Conservation Equations

Recap: Lecture 10/8/2018

Conservation Eq. for Interfaces

Eq. 2.2-15 $$\Big[\underbrace{(F - b\vec{v}_I)_B}_{} - \underbrace{(\vec{F} - b\vec{v}_I)_A}_{}\Big]\cdot\vec{n}_I = B_s$$

net fluxes of A and B relative to interface $\vec{n}_I$

rate of generation

Convective and Diffusive Flux Equations

Eq. 2.2-16 $$\vec{F} = b\vec{v} + \vec{f}$$

Flux relationship
$\vec{F}$ total flux
$\vec{f}$ diffusive (molecular) flux
$b\vec{v}$ convective (bulk motion) flux

(i) Combined with Conservation Eq. for Interior Points

$$\frac{\partial b}{\partial t} = -\vec{\nabla}\cdot\vec{F} + B_V$$

leads to:

Eq. 2.2-17 $$\frac{\partial b}{\partial t} + \vec{\nabla}\cdot(b\vec{v}) = -\vec{\nabla}\cdot\vec{f} + B_V$$ (interior point)

(ii) Combined with Conservation Eq. for Interfaces (see above Eq. 2.2-15)

Eq. 2.2-18 $$\Big[\big(\vec{f} + b(\vec{v} - \vec{v}_I)\big)_B - \big(\vec{f} + b(v - v_I)\big)_A\Big]\cdot\vec{n}_I = B_S$$

Conservation of Mass - Continuity

Eq. 2.3-1 $$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \quad ;$$

for $\rho = \text{const.}$
$\vec{\nabla}\cdot\vec{v} = 0$

Conservation Eq at Constant Density

Eq. 2.3-8 $$\frac{Db}{Dt} = -\vec{\nabla}\cdot\vec{f} + B_V \quad ;$$

Subst. Derivative
$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{V}\cdot\vec{\nabla}$$