Special Forms of Navier Stokes Eq.

$$\boxed{\rho \frac{D\vec{v}}{Dt} = \rho\vec{g} - \vec{\nabla}P + \mu\nabla^2\vec{v}}$$

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}$$

steady state: $\rho\vec{v}\cdot\vec{\nabla}\vec{v} = \rho\vec{g} - \vec{\nabla}P + \mu\nabla^2\vec{v}$

for $Re = \frac{UL}{\nu} \rightarrow 0$

$\vec{v}\cdot\vec{\nabla}\vec{v} \rightarrow 0$

inertial effects vanish

steady state and $Re \rightarrow 0$: $\underbrace{\vec{\nabla}P - \rho\vec{g}}$ $= \mu\nabla^2\vec{v}$

$$\boxed{\vec{\nabla}\mathcal{P} = \mu\nabla^2\vec{v}}$$ Stokes Eq. Eq. 8.2-1 or (Eq. 6.9-11 in dimensionless form)

$Re \rightarrow \infty$ inviscid fluid $\mu = 0$ $\}$

$$\boxed{\rho\frac{D\vec{v}}{Dt} = \rho\vec{g} - \vec{\nabla}P}$$ Eq. 9.2-1

Euler Eq. $\Leftrightarrow$ Gromeka-Lamb eq.

$\Leftrightarrow \frac{\partial\vec{v}}{\partial t} + \vec{w}\times\vec{v} = \vec{g} - \vec{\nabla}\left(\frac{P}{\rho} + \frac{v^2}{2}\right) - \frac{\mu}{\rho}(\vec{\nabla}\times i$

and steady state $\}$ $\rho\vec{v}\cdot\vec{\nabla}\vec{v} = -\vec{\nabla}\mathcal{P}$ Eq. 9.2-2

static fluid $\vec{v} = 0$

$$\boxed{\vec{\nabla}P = \rho\vec{g}}$$ Static Pressure Eq Eq. 6.3-11

Equivalent to Navier Stokes is the **Vorticity Transport Eq**

$$\boxed{\frac{D\vec{w}}{Dt} = \vec{w}\cdot\vec{\nabla}\vec{v} + \nu\nabla^2\vec{w}}, \quad \vec{w} = \vec{\nabla}\times\vec{v}$$

$$\frac{D\vec{w}}{Dt} = \frac{\partial\vec{w}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{w}$$

steady state and inviscid $\boxed{\vec{v}\cdot\vec{\nabla}\vec{w} = \vec{w}\cdot\vec{\nabla}\vec{v}}$ Eq. 9.2-11

inviscid and irrotation fluid steady state

from Gromeka-Lamb: $g - \vec{\nabla}\left(\frac{P}{\rho} + \frac{v^2}{2}\right) = 0$ Bernoulli Eq. Eq. 9.2-13

between 2Pts: $\overset{\text{integrale}}{\Longleftrightarrow}$ $\left|\frac{v_1^2 - v_2^2}{2} + \frac{P_1 - P_2}{\rho} + g(h_1 - h_2) = 0\right|$