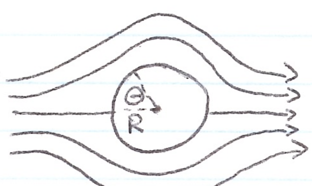
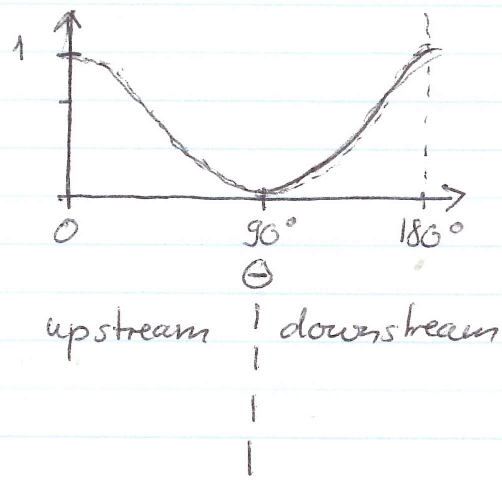
Potential Flow around Cylinder (crossflow)

Pressure coeff.
(dimensionless pressure)

$$C_p \equiv \frac{\Delta P}{\frac{1}{2}\rho U^2} = \left[1 - 4\sin^2\Theta\right]$$

based on potential flow theory





$\Delta P = p - P_\infty$

p ... mean surface pressure

P_∞ stream static pressure

Deen Eq. 9.3-16

$\Delta P \equiv \mathcal{P}(R, \Theta)$

$= \frac{F_{\text{pressure drag}}}{RL}$

(1) The pressure drag is given by:

$$C_{D,p} = \int_0^\pi C_p \cos\Theta \, d\Theta$$

For the idealized potential flow situation:

$$C_{D,p} = \int_0^\pi \left[1 - 4\sin^2\Theta\right] \cos\Theta \, d\Theta$$

$$= 0$$

This is an unrealistic result for the pressure drag coefficient. It is a consequence of the balanced pressures between the upstream and the downstream on the cylinder surface

Data fit for high Re and smooth surfaces with laminar BL (laminar regime)

pressure coeff:

$$C_p = \begin{cases} 1 - \frac{8}{3}\sin^2\Theta & \text{for } 0 \le \Theta \le \frac{\pi}{3} \\ -1 & \text{for } \frac{\pi}{3} \le \Theta \le \pi \end{cases}$$

HW 6 Problem 6-6

$\Rightarrow$ pressure drag

$$C_{D,P} = \int_0^{\pi/3} \left(1 - \frac{8}{3}\sin^2\Theta\right)\cos\Theta \, d\Theta - \int_{\pi/3}^{\pi} \cos\Theta \, d\Theta$$

$$= \frac{2}{\sqrt{3}}$$

used $\int_a^b \sin^2\Theta \cos\Theta \, d\Theta = \left.\frac{\sin^3\Theta}{3}\right|_a^b$

$\Rightarrow$ The pressure drag in stream direction is

$$F_P = (2RL)\left(\frac{\rho U^2}{2}\right) C_{D,P} \; ; \quad 2RL \text{ .. projected area of cylinder affected by cross flow}$$

$$= \frac{2}{\sqrt{3}} RL \rho U^2$$

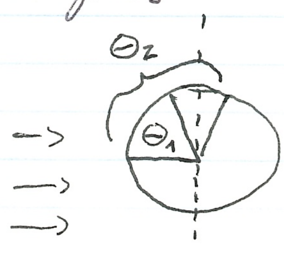
The fit above demonstrates that there is a critical angle $\Theta \neq \frac{\pi}{2}$ at which the functional behavior of the pressure drag is significantly changed. The reason lays in the separation of the boundary layer.

For a cylinder in crossflow, the boundary layer separates for

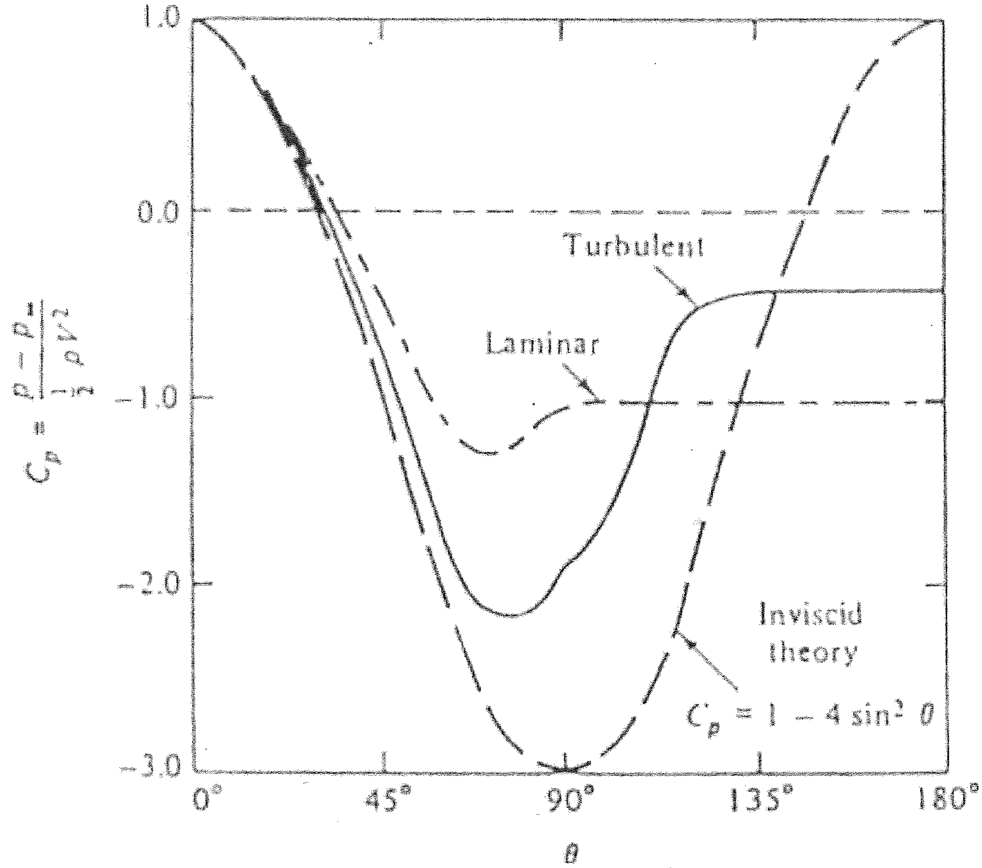
		Θ (separation point)
$Re \leq 1$	at $\approx$	60 degrees
$Re > \sim 5 \times 10^5$	at	107.7 degrees

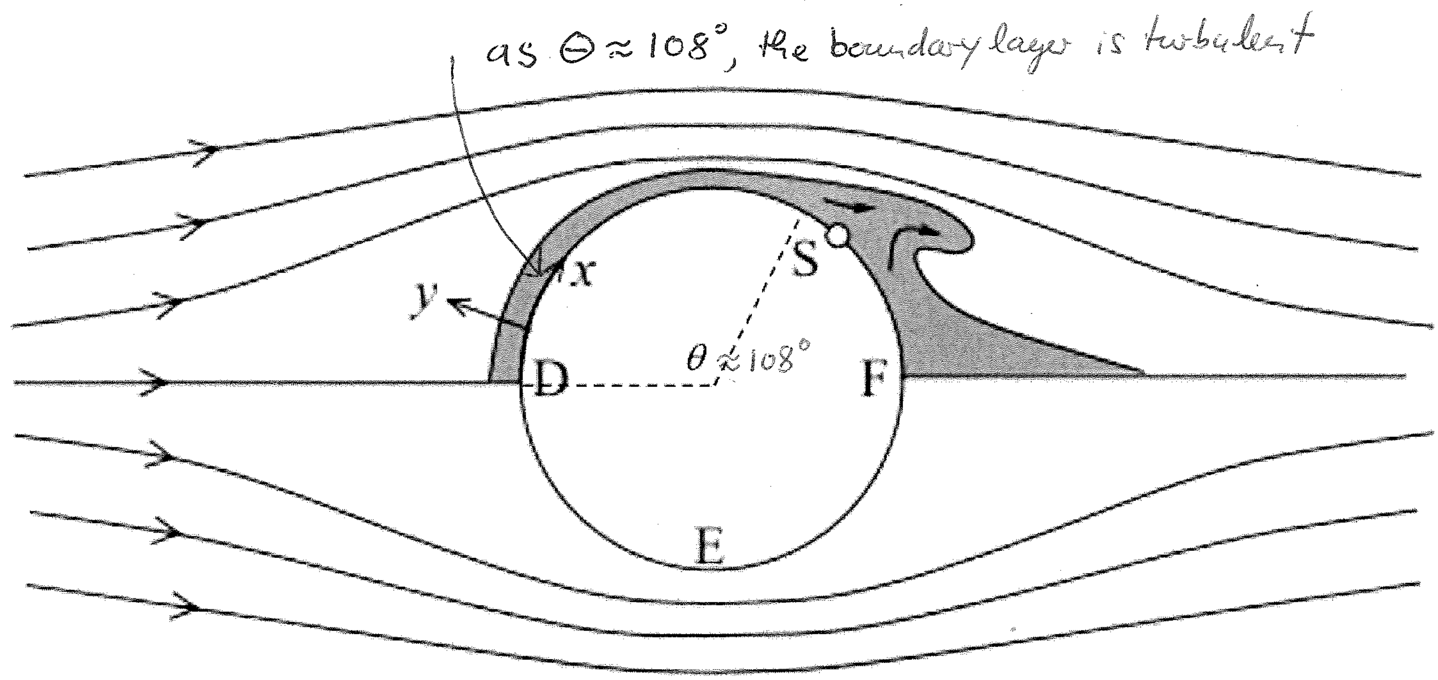
$\Theta_1 = 60^\circ \Rightarrow$ laminar boundary layer

$\Theta_2 = 107.7^\circ \Rightarrow$ turbulent boundary layer



Crossflow around Cylinder

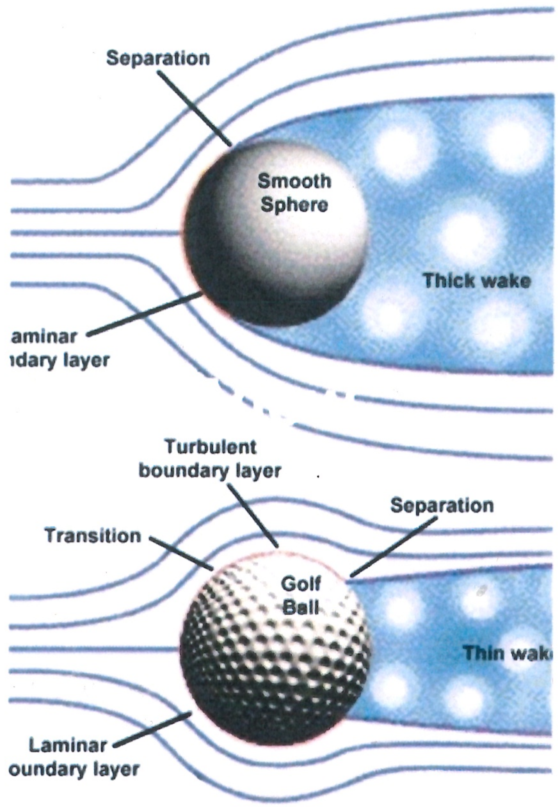




Boundary Layer Separation at θ

Laminar BL: $\theta \approx 80$ deg. (smooth cylinder surface)

Turbulent BL: $\theta = 107.7$ deg. (smooth cylinder surface)



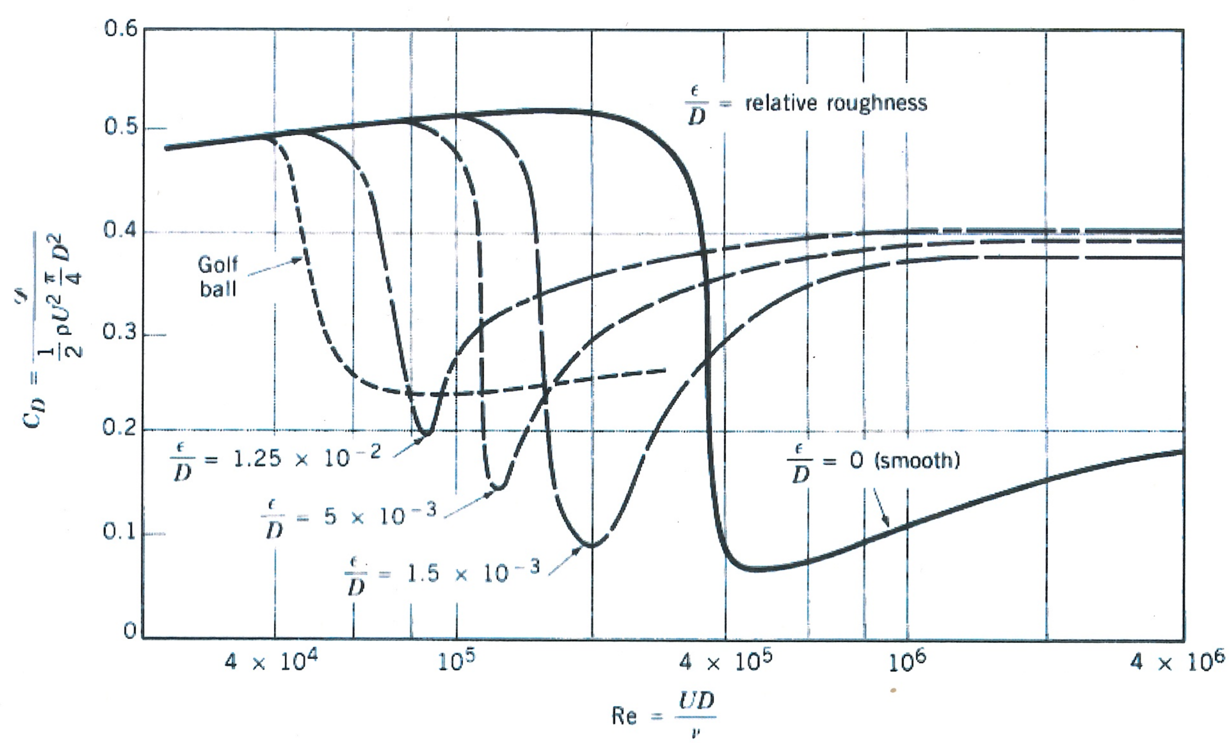


Figure 2. Effect of surface roughness on the drag coefficient of a sphere

https://www.mne.psu.edu/cimbala/me325web_Spring_2012/Labs/Drag/intro.pdf