

$$= 0$$

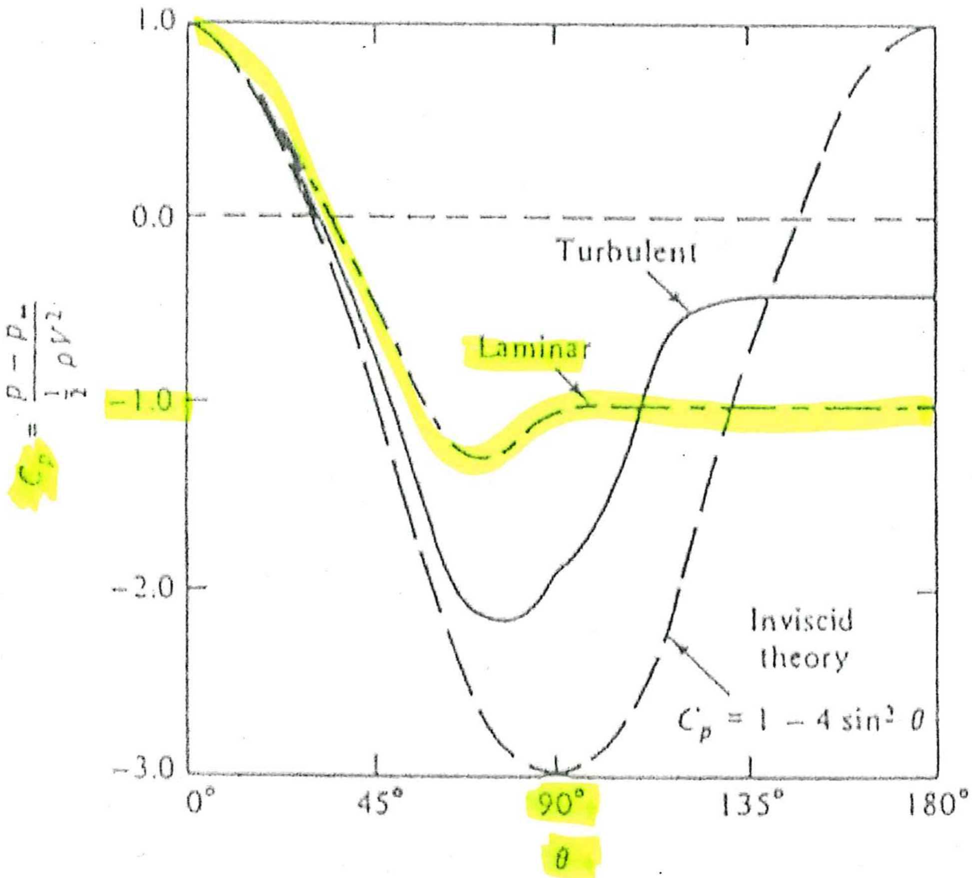
This is an unrealistic result for the pressure drag coefficient. It is a consequence of the balanced pressures between the upstream and the downstream on the cylinder surface

Data fit for $Re \ll 1$ yields for C_p
(laminar regime)

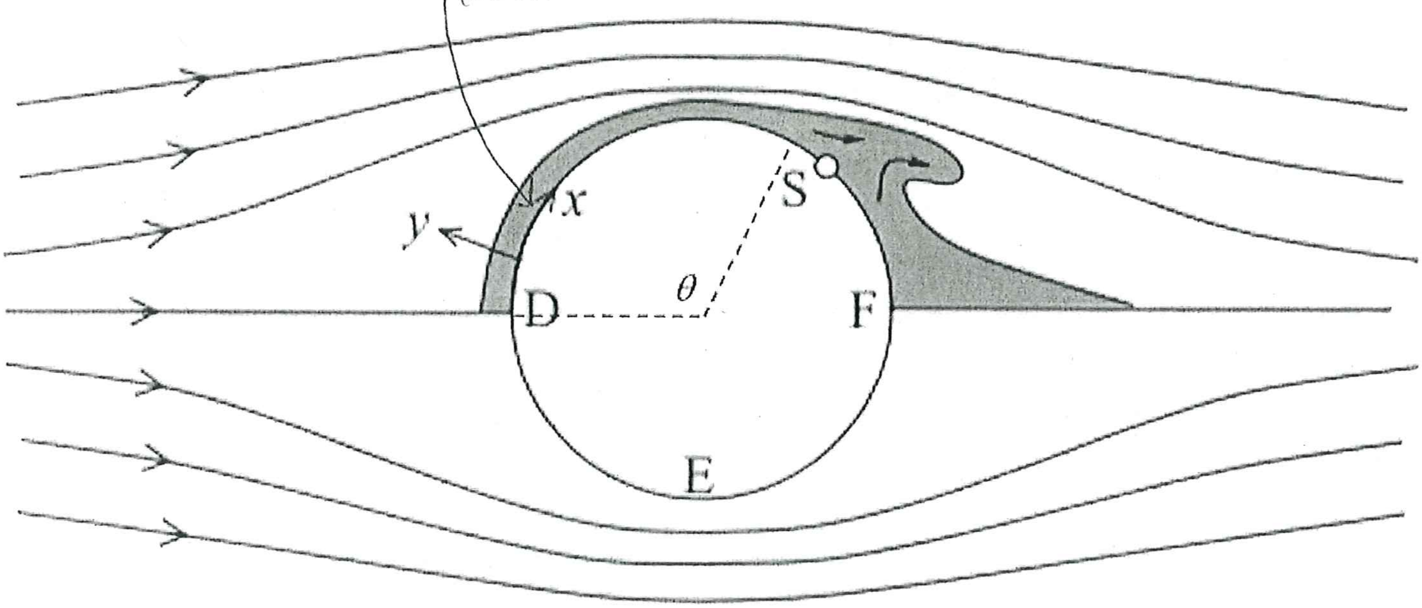
pressure coeff: $C_p = \begin{cases} 1 - \frac{8}{3}\sin^2\Theta & \text{for } 0 \le \Theta \le \frac{\pi}{3} \\ -1 & \text{for } \frac{\pi}{3} < \Theta \le \pi \end{cases}$

Crossflow around Cylinder

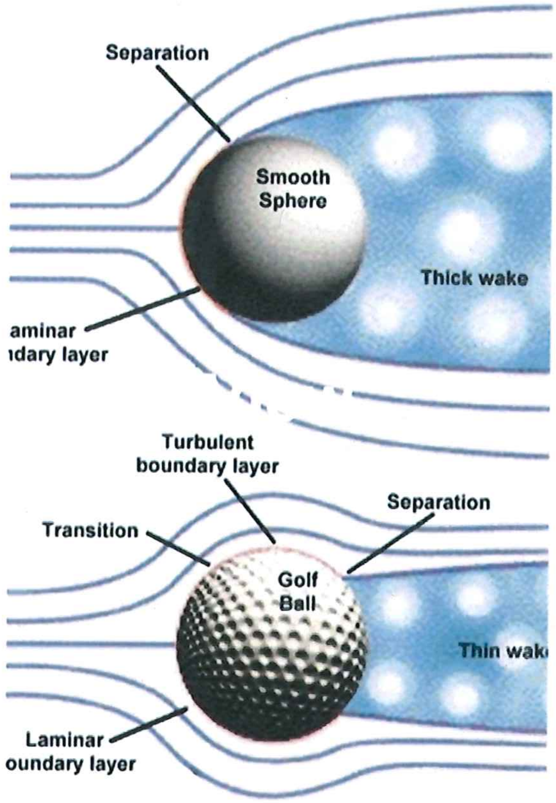
(2a))



laminar or turbulent



Boundary Layer Separation at θ
Laminar BL: $\theta \approx 80$ deg. (smooth cylinder surface)
Turbulent BL: $\theta = 107.7$ deg. (smooth cylinder surface)



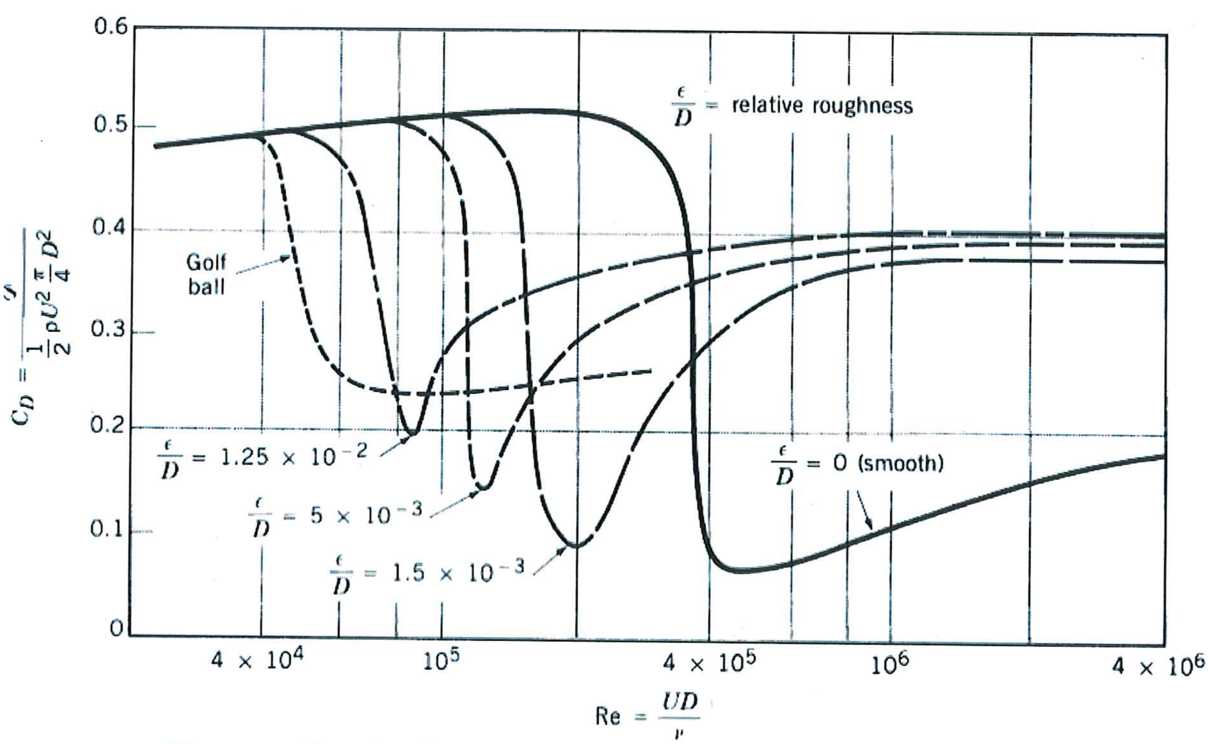


Figure 2. Effect of surface roughness on the drag coefficient of a sphere

https://www.mne.psu.edu/cimbala/me325web_Spring_2012/Labs/Drag/intro.pdf

$\Rightarrow$ pressure drag

$$\boxed{C_{D,P}} = \int_0^{\pi/3} \left(1 - \frac{8}{3}\sin^2\Theta\right)\cos\Theta \, d\Theta - \int_{\pi/3}^{\pi} \cos\Theta \, d\Theta$$

$$= \boxed{\frac{2}{\sqrt{3}}}$$ (see derivation in Exam 2 2017)

$\Rightarrow$ The pressure drag in stream direction is

$$\boxed{F_P} = (2RL)\left(\frac{\rho U^2}{2}\right) C_{D,P} \; ; \quad 2RL \text{.. projected area of cylinder affected by cross flow}$$

$$= \boxed{\frac{2}{\sqrt{3}} RL \rho U^2}$$

The fit above demonstrates that there is a critical angle $\Theta \neq \pi$ at which the functional behavior of the pressure drag is significantly changed. The reason lays in the separation of the boundary layer.

For a cylinder in crossflow, the boundary layer separates for

		Θ
$Re \leq 1$ (laminar)	at	$\approx$ 60-80 degrees
$Re > 4\times10^5$ (fully turbulent)	at	107.7 degrees

in addition to pressure (or form) drag, we consider (3)

(2) Skin friction drag

So far we have ignored shear forces in the liquid that are excerted from the solid surface. The so-called "skin friction drag" results from momentum transfer in a ultra thin boundary layer adjacent to the solid surface.

The BL is viscous (while our formerly discussed potential flow is not) and possesses a no-slip condition.

We can write the friction drag coefficient as

$$C_{D,f} = \int_0^{\Theta_s} C_f \sin\Theta \, d\Theta \quad ; \quad \Theta = 107.7 \text{ deg for high } Re$$

with $C_f = \dfrac{\tau_{wall}}{\frac{1}{2}\rho U^2}$ $\qquad Re = \dfrac{UD}{\nu}$

It is determined to be

dimensionless shear stress $\rightarrow$

$$C_f = \frac{4}{3} \, \frac{\lambda + 12}{3\sqrt{Re_D}} \, \sin\Theta \sqrt{\frac{\cos\Theta}{\lambda}}$$

$$\lambda = \frac{\delta^2}{\nu} \frac{dU(s)}{ds}$$

δ ... boundary layer thickness

ν ... kin. viscosity

$U(s)$ potential flow velocity outside BL

It can be shown that $C_{D,f} = \dfrac{5.786}{\sqrt{Re_D}}$ for $Re \gg 1$