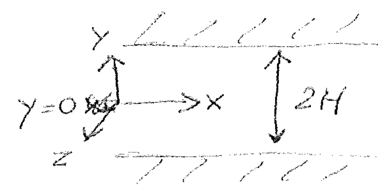
Recap

Navier Stokes Eq: $\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} P + \rho \vec{g} + \mu \nabla^2 \vec{v}$

ρ, μ const.

Ex: Flow in parallel-plate channel with $v_x = v_x(y)$, ρ, μ const.



Navier Stokes simplifies to

$$\frac{d^2 v_x}{dy^2} = \frac{1}{\mu} \frac{d\mathcal{P}}{dx} \quad ; \quad \mathcal{P} \text{ dyn. pressure}$$

BCs: $v_x \Big|_{y=\pm H} = 0$ no slip condition

$\frac{dv_x}{dy}\Big|_{y=0} = 0$ symm. cond

Velocity Profile

$$\Rightarrow v_x(y) = \frac{H^2}{2\mu} \frac{d\mathcal{P}}{dx} \left[1 - \left(\frac{y}{H}\right)^2\right] \quad \text{Plane Poiseuille Flow}$$

Mean velocity: $U \equiv \frac{1}{H} \int_0^H v_x \, dy = -\frac{H^2}{3\mu} \frac{d\mathcal{P}}{dx}$

$$\Rightarrow v_x(y) = \frac{3}{2} U \left[1 - \left(\frac{y}{H}\right)^2\right]$$

Max velocity: from $\frac{dv_x}{dy} = 0 \Rightarrow v_{max} = \frac{3}{2} U$

Volumetric Flow Rate: per unit width

$$q' \equiv U A' \quad ; \quad A' = 2H$$

$$= 2UH = -\frac{2H^3}{3\mu} \frac{d\mathcal{P}}{dx}$$