Stress Balance:

normal component: $P_1 - P_2 = -\tau_{nn}|_2 + \tau_{nn}|_1 - 2\mathcal{H}\gamma$

tangential component: $\tau_{nt}|_2 - \tau_{nt}|_1 = -\vec{t}\cdot\vec{\nabla}_s\,\gamma$

TABLE A-5 (p. 636)

Unit Normal, Surface Gradient, and Mean Curvature[a]

Surface given by $z = F(x,y)$

$$\mathbf{n} = \frac{1}{H}\left(-\frac{\partial F}{\partial x}\mathbf{e}_x - \frac{\partial F}{\partial y}\mathbf{e}_y + \mathbf{e}_z\right)$$

$$H = \left[\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + 1\right]^{1/2}$$

$$\nabla_s = \frac{1}{H^2}\left\{\left[\left(\frac{\partial F}{\partial y}\right)^2 + 1\right]\mathbf{e}_x - \frac{\partial F}{\partial x}\frac{\partial F}{\partial y}\mathbf{e}_y + \frac{\partial F}{\partial x}\mathbf{e}_z\right]\frac{\partial}{\partial x}$$
$$+ \frac{1}{H^2}\left[-\frac{\partial F}{\partial x}\frac{\partial F}{\partial y}\mathbf{e}_x + \left[\left(\frac{\partial F}{\partial x}\right)^2 + 1\right]\mathbf{e}_y + \frac{\partial F}{\partial y}\mathbf{e}_z\right]\frac{\partial}{\partial y}$$
$$+ \frac{1}{H^2}\left[\frac{\partial F}{\partial x}\mathbf{e}_x + \frac{\partial F}{\partial y}\mathbf{e}_y + \left[\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2\right]\mathbf{e}_z\right]\frac{\partial}{\partial z}$$

$$2\mathcal{H} = \frac{1}{H^3}\left\{\left[\left(\frac{\partial F}{\partial y}\right)^2 + 1\right]\frac{\partial^2 F}{\partial x^2} - 2\frac{\partial F}{\partial x}\frac{\partial F}{\partial y}\frac{\partial^2 F}{\partial x \partial y}\right.$$
$$\left. + \left[\left(\frac{\partial F}{\partial x}\right)^2 + 1\right]\frac{\partial^2 F}{\partial y^2}\right]$$

Surface given by $z = F(r)$

$$\mathbf{n} = \frac{1}{H}\left(-\frac{dF}{dr}\mathbf{e}_r + \mathbf{e}_z\right)$$

$$H = \left[\left(\frac{dF}{dr}\right)^2 + 1\right]^{1/2}$$

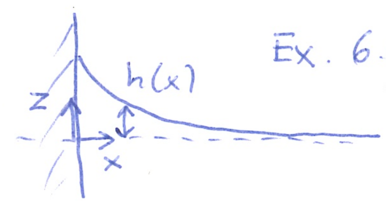
$$\nabla_s = \frac{1}{H^2}\left(\mathbf{e}_r + \frac{dF}{dr}\mathbf{e}_z\right)\frac{\partial}{\partial r} + \left(\frac{1}{r}\mathbf{e}_\theta\right)\frac{\partial}{\partial \theta}$$
$$+ \frac{1}{H^2}\frac{dF}{dr}\left(\mathbf{e}_r + \frac{dF}{dr}\mathbf{e}_z\right)\frac{\partial}{\partial z}$$

$$2\mathcal{H} = \frac{1}{H^3}\left(\frac{d^2F}{dr^2} + \frac{H^2}{r}\frac{dF}{dr}\right)$$

[a] In these expressions the unit normal $\mathbf{n}$ points toward positive z. Reversing the direction of $\mathbf{n}$ will change the sign of $\mathcal{H}$.

Example: $F(x,y) \rightarrow h(x)$

$z = h(x)$



Ex. 6.6-2: Wetted wall

from Table A-5:

$$\vec{n} = \frac{1}{H}\begin{pmatrix} -\frac{\partial h}{\partial x} \\ 0 \\ 1 \end{pmatrix}; \quad H = \left[\left(\frac{\partial h}{\partial x}\right)^2 + 1\right]^{\frac{1}{2}}$$

$$\vec{\nabla}_s = \frac{1}{H^2}\begin{pmatrix} \frac{\partial}{\partial x} + \frac{\partial h}{\partial x}\frac{\partial}{\partial z} \\ \left\{\left(\frac{\partial h}{\partial x}\right)^2 + 1\right\}\frac{\partial}{\partial y} \\ \frac{\partial h}{\partial x}\frac{\partial}{\partial x} + \left(\frac{\partial h}{\partial x}\right)^2\frac{\partial}{\partial z} \end{pmatrix}$$

$$2\mathcal{H} = \frac{\left(\frac{\partial h}{\partial x}\right)^2}{H^3} = \frac{\frac{d^2h}{dx^2}}{\left[1 + \left(\frac{dh}{dx}\right)^2\right]^{3/2}}$$

Eq 6.6-16