Recap : High Reynolds Number Bi-Directional Flow

Stream fct : $\psi(x,y)$ $\quad v_x = \frac{\partial \psi}{\partial y}$; $v_y = -\frac{\partial \psi}{\partial x}$

Velocity potential : (in 2D) $\phi(x,y)$ $\quad v_x = \frac{\partial \phi}{\partial x}$; $v_y = \frac{\partial \phi}{\partial y}$

} in rect. coord.

$$\Rightarrow \quad \boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \; ; \; \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}}$$

equivalent to Cauchy-Riemann equations

Irrotational flow is equivalent to potential flow, governed by the Laplace equations of the stream fct and velocity potential

$$\boxed{\begin{aligned} \nabla^2 \phi &= 0 \\ \nabla^2 \psi &= 0 \end{aligned}}$$

(Note most generally ϕ is defined via : $\vec{V} = \vec{\nabla}^2 \phi$)

Equivalent to these eqs is the Bernoulli eq.

$$\frac{\Delta P}{\rho} + \frac{1}{2}\left(v_1^2 - v_2^2\right) + g\left(h_1 - h_2\right) = 0 \; ; \; \Delta P = P_1 - P_2$$

Def : Streamline : $\psi(x,y) = c$

Equipot. line : $\phi(x,y) = d$

c, d are constants

Streamlines and equipot. lines are perpendicular.

Recap cont.

We discussed irrotat. flow past a cylinder and considered

$$\boxed{\nabla^2 \phi = 0} \quad \text{Governing Eq.}$$

resulting in

$$\boxed{\begin{aligned} v_r(r,\theta) &= U\cos\theta\left[1-\left(\frac{R}{r}\right)^2\right] \\ v_\theta(r,\theta) &= -U\sin\theta\left[1+\left(\frac{R}{r}\right)^2\right] \end{aligned}}$$

$\Rightarrow$ a non-zero tangential velocity at cylinder surface

$$v_\theta(R,\theta) = -2U\sin\theta$$

Bernoulli eq. yielded the dyn. pressure

$$\boxed{P(R,\theta) = \frac{\rho U^2}{2}\left[1-4\sin^2\theta\right]}$$

at cyl. surface