A first-order reaction $A \xrightarrow{k_1} P$ takes place inside a spherical catalyst. The catalyst is 6×10^{-3} m in diameter and has a surface area of 2×10^{8} m²/m³ catalyst. (a) Obtain an expression for the concentration profile of A inside the catalyst, and (b) determine the value of the effectiveness factor. Data:

Effective diffusivity $D_{Ae} = 1.26 \times 10^{-4}$ m²/h

Reaction rate constant $= 1.728 \times 10^{-7} \frac{1}{(\text{h})(\text{m}^2/\text{m}^3 \text{ cat.})}$

Concentration of A at the catalyst surface is 28 kgmol/m³

$$\phi = \frac{V_p}{S_x}\sqrt{\frac{a k_{1s}}{D_{Ae}}} \quad (3.105)$$

$$\frac{C_A}{C_{AS}} = \frac{R \sinh(3\phi r/R)}{r \sinh(3\phi)} \quad (3.106)$$

(a) $A \xrightarrow{k_1} P$

Data: $D_{Ae} = 1.26 \times 10^{-4}$ m²/h

$k_1 = 1.728 \times 10^{-7} \frac{1}{\text{h}\left(\frac{\text{m}^2}{\text{m}^3 \text{ cat.}}\right)}$

$C_{AS} = 28$ kg mol/m³, $R = 3 \times 10^{-3}$ m

$a = 2 \times 10^{8}$ m²/m³ cat. (surface area per unit volume of cat.)

From Eqs. (3-105 and 3-106) we have

$$C_A = C_{AS} \frac{R}{r} \frac{\sinh(3\phi r/R)}{\sinh(3\phi)}$$

$$\phi = \frac{V_p}{S_x}\sqrt{\frac{a k_1^s}{D_{Ae}}}$$

plug into the data gives

$$\phi = \frac{\frac{4}{3}\pi R^3}{4\pi R^2}\sqrt{\frac{a k_1^s}{D_{Ae}}}$$

$$= \frac{3 \times 10^{-3}}{3}\sqrt{\frac{2 \times 10^{+8}(1.728 \times 10^{-7})}{1.26 \times 10^{-4}}}$$

$$= 0.5237$$

thus

$$C_A = (28)\frac{3 \times 10^{-3}}{r}\frac{\sinh[3(0.5237)r/3 \times 10^{-3}]}{\sinh[(3)(0.5237)]}$$

$$= \frac{0.0365}{r}\sinh(523.7r)$$

(b) $$\eta = \frac{1}{3\phi^2}(3\phi \coth 3\phi - 1)$$

$$= \frac{1}{3(0.5237)^2}[3(0.5237)\coth[3(0.5237)] - 1]$$

$$= 0.867$$