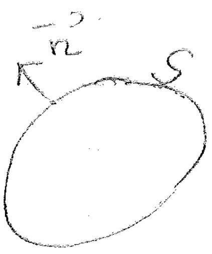
Force Calculation

The pressure force on a surface is given by



$$\boxed{\vec{F}_P = -\int_S P\vec{n}\,dS} \qquad \text{Eq. 6.7-1}$$

where S is a fluid-solid or fluid-fluid interface (or imaginary)

If we consider fluids in motion, the forces on an object are

$$\vec{F} = \vec{F}_g + \vec{F}_p + \vec{F}_\tau = \rho_0 \vec{g} V - \int_S P\vec{n}\,dS + \underbrace{\int_S \vec{n}\cdot\underline{\underline{\tau}}\,dS}_{\text{viscous stress of moving fluid}}$$

with ρ_0 object density

If we introduce the dynamic pressure force

$$\vec{F}_{\mathcal{P}} \equiv \vec{F}_p + \rho_0\vec{g}V \equiv \vec{F}_p - \vec{F}_0 \; ; \quad \vec{F}_0 = -\rho_0\vec{g}V$$

static pressure ~~due to~~ on submerged object

$$\Rightarrow \boxed{\vec{F}_{\mathcal{P}} = -\int_S \mathcal{P}\vec{n}\,dS} \qquad \text{Eq. 6.7-11}$$

The drag force in flow direction (e.g. $\vec{e}_z$) is given as given by the pressure force and shear force.

$$\text{Eq. 6.7-13} \quad \boxed{F_D = \vec{e}_z\cdot(\vec{F}_{\mathcal{P}} + \vec{F}_\tau) = -\int_S \mathcal{P}\vec{n}\cdot\vec{e}_z\,dS + \int_S \vec{n}\cdot\underline{\underline{\tau}}\cdot e_z\,dS}$$

HW 6 / Problem 6-6

Drag on cylinder at high Reynolds Number

(a) Determine $F_{\mathcal{P}}$ for

$$\frac{\mathcal{P}(R,\Theta)}{\rho V^2/2} = \begin{cases} 1 - \frac{8}{3}\sin^2\Theta & ; 0 \le \Theta \le \frac{\pi}{3} \\ -1 & ; \frac{\pi}{3} \le \Theta \le \pi \end{cases}$$

form drag in $\vec{e}_x$: $\overrightarrow{F_{\mathcal{P}}} = -\vec{e}_x \cdot \left(- \int_S \vec{n}\, \mathcal{P}\, dS \right)$

$$\vec{n} = \vec{e}_r = \cos\Theta\, \vec{e}_x + \sin\Theta\, \vec{e}_y$$

$$\Rightarrow \vec{e}_x \cdot \vec{n} = \cos\Theta$$

$$dS = RL\, d\Theta \quad (\text{diff. of cylinder surface})$$

$$\Rightarrow F_{\mathcal{P}} = RL \int_0^{2\pi} \mathcal{P}(R,\Theta) \cos\Theta\, d\Theta$$

$$= 2RL \int_0^{\pi} \mathcal{P}(R,\Theta) \cos\Theta\, d\Theta$$

subst $\mathcal{P}(R,\Theta)$ from above

for $\int_0^{\pi/3}$ and $\int_{\pi/3}^{\pi}$

Definition of Drag Coefficient:

$$\boxed{C_D = \frac{F_{\mathcal{P}}}{RL\rho V^2}}$$