Recap Nov. 21

Bidirectional Flow

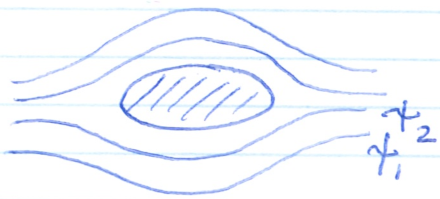
$\vec{v} = (v_x, v_y, 0)$ or $\vec{v}(v_r, v_\theta, 0)$

$\psi(x, y)$ or $\psi(r, \theta)$

Def. Streamline :

$$v_x = \frac{\partial \psi}{\partial y}, \quad v_y = -\frac{\partial \psi}{\partial x} \quad (\text{cart.})$$

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}; \quad v_\theta = -\frac{\partial \psi}{\partial r} \quad (\text{polar})$$



$\psi(x, y) = \text{const}$ defines a distinct streamline (e.g. ψ_1)

Planar flow : $\boxed{w_z = -\nabla^2 \psi}$; planar and irrot. $\Rightarrow \boxed{\nabla^2 \psi = 0}$

(Laplace equation)

Axisymm. flow: $w_\theta = -\frac{1}{r} E^2 \psi$ (cyl.) (r, z)

$$w_\phi = -\frac{1}{r \sin\theta} E^2 \psi \quad (\text{sph. } r, \theta)$$

axisymm and irrot. $\Rightarrow \boxed{E^2 \psi = 0}$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2} \; (\text{cart.}); \quad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \; (\text{polar})$$

$$E^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\right)$$

Note: Irrotational flow is pseudosteady $\Leftrightarrow$ $\boxed{\begin{array}{c} \nabla^2 \psi = 0 \\ E^2 \psi = 0 \end{array}}$

applicable to steady and unsteady flow.

Navier Stokes Eq for Bidirectional Flow

$$\frac{\partial}{\partial t}(\nabla^2 \psi) - \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, y)} = \nu \nabla^4 \psi \quad (\text{planar flow, cart. coord})$$

see Table 6-12 : for cyl. coord and axisymm. flow

subst $w_z = -\nabla^2 \psi \Rightarrow \dfrac{Dw_z}{Dt} = \nu \nabla^2 w_z$

Planar and irrotational flow governed by Laplace Eq.

$\boxed{\nabla^2 \psi = 0}$