(found in W.M. Deen, Analysis of Transport Phenomena)

Fluid Properties	**Continuity Eq.**	**Equation of Momentum**
general stress vs. strain rate relationship: $\underline{\underline{\tau}}\left(\underline{\underline{\Gamma}}\right)$ $\underline{\underline{\Gamma}} = \frac{1}{2}[\vec{\nabla}\vec{v} + (\vec{\nabla}\vec{v})^t]$	$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$ (2.3-1)	Cauchy's Momentum Eq. $\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} - \vec{\nabla} P + \vec{\nabla} \cdot \underline{\underline{\tau}}$ (6.3-10)
assume the following stress vs. strain-rate relationship: $\underline{\underline{\tau}} = 2\mu(\Gamma)\underline{\underline{\Gamma}}$ $\underline{\underline{\Gamma}} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right)_{ij}$ 9 element symm. tensor Static Fluid: $v = 0; \quad \underline{\underline{\tau}} = 0$ Fully Developed Creeping Flow (Re « 1)		Generalized Momentum Eq. for Newtonian and non-Newtonian Fluids $\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} - \vec{\nabla} P + \vec{\nabla} \cdot 2\mu(\Gamma)\underline{\underline{\Gamma}}$ (6.5-16) magnitude of the strain-rate tensor $\Gamma \equiv \left[\frac{1}{2}\left(\underline{\underline{\Gamma}} : \underline{\underline{\Gamma}}\right)\right]^{1/2}$ (6.5-12) dissipation function $\Phi = \left(2\Gamma\right)^2$ Table 6-10 Static Pressure Eq. $\vec{\nabla} P = \rho \vec{g}$ (6.3-11) Stokes Equation $\vec{\nabla} P + \rho \vec{g} = \mu \nabla^2 \vec{v}$ (8.2-1) dynamic pressure: $\vec{\mathcal{P}} = \vec{\nabla} P - \rho \vec{g}$
Newtonian: stress vs. strain-rate relationship $\underline{\underline{\tau}} = 2\mu\underline{\underline{\Gamma}}$ neglected molecular resistance to isotropic expansion or compression, $\kappa << \mu$ assumed μ = constant, and incompressibility (ρ = const.)	$\vec{\nabla} \cdot \vec{v} = \mathrm{div}\vec{v} = 0$	Navier-Stokes Representations: $\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v}\right) = \rho \vec{g} - \vec{\nabla} P + \mu \nabla^2 \vec{v}$ (6.5-10) Vorticity Transport Equation (equivalent to Navier-Stokes) $\frac{D\vec{w}}{Dt} = \vec{w} \cdot \vec{\nabla}\vec{v} + \frac{\mu}{\rho}\nabla^2 \vec{w}$ (6.8-7) for inviscid flow ($\mu = 0$ or Re $\to \infty$) $\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} + \frac{1}{\rho}\vec{\nabla} P - \vec{g} = 0$ (Euler Eq.) (9.2-2) Gromeka-Lamb Equation (equivalent to Euler) $\frac{\partial \vec{v}}{\partial t} + \vec{w} \times \vec{v} = \vec{g} - \vec{\nabla}\left(\frac{P}{\rho} + \frac{v^2}{2}\right) - \nu(\vec{\nabla} \times \vec{w})$
Planar Flow (equivalent to Navier-Stokes in 2D)		Stream Function Equation $\frac{\partial}{\partial t}(\nabla^2 \psi) - \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, y)} = \nu \nabla^4 \psi$ Table (6-12) Jacobian determinant $\frac{\partial(f, g)}{\partial(x, y)} = \begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix}$

cont. **Fluid Properties**	**Continuity Equation**	**Equation of Momentum**
2D Boundary Layer Equation $v_x(x,y) >> v_y$, $dP/dx=f(x)$ neglect derivatives involving v_y	$\frac{\partial}{\partial x}v_x + \frac{\partial}{\partial y}v_y = 0$	$\rho\left(v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y}\right) = -\frac{dP}{dx} + \mu\frac{\partial^2}{\partial y^2}v_x$
for known pressure gradient from the outer inviscid flow region (any geometry)		Prandtl Equation: $v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} - u\frac{du}{dx} = \upsilon\frac{\partial^2}{\partial y^2}v_x$ (9.4-1)
restriction to planar geometry (pressure gradient vanishes)		Blasius solution for laminar boundary layer flow over a flat plate: Prandtl-Blasius eq. $v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} = \upsilon\frac{\partial^2}{\partial y^2}v_x$ (9.2-25) or with dimensionless stream functions (to reduce variables): $\frac{\partial\hat\psi}{\partial\hat y}\frac{\partial^2\hat\psi}{\partial\hat x\partial\hat y} - \frac{\partial\hat\psi}{\partial\hat x}\frac{\partial^2\hat\psi}{\partial\hat y^2} = \upsilon\frac{\partial^3\hat\psi}{\partial\hat y^3}$ (9.4-4)
Lubrication Approx.(2D flow) $v_x(x,y) >> v_y$, $dP/dx=f(x)$; neglect v_y		$\frac{\partial^2}{\partial y^2}v_x = \frac{1}{\mu}\frac{dP}{dx}$
steady state, inviscid, incompressible constant density	$\vec\nabla\cdot\vec v = \mathrm{div}\vec v = 0$	Bernoulli Equation (true along a given streamline) $\frac{v_2^2 - v_1^2}{2} + \int_1^2\frac{dp}{\rho} + g(h_2 - h_1) = 0$ (9.2-6) Differential Form of Bernoulli equation: Lamb's Equation (steady state and inviscid Gromeka-Lamb Eq.) $\vec w\times\vec v = -\vec\nabla\left(\frac{P}{\rho} + \frac{v^2}{2} + \phi_g\right)$ gravitational potential: $\vec\nabla\phi_g \equiv \vec g$
irrotational, steady state, inviscid, incompressible, constant density		Bernoulli Equation (true everywhere in the liquid) $\frac{v_2^2 - v_1^2}{2} + \frac{P_2 - P_1}{\rho} + g(h_2 - h_1) = 0$ (9.2-13)