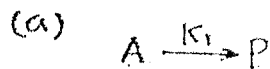


A first-order reaction  $A \xrightarrow{k_1} P$  takes place inside a spherical catalyst. The catalyst is  $6 \times 10^{-3}$  m in diameter and has a surface area of  $2 \times 10^8$  m<sup>2</sup>/m<sup>3</sup> catalyst.  
 (a) Obtain an expression for the concentration profile of A inside the catalyst, and  
 (b) determine the value of the effectiveness factor. Data:

Effective diffusivity  $D_{Ae} = 1.26 \times 10^{-4}$  m<sup>2</sup>/h

Reaction rate constant =  $1.728 \times 10^{-7} \frac{1}{(h)(m^2/m^3 \text{ cat.})}$

Concentration of A at the catalyst surface is 28 kgmol/m<sup>3</sup>



Data:  $D_{Ae} = 1.26 \times 10^{-4} \text{ m}^2/\text{h}$

$k_1 = 1.728 \times 10^{-7} \frac{1}{h(\frac{m^2}{m^3 \text{ cat.}})}$

$C_{As} = 28 \text{ kg mol/m}^3, R = 3 \times 10^{-3} \text{ m}$

$a = 2 \times 10^8 \text{ m}^2/\text{m}^3 \text{ cat.}$  (surface area per unit volume)

From Eqs. (3-105 and 3-106) we have

$$C_A = C_{As} \frac{R}{r} \frac{\sinh(3\phi r/R)}{\sinh(3\phi)}$$

$$\phi = \frac{V_p}{S_x} \sqrt{\frac{a k_1^s}{D_{Ae}}}$$

plug into the data gives

$$\begin{aligned} \phi &= \frac{\frac{4}{3} \pi R^3}{4 \pi R^2} \sqrt{\frac{a k_1^s}{D_{Ae}}} \\ &= \frac{3 \times 10^{-3}}{3} \sqrt{\frac{2 \times 10^8 (1.728 \times 10^{-7})}{1.26 \times 10^{-4}}} \\ &= 0.5237 \end{aligned}$$

thus

$$\begin{aligned} C_A &= (28) \frac{3 \times 10^{-3}}{r} \frac{\sinh[3(0.5237)/3 \times 10^{-3}]}{\sinh[(3)(0.5237)]} \\ &= \frac{0.0365}{r} \sinh(523.7r) \end{aligned}$$

(b)  $\eta = \frac{1}{3\phi^2} (3\phi \coth 3\phi - 1)$

$$= \frac{1}{3(0.5237)^2} [3(0.5237) \coth[3(0.5237)] - 1]$$

$= 0.867$

$$\phi = \frac{V_p}{S_x} \sqrt{\frac{a k_1^s}{D_{Ae}}} \quad (3.105)$$

$$\frac{C_A}{C_{As}} = \frac{R \sinh(3\phi r/R)}{r \sinh(3\phi)} \quad (3.106)$$