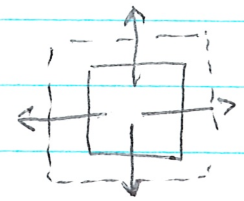
Recap

Deformation in Fluids



- Dilatation (volume changing deformation)

  It preserves "shape" but not "volume"

  The rate of strain from dilatation process is:

$$\boxed{\frac{|\dot{\Delta r}|}{\Delta r} = \frac{1}{3}\left(\vec{\nabla}\cdot\vec{v}\right)} \qquad \text{Eq. 6.4-14}$$

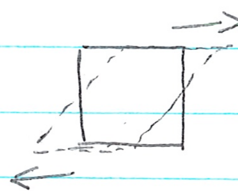
in matrix form: $\left(\frac{|\dot{\Delta r}|}{\Delta r}\right) = \frac{1}{3}\left(\vec{\nabla}\cdot\vec{v}\right)\underline{\underline{\delta}}$

$$= \frac{1}{3}\begin{pmatrix} \vec{\nabla}\cdot\vec{v} & & 0 \\ & \vec{\nabla}\cdot\vec{v} & \\ 0 & & \vec{\nabla}\cdot\vec{v} \end{pmatrix}$$

Shear

- Deformation (shape changing deformation)

  It preserves "volume" but not "shape"



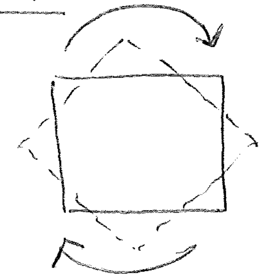
  Difference between total deformation $\underline{\underline{\Gamma}}$ and dilatation ε_{ii}

$$\underline{\underline{\Gamma}} - \frac{1}{3}\left(\vec{\nabla}\cdot\vec{v}\right)\underline{\underline{\delta}} = \begin{pmatrix} \frac{\partial v_1}{\partial x_1} - \frac{1}{3}\left(\vec{\nabla}\cdot\vec{v}\right) & & \\ & \frac{\partial v_2}{\partial x_2} - \frac{1}{3}\left(\vec{\nabla}\cdot\vec{v}\right) & \\ & & \frac{\partial v_3}{\partial x_3} - \frac{1}{3}\left(\vec{\nabla}\cdot\vec{v}\right) \end{pmatrix}$$

$$\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right)$$

Recap cont.

Other stress-induced fluid motion



– Rigid Body Rotation

The rigid body rotation is fully expressed by the vorticity tensor

$$\underline{\underline{\Omega}} = \frac{1}{2}\left[\underline{\underline{\nabla V}} - (\underline{\underline{\nabla V}})^t\right] = \frac{1}{2}\begin{pmatrix} 0 & w_z & -w_y \\ -w_z & 0 & w_x \\ w_y & -w_x & 0 \end{pmatrix}$$

i.e., the antisymmetric component of the velocity gradient $\underline{\underline{\nabla V}}$.

w_x, w_y, w_z are the component of the vorticity vector $\vec{w}(w_x, w_y, w_z) = \vec{\nabla} \times \vec{V}$ given as

$$w_x = \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}$$

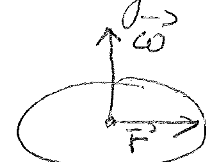
$$w_y = \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}$$

$$w_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial z}$$

Note:

$\vec{w} = 2\vec{\omega}$

$\vec{\omega}$ angular velocity



It turns out that rate of strain of the rigid body rotation is zero.

Recap cont.

Constitutive Equation for viscous Stress

We search for relationships between stress and strainrate, i.e., $\underline{\underline{\tau}} = f(\underline{\underline{\Gamma}})$

Assume viscous stresses are linearly related to the two strain rate components for (a) dilatation, and (b) shape changing shear deformation

$\Rightarrow$ Newtonian Fluid (Constitutive Eq.)

$$\boxed{\underline{\underline{\tau}} = \underbrace{2\mu}_{\text{shear viscosity}} \underbrace{\Big[\underbrace{\underline{\underline{\Gamma}}}_{\text{total deformation}} - \frac{1}{3}\underbrace{(\vec{\nabla}\cdot\vec{V})}_{\text{volume change}}\underline{\underline{\delta}}\Big]}_{\text{shape changing shear deformation}} + \underbrace{3K}_{\substack{\text{dilatational viscosity}\\\text{("bulk viscosity")}}} \underbrace{\Big[\frac{1}{3}(\vec{\nabla}\cdot\vec{V})\underline{\underline{\delta}}\Big]}_{\text{dilatation}}}$$

$$\left.\begin{aligned} \mu &= f(T,P) \\ K &= g(T,P) \end{aligned}\right\} \text{ but not dependent on } |\underline{\underline{\Gamma}}| \text{ or } \vec{\nabla}\cdot\vec{V}$$

As $K \ll \mu$ $\Rightarrow$ $\boxed{\underline{\underline{\tau}} \approx 2\mu\Big[\underline{\underline{\Gamma}} - \frac{1}{3}(\vec{\nabla}\cdot\vec{V})\underline{\underline{\delta}}\Big]}$

for $\rho = \text{const}$ (incompressible fluid) $\Rightarrow$ $\vec{\nabla}\cdot\vec{V} = 0$ (continuity eq.)

$\Rightarrow$ $\boxed{\underline{\underline{\tau}} \cong 2\mu\underline{\underline{\Gamma}}}$ Newtonian fluid for $K \to 0$ and $\rho = \text{const}$ (incompressible)

Recap cont.

Navier - Stokes Eq. :

<u>Conservation of Momentum for Newtonian incompressible fluids</u>

Subst. in Cauchy Eq.

$$\rho \frac{D\vec{V}}{Dt} = -\vec{\nabla} P + \rho \vec{g} + \vec{\nabla} \cdot \underline{\underline{\tau}}$$

for $\underline{\underline{\tau}} = 2\mu \underline{\underline{\Gamma}} = \mu \left[\vec{\nabla}\vec{V} + (\vec{\nabla}\vec{V})^{t} \right]$

$$\Rightarrow \boxed{\rho \frac{D\vec{V}}{Dt} = -\vec{\nabla} P + \rho \vec{g} + \mu \nabla^2 \vec{V}}$$

∇^2... Laplace Operator

Navier - Stokes Eq.