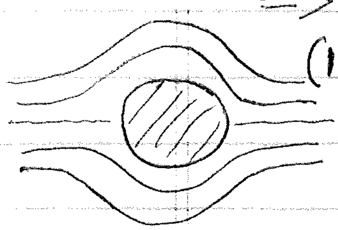
Recap

Potential Flow around Cylinder (crossflow)

Pressure coeff.: $C_p(\theta) \equiv \dfrac{\Delta P}{\frac{1}{2}\rho U^2} = \underbrace{(1-4\sin^2\theta)}$

$\Rightarrow$ unrealistic pressure drag:

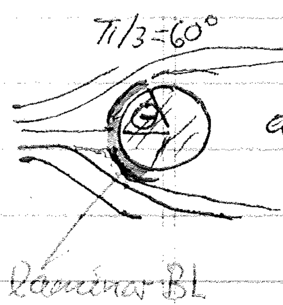


(1) Pressure Drag

symm.

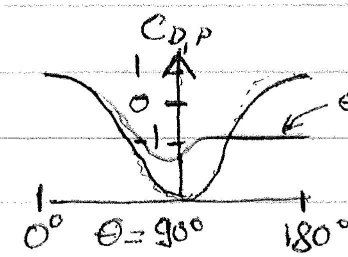
$$C_{D,P} = \int_0^{\pi} C_p \cos\theta \, d\theta = 0$$

Fit based on experiments (laminar BL)



asymm.

$$C_{D,P} = \begin{cases} 1 - \frac{8}{3}\sin^2\theta & ; \; 0 \le \theta \le \frac{\pi}{3} \\ -1 & ; \; \frac{\pi}{3} \le \theta \le \pi \end{cases}$$



empirical with laminar BL

from HW6: $C_{D,P} = \dfrac{2}{\sqrt{3}}$ (laminar)

(2) Skin Friction Drag: $C_{D,f} = \displaystyle\int_0^{\theta_s} C_f \sin\theta \, d\theta$

$$C_f \equiv \frac{\tau_{wall}}{\frac{1}{2}\rho U^2} = f(\theta, u, Re_D) \qquad Re_D = \frac{UL}{\nu}$$

u velocity at "interface" between BL and potential flow regime.

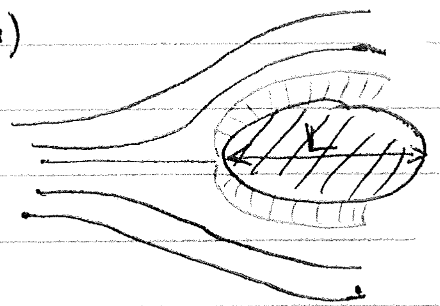
(3) Total Drag: $C_D = C_{D,f} + \underbrace{C_{D,P}}_{\text{laminar + correction}}$

$$\cong \frac{5.786}{\sqrt{Re_D}} + \frac{2}{\sqrt{3}} + \frac{1.26}{Re_D}$$

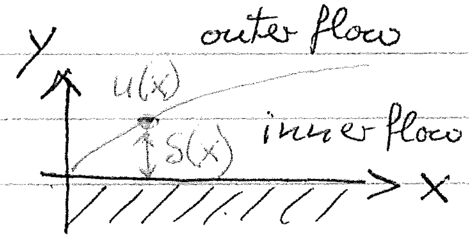
Recap cont

Flow around object $\begin{cases} \text{outer potential flow region} \\ \text{inner boundary layer (BL) flow region} \end{cases}$

(a)



(b)



asymptotic velocity: $u(x) = \begin{cases} \lim\limits_{y \to 0} v_x^o(x,y) \\ \lim\limits_{y \to \infty} v_x^i(x,y) \end{cases}$

local boundary layer thickness: $\delta(x)$

Assume: $\mu = const$, $\rho = const$:

<u>Navier Stokes Eq.</u> (within BL regime)

(a) <u>Prandtl Eq.</u> (Eq. 9.2-17) valid for $\delta(x) \ll L$, $v_x >> v_y$

$$\underbrace{v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y}}_{\substack{\text{advection terms} \\ \text{(inertial terms)}}} = \underbrace{-\frac{1}{\rho}\frac{d\mathcal{P}}{dx}}_{\substack{\text{pressure} \\ \text{term}}} + \underbrace{\nu \frac{\partial^2 v_x}{\partial y^2}}_{\substack{\text{viscous} \\ \text{term}}}$$

also fullfilled has to be the <u>continuity</u> eq. (Eq. 9.2-16)

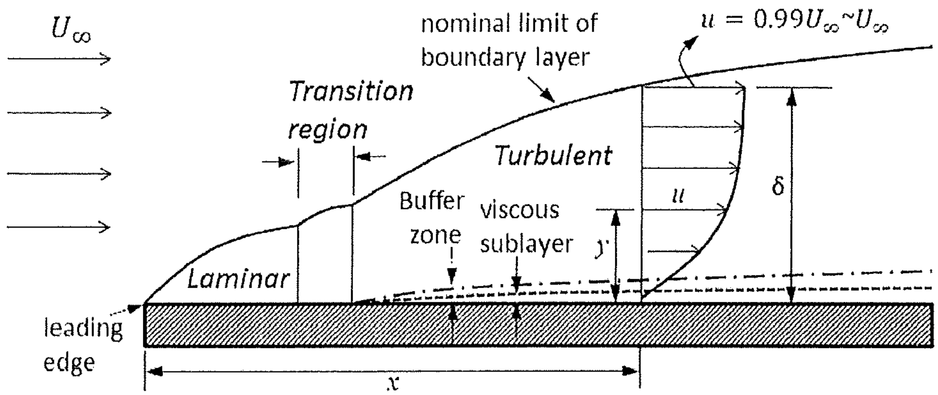
$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

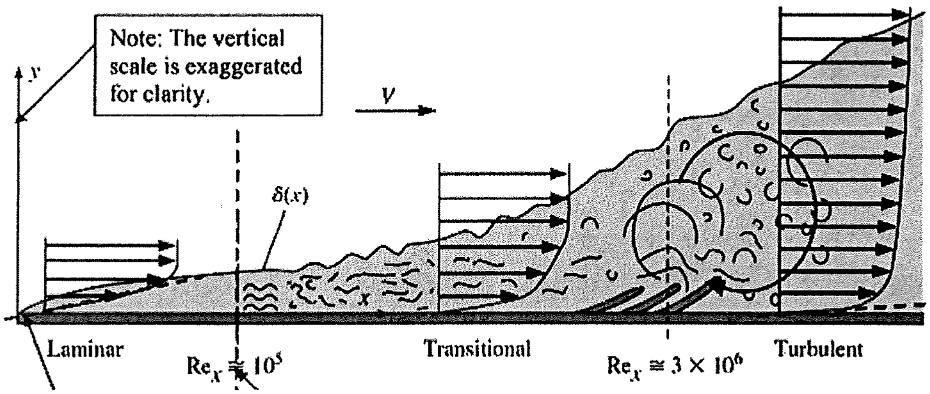
(b) planar surface ($\delta(x) \ll L$): Blasius Eq. (Eq. 9.4-1)
ignore pressure term

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

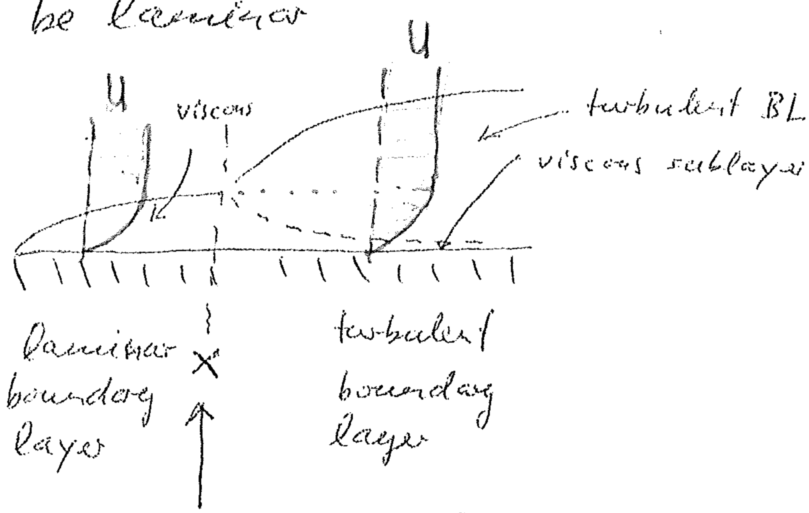
Boundary Layer

1. Flat Plat





It is important to note that the assumption of a very thin boundary layer, requires the flow to be laminar



$Re_x \approx 5 \times 10^5$; $Re_x = \frac{Ux}{\nu}$