Tensor Rank/order 3^n | n is the rank describes the # of elements in tensor

Scalar = 0 e.g. T, ρ → magnitude

Vector = 1 e.g $\vec{v}, \vec{E}$ → magnitude/direction

matrix = 2 e.g stress, flux → magnitude/direction/orientation
↖ also called tensor

> Determine rank/order
> 1) sum rank/order of all components
> 2) for every dot product sub. 2
> 3) for every cross product sub 1

Various Representations

Vector: $\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \vec{V} = \underline{V} = \underbrace{\sum_i^3 V_i \vec{\delta}_i}_{\text{Einstein Notation}}$ ← basis vector, $\vec{\delta}_i$

tensor: $\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \underline{\underline{\sigma}} = \sum_i^3 \sum_j^3 \sigma_{ij} \vec{\delta}_i \vec{\delta}_j$

★ # of basis vectors $\vec{\delta}_n$ represents rank/order

Kronecker Delta

$\vec{\delta}_i \cdot \vec{\delta}_j = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$ used to eliminate index of dot product when $\vec{\delta}_i, \vec{\delta}_j$ are orthogonal unit vectors

$\vec{a} \cdot \vec{b} = \sum_i \sum_j a_i b_j (\vec{\delta}_i \cdot \vec{\delta}_j) = \sum_i \sum_j a_i b_j \delta_{ij} = \sum_i a_i b_i$ $\delta_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Gradient/Del operator: $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$ in cartesian coordinates

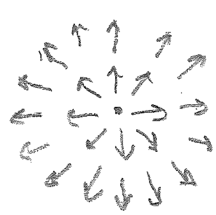
$\vec{\nabla}$ of scalar → vector $\vec{\nabla}$ of vector → dyad (Tensor)

Scalar field

```
   2 2 2
2 1 1 1 2
2 1 0 1 2
2 1 1 1 2
   2 2 2
```

$\xrightarrow{\vec{\nabla}}$

Vector field



$\xrightarrow{\vec{\nabla}}$

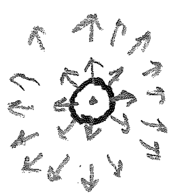
Dyad (e.g $\vec{\nabla}\vec{v}$)

$$\begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

Divergence: Dot product between $\vec{\nabla}$ and vector field, $\vec{A}$

$\vec{\nabla} \cdot \vec{A}$ → in (x, y, z) $\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

– physically represents the density of a vector quantity and determines the flux



- if we imagine a circle in a vector field
  ↳ flow appears to be emerging from center

★ More operators will be introduced later
See Coordinate transform Eqn Sheet on Website