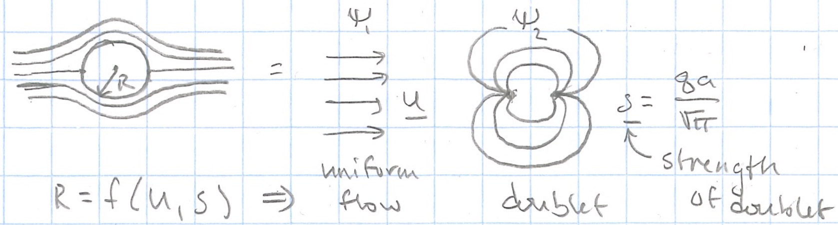
Flow around a cylinder:



$$\psi(r,\theta) = \underbrace{Ur\sin\theta}_{\psi_1} - \underbrace{S\frac{\sin\theta}{r}}_{\psi_2}$$

$R = f(u, s) \Rightarrow$

$$\phi(r,\theta) = \underbrace{Ur\cos\theta}_{\phi_1} + \underbrace{S\frac{\cos\theta}{r}}_{\phi_2}$$

Streamlines must never cross

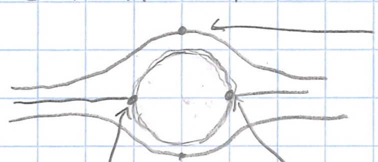
$\therefore \uparrow S \uparrow R$ of cylinder and vice versa

$$\Rightarrow v_r = \frac{1}{r}\frac{\partial\psi}{\partial r} = \cos\theta\left(u - S/r^2\right) \qquad v_\theta = -\frac{\partial\psi}{\partial r} = -\sin\theta\left(u + S/r^2\right)$$

No penetration condition: $v_r\Big|_R = 0 \Rightarrow \cos\theta\left(u - S/r^2\right)\Big|_{r=R} = 0$

$\Rightarrow S/R^2 = u \Rightarrow \boxed{R^2 = S/u}$ ← forces streamlines to match cylinder

@ $r = R$: $\psi = 0$ $\quad \phi = 2Ur\cos\theta$ $\quad v_r = 0$ $\quad v_\theta = -2u\sin\theta$ ← v_θ exist b/c no-slip not employed



max surf velocity $v_\theta\Big|_{r=R} = 2u$

stagnation points $\quad v_\theta(R,\pi) = -v_\theta(R,0) = 0$

Applying Bernoulli eqn: $\boxed{C_{p,D} = 1 - 4\sin^2\theta}$ $\qquad V_{\theta R} = -2\sin\theta$

(1) <u>Pressure Drag</u> $\quad C_{D,P} = \int_0^\pi C_p \cos\theta \, d\theta \quad \Big| \; C_p$ is drag coeff

for potential flow $\quad = \int_0^\pi [1 - 4\sin^2\theta]\cos\theta \, d\theta = 0$ not realistic, enforces pressure symm.

Data fitting for Re. << 1 (laminar): $C_p = \begin{cases} 1 - \frac{8}{3}\sin^2\theta & 0 < \theta \le \pi/3 \\ -1 & \pi/3 < \theta \le \pi \end{cases}$

$\therefore C_{D,P}(\text{laminar}) = \frac{2}{\sqrt{3}}$

$\Rightarrow F_p = (2RL)(\rho u^2/2)\, C_{D,P} = \frac{2}{\sqrt{3}} RL\rho u^2$

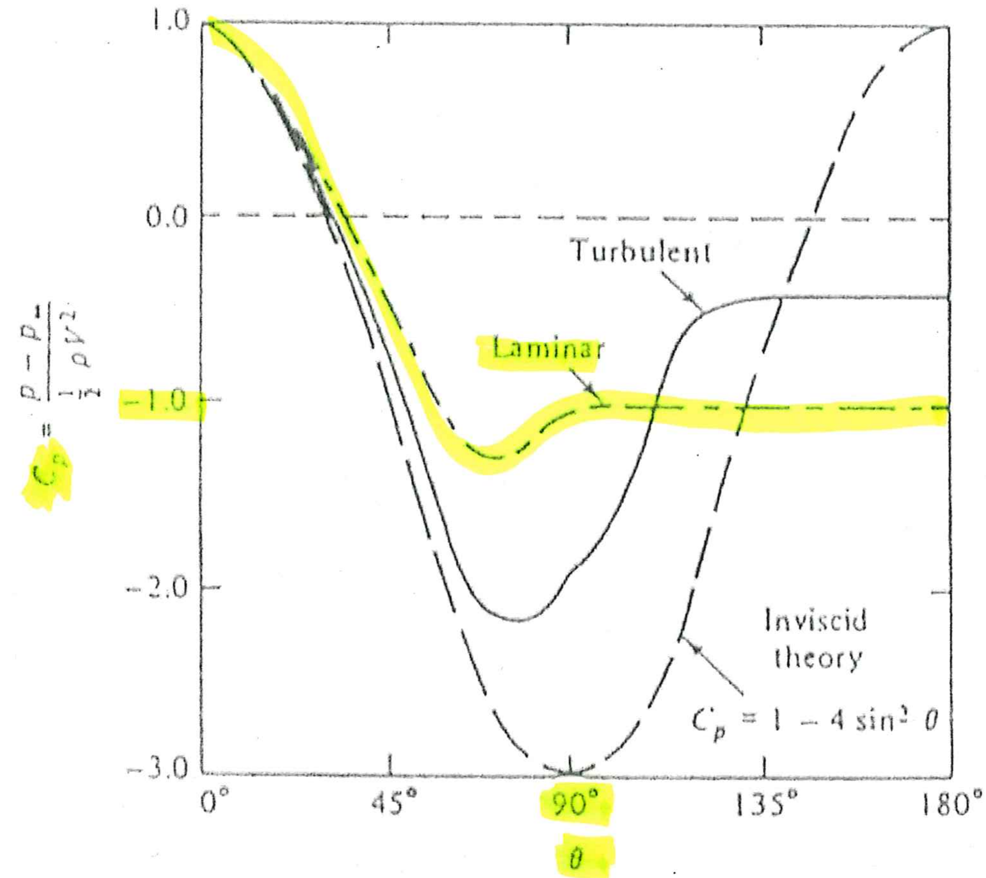
(2RL ↑ projected area)

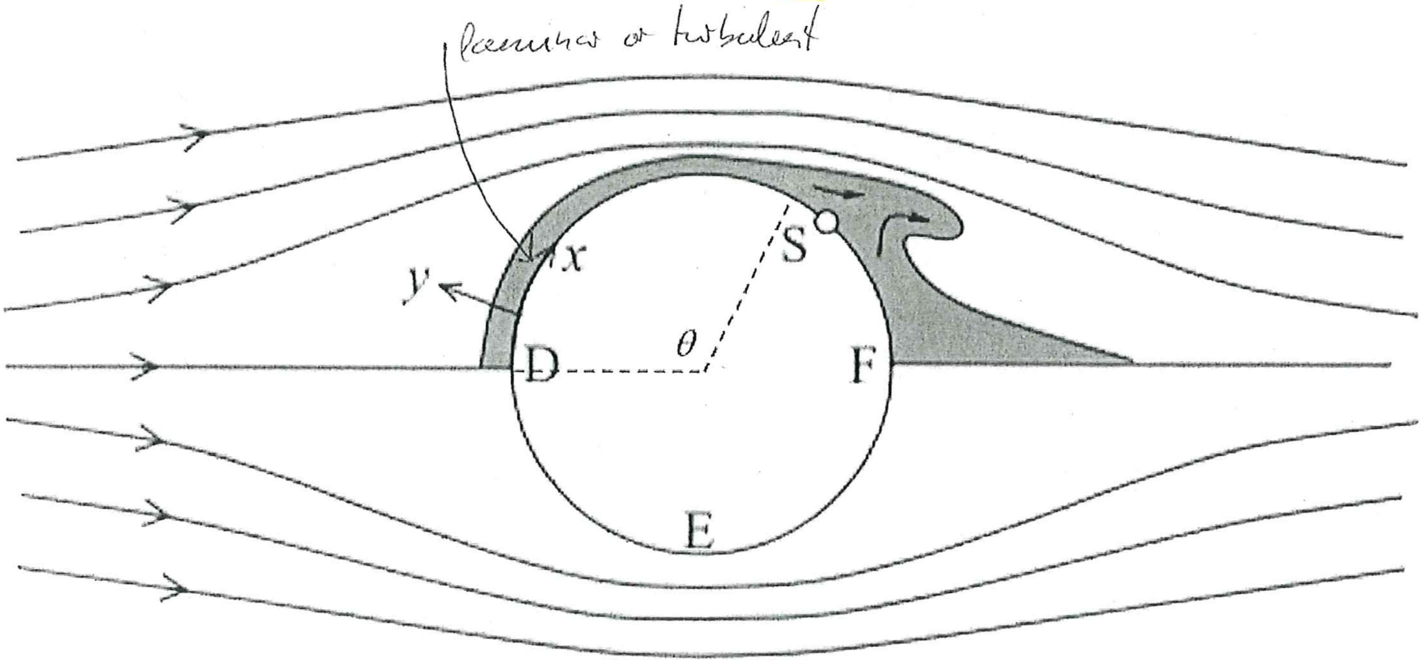
	Critical Angle θ
laminar Re ≈ 1	60-80
turbulent Re > 4x10^5	107.7

critical angle <=> boundary layer separation

Crossflow around Cylinder

(2a))

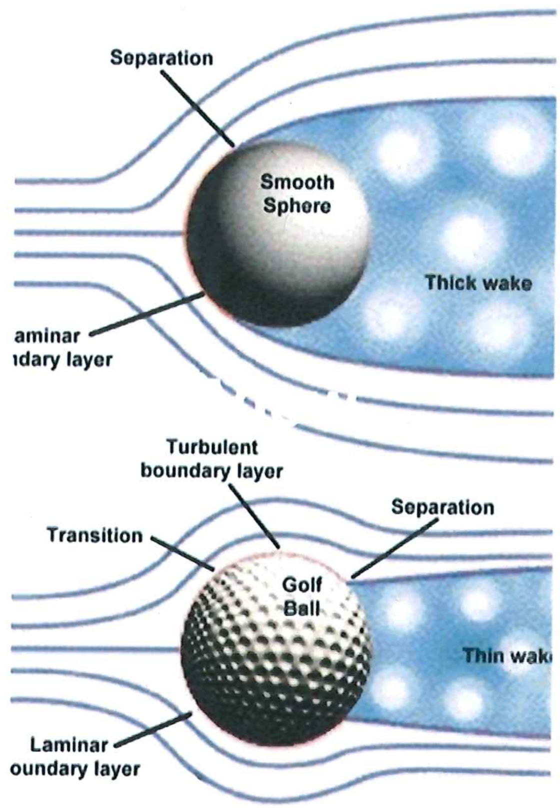




Boundary Layer Separation at θ

Laminar BL: $\theta \approx 80$ deg. (smooth cylinder surface)

Turbulent BL: $\theta = 107.7$ deg. (smooth cylinder surface)



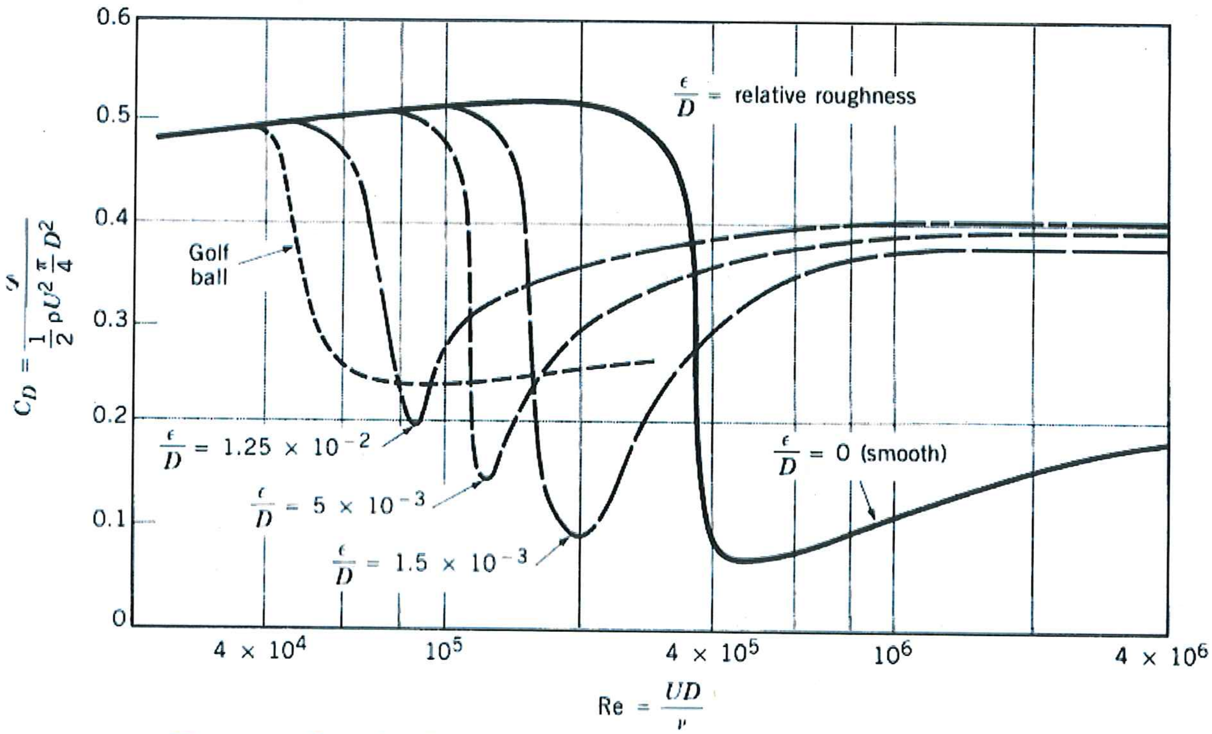


Figure 2. Effect of surface roughness on the drag coefficient of a sphere

https://www.mne.psu.edu/cimbala/me325web_Spring_2012/Labs/Drag/intro.pdf