Recap

Conservation of momentum

Cauchy: $$\boxed{\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} - \vec{\nabla} P + \vec{\nabla} \cdot \underline{\underline{\tau}}} \qquad (1)$$

Newtonian Fluid using the constitutive eq $\underline{\underline{\tau}} = 2\mu \underline{\underline{\Gamma}} = \mu\left[\vec{\nabla}\vec{v} + (\vec{\nabla}\vec{v})^t\right]$

$$\boxed{\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} - \vec{\nabla} P + \vec{\nabla} \cdot 2\mu \underline{\underline{\Gamma}}} \qquad (2)$$

μ const.

Non-Newtonian Fluid: set $\mu = f(\Gamma)$

Generalized Newtonian Fluid Momentum Conservation Equation

$$\boxed{\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} - \vec{\nabla} P + \vec{\nabla} \cdot \{2\mu(\Gamma)\underline{\underline{\Gamma}}\}} \qquad (3)$$

Strain rate: $\underline{\underline{\Gamma}} \equiv \frac{1}{2}\left[\vec{\nabla}\vec{v} + \vec{\nabla}\vec{v})^t\right]$

Magnitude of strain rate: $\Gamma \equiv \left[\frac{1}{2}\left(\underline{\underline{\Gamma}} : \underline{\underline{\Gamma}}\right)\right]^{1/2}$; Dissipation fct $\Phi = (2\Gamma)^2$ (Table 6-10)

$\mu(\Gamma)$ empirical relationships (e.g. Bingham Model)

Newtonian Fluid with ρ = const and μ = const.

Navier Stokes Eq.

$$\boxed{\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} - \vec{\nabla} P + \mu \nabla^2 \vec{v}}$$

Continuity eq. for Newtonian Fluid and ρ = const. and μ = const

$$\vec{\nabla} \cdot \vec{v} = 0$$

(general: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$)