Recap: Thermal Energy Conservation ( )

Thermal Energy Conservation Eq (ρ const.):

Eq. 2.4-1: $\rho C_p \dfrac{DT}{Dt} = -\vec{\nabla}\cdot q + H_V$ (const. ρ and C_p)

with Fourier:

Eq. 2.4-3: $\rho C_p \dfrac{DT}{Dt} = k\, \nabla^2 T + H_V$ (const. ρ, C_p, k)

or $\dfrac{DT}{Dt} = \alpha\, \nabla^2 T + \dfrac{H_V}{\rho C_p}$; $\alpha \equiv \dfrac{k}{\rho C_p}$

At Interfaces: normal flux: $q_n|_{1,2}$; phases 1 and 2 separated by interface

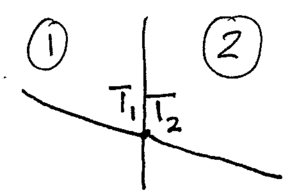
Eq. 2.5-1 $q_n(\vec{r}_s, t)|_2 - q_n(\vec{r}_s, t)|_1 = H_s(\vec{r}_s, t)$

($\vec{r}_s$: interfacial location)

usual situation $H_s = 0$

$q_n|_1 = q_n|_2$ heat flux continuous at interface

Zero resistance interface: $T_1 = T_2$

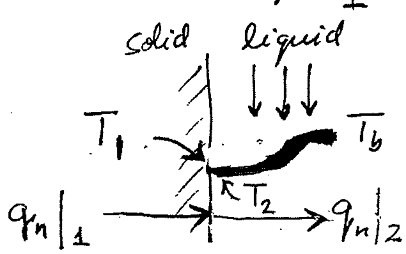


(applicable to liquid/solid interface)
(not applicable to solid/solid interface)

Convection Boundary Condition

Eq. 2.5-5: $q_n|_1 \equiv h\,(T_2 - T_b)$

- h: convection heat transfer coeff.
- T_2: interfacial temp. of liquid
- T_b: bulk liquid temp.



in solid: $q_n|_1 = -k_1(\vec{n}\cdot\vec{\nabla}T)_1$

$q_n|_1 = q_n|_2 \implies -k_1(\vec{n}\cdot\vec{\nabla}T)_1 = h\,(T_2 - T_b)$

Phase Change

mass conservation: $\rho_1 \left(v_n|_1 - v_{In} \right) = \rho_2 \left(v_n|_2 - v_{In} \right)$ Eq. 2.5-10

energy conservation: $\rho_1 \hat{H}_1 \left(v_n|_1 - v_{In} \right) + q_n|_1 =$

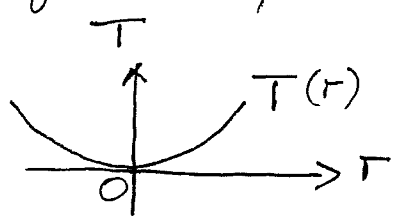
$$= \underbrace{\rho_2 \hat{H}_2 \left(v_n|_2 - v_{In} \right)}_{\text{convective contribution}} + q_n|_2 \qquad \text{Eq. 2.5-11}$$

See textbook for combined Eq. 2.5-12

Symmetry

At symmetry planes or symmetry points the normal fluxes vanish, i.e. $F_n = 0$

Ex: Axisymmetric point in Heat Transfer



as $q_r = \begin{cases} -k \dfrac{\partial T}{\partial r} & ; \text{ Fourier} \\ 0 & ; \text{ Symmetry} \end{cases}$

$$\Longrightarrow \left. \frac{\partial T}{\partial r} \right|_{r=0} = 0$$