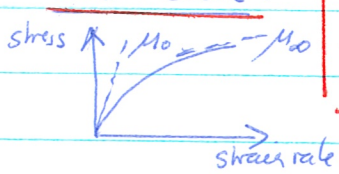
Recap (Lecture 11/13/2019)

Empirical models for $\mu(\dot{\Gamma})$:

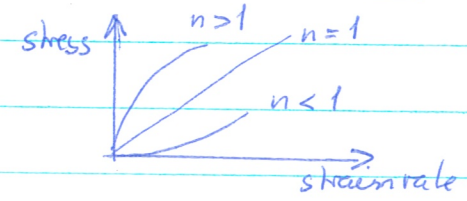
Carreau:



$$\frac{\mu(\dot{\Gamma}) - \mu_\infty}{\mu_0 - \mu_\infty} = \left[1 + (2\lambda\dot{\Gamma})^2\right]^{\frac{n-1}{2}}$$

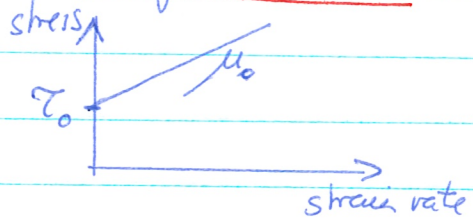
Power-Law Model:

$$\mu(\dot{\Gamma}) = m(2\dot{\Gamma})^{n-1}$$



$n = 1$ Newtonian
$n < 1$ pseudoplastic (shear thinning)
$n > 1$ shear thickening

Bingham Model:



$$\mu = \begin{cases} \infty & \text{for } \tau \le \tau_0 \\ \mu_0 + \dfrac{\tau_0}{2\dot{\Gamma}} & \text{for } \tau > \tau_0 \end{cases}$$

Drag force in flow direction $\vec{e}_z$

$$F_D = \vec{e}_z\left(\vec{F}_p + \vec{F}_\tau\right) = -\int_S p\vec{n}\cdot\vec{e}_z\,dS + \int_S \vec{n}\cdot\underline{\underline{\tau}}\cdot\vec{e}_z\,dS$$

$\vec{F}_p$: dynamic pressure force; $\vec{F}_\tau$: viscous force

$\int_S p\vec{n}\cdot\vec{e}_z\,dS$: form drag; $\int_S \vec{n}\cdot\underline{\underline{\tau}}\cdot\vec{e}_z\,dS$: friction drag

Form Drag coefficient:

$$C_D = \frac{F_p}{A_p\left(\frac{1}{2}\rho v^2\right)}$$

Projected area in flow direction
cyl.: $A_p = 2RL$

Recap cont.

Stress Balance at Interfaces with line tension

γ ... surface tension; $\mathcal{H}$ mean curvature

$$\underbrace{\vec{s}(\vec{n})\big|_2 - \vec{s}(\vec{n})\big|_1}_{\text{stress difference at either side}} = \underbrace{-\vec{\nabla}_s \gamma}_{\text{stress due to tension}} + \underbrace{2\mathcal{H}\vec{n}\gamma}_{\text{geometric (curvature) stress}}$$

$\vec{s}(\vec{n}) = \vec{n}\cdot\underline{\underline{\sigma}} = \vec{n}\cdot(-p\underline{\underline{S}} + \underline{\underline{\tau}})$

$\vec{\nabla}_s \gamma = 0$ for γ uniform

$\mathcal{H} = 0$ for planar interface

$\mathcal{H} = \frac{-1}{2R}$ cyl. int.

$\mathcal{H} = -\frac{1}{R}$ sph. int.

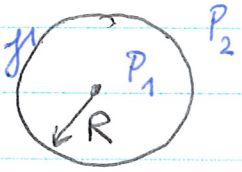
normal stress component:

$$p_1 - p_2 = \tau_{nn}\big|_1 - \tau_{nn}\big|_2 - 2\mathcal{H}\gamma$$

tangential stress component: $\tau_{nt}\big|_2 - \tau_{nt}\big|_1 = -\vec{t}\cdot\vec{\nabla}_s\gamma$

Young-Laplace Eq.

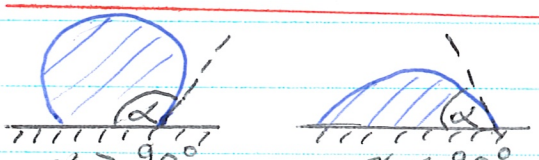
$$p_1 - p_2 = \frac{2\gamma}{R}$$



Young Equation

$$\gamma_{LG}\cos\alpha + \gamma_{SL} = \gamma_{SG}$$

$\gamma \equiv \gamma_{LG}$ (surface tension of liquid)



$\alpha > 90°$ poorly wetting

$\alpha < 90°$ well wetting

water: hydrophobic hydrophilic