Multiple Operations involving the **del Operator** (∇) in Cartesian, Cylindrical Coordinates

https://en.wikipedia.org/wiki/Del_in_cylindrical_and_spherical_coordinates

Del formula [edit]

Table with the del operator in cartesian, cylindrical and spherical coordinates

| Operation | Cartesian coordinates (x, y, z) | Cylindrical coordinates $(ho, arphi, z)$ | Spherical coordinates $(r,	heta, arphi)$, where $	heta$ is the polar and $arphi$ is the azimuth $	ext{angle}^{lpha}$ |
|---|--|--|---|
| A vector field A | $A_x\hat{f x}+A_y\hat{f y}+A_z\hat{f z}$ | $A_{ ho}\hat{oldsymbol{ ho}}+A_{arphi}\hat{oldsymbol{arphi}}+A_{z}\hat{f z}$ | $A_{	au}\hat{f r}+A_{	heta}\hat{m	heta}+A_{arphi}\hat{marphi}$ |
| Gradient $ abla f$ | $rac{\partial f}{\partial x}\hat{\mathbf{x}} + rac{\partial f}{\partial y}\hat{\mathbf{y}} + rac{\partial f}{\partial z}\hat{\mathbf{z}}$ | $rac{\partial f}{\partial ho}\hat{oldsymbol{ ho}} + rac{1}{ ho}rac{\partial f}{\partial arphi}\hat{oldsymbol{arphi}} + rac{\partial f}{\partial z}\hat{f z}$ | $rac{\partial f}{\partial r}\hat{\mathbf{r}} + rac{1}{r}rac{\partial f}{\partial 	heta}\hat{oldsymbol{	heta}} + rac{1}{r\sin	heta}rac{\partial f}{\partial arphi}\hat{oldsymbol{arphi}}$ |
| Divergence $ abla \cdot \mathbf{A}$ | $rac{\partial A_x}{\partial x} + rac{\partial A_y}{\partial y} + rac{\partial A_z}{\partial z}$ | $rac{1}{ ho}rac{\partial\left(ho A_{ ho} ight)}{\partial ho}+rac{1}{ ho}rac{\partial A_{arphi}}{\partialarphi}+rac{\partial A_{z}}{\partial z}$ | $rac{1}{r^2}rac{\partial \left(r^2A_r ight)}{\partial r}+rac{1}{r\sin	heta}rac{\partial}{\partial	heta}\left(A_	heta\sin	heta ight)+rac{1}{r\sin	heta}rac{\partial A_arphi}{\partialarphi}$ |
| Curl $ abla 	imes \mathbf{A}$ | $egin{aligned} &\left(rac{\partial A_z}{\partial y} - rac{\partial A_y}{\partial z} ight)\hat{\mathbf{x}} \ + \left(rac{\partial A_x}{\partial z} - rac{\partial A_z}{\partial x} ight)\hat{\mathbf{y}} \ + \left(rac{\partial A_y}{\partial x} - rac{\partial A_x}{\partial y} ight)\hat{\mathbf{z}} \end{aligned}$ | $egin{aligned} \left(rac{1}{ ho}rac{\partial A_z}{\partial arphi}-rac{\partial A_{arphi}}{\partial z} ight)\hat{oldsymbol{ ho}}\ +\left(rac{\partial A_ ho}{\partial z}-rac{\partial A_z}{\partial ho} ight)\hat{oldsymbol{arphi}}\ +rac{1}{ ho}\left(rac{\partial \left(ho A_{arphi} ight)}{\partial ho}-rac{\partial A_ ho}{\partial arphi} ight)\hat{oldsymbol{z}} \end{aligned}$ | $egin{aligned} &rac{1}{r\sin	heta}\left(rac{\partial}{\partial	heta}\left(A_{arphi}\sin	heta ight)-rac{\partial A_{	heta}}{\partialarphi} ight)\hat{\mathbf{r}}\ &+rac{1}{r}\left(rac{1}{\sin	heta}rac{\partial A_{r}}{\partialarphi}-rac{\partial}{\partial r}\left(rA_{arphi} ight) ight)\hat{oldsymbol{	heta}}\ &+rac{1}{r}\left(rac{\partial}{\partial r}\left(rA_{	heta} ight)-rac{\partial A_{r}}{\partial	heta} ight)\hat{oldsymbol{arphi}} \end{aligned}$ |
| Laplace operator $\nabla^2 f \equiv \Delta f$ | $rac{\partial^2 f}{\partial x^2} + rac{\partial^2 f}{\partial y^2} + rac{\partial^2 f}{\partial z^2}$ | $rac{1}{ ho}rac{\partial}{\partial ho}\left(horac{\partial f}{\partial ho} ight)+rac{1}{ ho^2}rac{\partial^2 f}{\partialarphi^2}+rac{\partial^2 f}{\partial z^2}$ | $rac{1}{r^2}rac{\partial}{\partial r}igg(r^2rac{\partial f}{\partial r}igg) + rac{1}{r^2\sin	heta}rac{\partial}{\partial	heta}igg(\sin	hetarac{\partial f}{\partial	heta}igg) + rac{1}{r^2\sin^2	heta}rac{\partial^2 f}{\partialarphi^2}$ |
| Vector Laplacian $\nabla^2 \mathbf{A} \equiv \Delta \mathbf{A}$ | $ abla^2 A_x \hat{\mathbf{x}} + abla^2 A_y \hat{\mathbf{y}} + abla^2 A_z \hat{\mathbf{z}}$ | — View by clicking [show] — [show] | — View by clicking [show] — [sh |
| Material derivative $^{\alpha[1]}$ $(\mathbf{A}\cdot \nabla)\mathbf{B}$ | $\mathbf{A} \cdot abla B_x \hat{\mathbf{x}} + \mathbf{A} \cdot abla B_y \hat{\mathbf{y}} + \mathbf{A} \cdot abla B_z \hat{\mathbf{z}}$ | $\begin{split} &\left(A_{\rho}\frac{\partial B_{\rho}}{\partial \rho} + \frac{A_{\varphi}}{\rho}\frac{\partial B_{\rho}}{\partial \varphi} + A_{z}\frac{\partial B_{\rho}}{\partial z} - \frac{A_{\varphi}B_{\varphi}}{\rho}\right)\hat{\boldsymbol{\rho}} \\ &+ \left(A_{\rho}\frac{\partial B_{\varphi}}{\partial \rho} + \frac{A_{\varphi}}{\rho}\frac{\partial B_{\varphi}}{\partial \varphi} + A_{z}\frac{\partial B_{\varphi}}{\partial z} + \frac{A_{\varphi}B_{\rho}}{\rho}\right)\hat{\boldsymbol{\varphi}} \\ &+ \left(A_{\rho}\frac{\partial B_{z}}{\partial \rho} + \frac{A_{\varphi}}{\rho}\frac{\partial B_{z}}{\partial \varphi} + A_{z}\frac{\partial B_{z}}{\partial z}\right)\hat{\mathbf{z}} \end{split}$ | — View by clicking [show] — [sh |
| Tensor divergence $\nabla \cdot \mathbf{T}$ | — View by clicking [show] — [show] | — View by clicking [show] — [show] | — View by clicking [show] — [sh |
| Differential displacement $d\ell$ | $dx\hat{f x} + dy\hat{f y} + dz\hat{f z}$ | $d ho\hat{oldsymbol{ ho}}+ hodarphi\hat{oldsymbol{arphi}}+dz\hat{f z}$ | $dr\hat{f r} + rd	heta\hat{m 	heta} + r\sin	hetadarphi\hat{m arphi}$ |
| | $dydz\hat{\mathbf{x}}$ | $ hodarphidz\hat{m{ ho}}$ | $r^2\sin	hetad	hetadarphi\hat{\mathbf{r}}$ |
| Differential normal area dS | $+dxdz\hat{\mathbf{y}}\ +dxdy\hat{\mathbf{z}}$ | $egin{aligned} &+d hodz\hat{oldsymbol{arphi}}\ &+ hod hodarphi\hat{f z} \end{aligned}$ | $+r\sin	hetadrdarphi\hat{m{	heta}}\ +rdrd	heta\hat{m{arphi}}$ |
| Differential volume dV | dxdydz | $ ho\mathrm{d} ho\mathrm{d}arphi\mathrm{d}z$ | $r^2\sin	hetadrd	hetadarphi$ |
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 $^{^{\}alpha}$ This page uses θ for the polar angle and φ for the azimuthal angle, which is common notation in physics. The source that is used for these formulae uses θ for the azimuthal angle and φ for the polar angle, which is common mathematical notation. In order to get the mathematics formulae, switch θ and φ in the formulae shown in the table above.