Chapter 5

Quarks and hadrons

Every atom has its ground state — the lowest energy state of its electrons in the presence of the atomic nucleus — as well as many excited states which can decay to the ground state via emission of photons. Nuclei composed of multiple protons and neutrons also have their ground state plus various excited nuclear energy levels, which typically also decay via emission of photons (or in some cases, α or β radiation). But what about individual protons or neutrons?

It was asserted earlier that individual nucleons are also composite objects, and may be viewed as bound states of quarks. And just as atoms and nuclei have excited states, so do individual nucleons.

The force which binds quarks together into bound states is known as the *strong interaction*, and the theory which describes strong interactions is called *quantum chromodynamics*, often abbreviated as QCD. We will have more to say about QCD as we progress. But the justification for the validity of the following qualitative description of quarks and their bound states lies in the success of QCD. Using this theory, one can do detailed quantitative calculations of the masses and other properties of bound states of quarks and compare with experimental results. The theory works.

5.1 Quark flavor and color

Quarks are spin-1/2 particles (fermions) which come in various species, referred to as *flavors*. Different quark flavors have been given somewhat whimsical names, as shown in Table 5.1 . In addition to the curious names, two other things in Table 5.1 should strike you as odd: the enormous disparity of masses of different quarks, spanning five orders of magnitude, and the fact that quarks have fractional charge (in units of $|e|$). The quark masses listed in this table must be interpreted with some care, as *isolated* quarks are never observed experimentally. The mass, or rest energy, of observed particles which are bound states of quarks (like the proton) largely reflects the binding energy of the quarks, and is not just the sum of the intrinsic quark masses. Nevertheless, it is remarkable that quark masses vary over such a wide range, from a few MeV to hundreds of GeV. The three lightest quark flavors, denoted u, d and s, have masses which are small relative to the proton mass; the three heavy flavors, c, b and t, have masses which are comparable or large relative to the proton mass.

Along with quarks, there are also *antiquarks*, denoted $\bar{u}$, $\bar{d}$, $\bar{s}$, etc., with the same masses but opposite electric charge as their partner. (So, for example, the $\bar{u}$ antiquark has charge $-2/3$ and the $\bar{d}$ has charge $+1/3$.)

flavor	**symbol**	**mass**	**charge**
up	u	≈ 2 MeV/c^2	$\frac{2}{3}\,\|e\|$
down	d	≈ 5 MeV/c^2	$-\frac{1}{3}\,\|e\|$
strange	s	≈ 95 MeV/c^2	$-\frac{1}{3}\,\|e\|$
charm	c	≈ 1.3 GeV/c^2	$\frac{2}{3}\,\|e\|$
bottom	b	≈ 4.2 GeV/c^2	$-\frac{1}{3}\,\|e\|$
top	t	≈ 173 GeV/c^2	$\frac{2}{3}\,\|e\|$

Table 5.1: Known quark flavors

Quarks have an additional attribute, analogous to but different from electric charge, which is termed *color* charge. The color charge of a quark can have three possible values which may be denoted as 'red', 'green', or 'blue'. These names are simply labels for different quantum states of the quark.[1] Antiquarks carry opposite electric and color charge as the corresponding quarks; color states of antiquarks can be called 'anti-red', 'anti-green' or 'anti-blue'.

Since quarks (and antiquarks) have spin 1/2, so they can also be labeled by their spin projection, $\uparrow$ or $\downarrow$, along any chosen spin quantization axis. Hence, for each quark flavor, there are really six different types of quark, distinguished by the color (red, blue, green) and spin projection (up, down).

5.2 Hadrons

No (reproducible) experiments have detected any evidence for free (*i.e.*, isolated) quarks. Moreover, there is no evidence for the existence of any isolated charged particle whose electric charge is not an integer multiple of the electron charge. This is referred to as *charge quantization*. Consistent with these observational facts, the theory of strong interactions predicts that quarks will always be trapped inside bound states with other quarks and antiquarks.[2] Bound states produced by the strong interactions are called *hadrons* (*hadros* is Greek for 'strong').

Quantum chromodynamics predicts that only certain types of bound states of quarks can exist, namely those which are "colorless". (This can be phrased in a mathematically precise fashion in terms of the symmetries of the theory. More on this later.) Recall that to make white light, one mixes together red, blue, and green light. Similarly, to make a colorless bound state of quarks one sticks together three quarks, one red, one blue, and one green. But this is not the only way. Just as

[1] These names are purely conventional — one could just as well label the different "color" states as 1, 2, and 3. But the historical choice of names explains why the theory of strong interactions is called quantum *chromo*dynamics: a quantum theory of the dynamics of "color" — although this color has nothing to do with human vision!

[2] Except at sufficiently high temperatures. Above a temperature of $T_c \approx 2 \times 10^{12}$ K (or $kT \approx 170$ MeV), hadrons "melt" or "vaporize" and quarks are liberated. This is important in the physics of the early universe, since temperatures are believed to have exceeded this value in the earliest moments of the big bang. Temperatures above T_c can also be produced, briefly, in heavy ion collisions. A nice overview of heavy ion collisions and quark gluon plasma may be found at `www.bnl.gov/rhic/heavy_ion.htm`. There is an ongoing program studying heavy ion collisions both at the RHIC accelerator on Long Island, and at the LHC where some running time is devoted to colliding heavy nuclei rather than protons.

antiquarks have electric charges which are opposite to their partner quarks, they also have "opposite" color: anti-red, anti-blue, or anti-green. Another way to make a colorless bound state is to combine three antiquarks, one anti-red, one anti-blue, and one anti-green. A final way to make a colorless bound state is to combine a quark and an antiquark (in the quantum superposition $r\bar{r} + g\bar{g} + b\bar{b}$ of correlated color states which is overall colorless).

Bound states of three quarks are called *baryons*, bound states of three antiquarks are called *antibaryons*, and quark-antiquark bound states are called *mesons*. Baryons and antibaryons, as bound states of three spin-1/2 quarks, necessarily have half-integer values of spin, and are fermions. Mesons, as bound states of two spin-1/2 constituents, have integer values of spin, and are bosons.

Strong interactions are effectively flavor-blind; except for the difference in mass, quarks of different flavors have identical strong interactions. Strong interaction processes cannot change the net number of quarks of a given flavor (*e.g.*, the number of up quarks minus up antiquarks, *etc*).[3]

How these rules emerge from QCD will be described in a bit more detail later. For now, let's just look at some of the consequences. The prescription that hadrons must be colorless bound states says nothing about the flavors of the constituent quarks and antiquarks. Since quarks come in multiple flavors, listed in Table 5.1, one can (and we will) enumerate the various possibilities.

The rest energy of a hadron (like any bound state) may be regarded as the sum of the rest energies of its constituents plus a binding energy. For some hadrons, including nucleons, we will see that most of their total energy comes from binding energy. But the masses of quarks also play a part. Looking at the quark masses listed in Table 5.1 , it is apparent that u, d and s quarks are quite light compared to the mass (≈ 1 GeV/c^2) of a nucleon, while the other quark flavors are considerably heavier. So it should not be surprising that the lightest hadrons will be those which are bound states of u and d quarks. Substituting a strange quark for a u or d quark should be expected to raise the mass of the resulting bound by roughly 100 MeV. And hadrons containing the other quark flavors (c, b, or t) should be substantially heavier.

When enumerating possible combinations of quarks which could form hadrons, one must also think about spin and electric charge. Combining electric charge is easy: the charge of a hadron is just the sum of the charges of its constituent quarks. Combining the angular momentum of two spin 1/2 particles can yield either spin 1 or 0 (depending on whether the spin wavefunction is symmetric or antisymmetric). Three spin 1/2 particles can combine to form either spin 3/2 or 1/2.[4] In addition to the combined total spin $\vec{S}$ of the constituents, the total angular momentum $\vec{J}$ of a multi-particle bound state can also receive a contribution from the orbital angular momentum $\vec{L}$ which arises due to the internal motion of the constituents. So, in general, $\vec{J} = \vec{L} + \vec{S}$. For the lowest mass hadrons, of a given spin and flavor content, one may regard the quarks as nearly at rest (within the constraints imposed by quantum mechanics) with vanishing orbital angular momentum, $\vec{L} = 0$.

[3] In contrast, weak interactions (whose details will be discussed further in a later chapter) can change a quark of one flavor into a quark of a different flavor. Hence weak interactions need not conserve the net number of quarks of a given flavor.

[4] More generally, recall that when a system with spin S_1 is combined with a system with spin S_2, the result can have a total spin which ranges from a minimum of $|S_1 - S_2|$ to a maximum of $S_1 + S_2$ in unit steps (when all spins are measured in units of $\hbar$). For any system with spin S, there are $2S + 1$ possible values for the projection $\vec{S} \cdot \hat{n}$ of the spin vector along some chosen spin quantization axis $\hat{n}$ ranging from $-S$ to $+S$ in unit steps.

5.3 Mesons

Let us start with mesons and (for the moment) consider just the three lightest quark flavors, u, d, and s. Since a meson is a bound state of a quark and antiquark, there are nine different possible flavor combinations. Table 5.2 displays these possibilities arranged according to the resulting electric charge Q as well as the number of strange (or antistrange) quarks. For each combination, there will be one spin zero state, and one spin one state. Reassuringly, the lightest observed mesons are either spin zero or spin one. Table 5.3 lists the light spin zero mesons, along with their dominant decay modes, while Table 5.4 does the same for light spin one mesons.[5] These are the lightest mesons.

As Tables 5.3 and 5.4 show, the numbers and charges of the lightest spin zero and spin one mesons precisely match what is expected based on the possible combinations of a quark and antiquark. In general, mesons containing strange quarks are heavier than those without. But among the neutral mesons, it is noteworthy that it is certain linear combinations of the $Q = 0$ quark combinations listed in Table 5.2 which correspond to distinct particles. This reflects the possibility of quantum mechanical mixing among states with identical quantum numbers. In other words, eigenstates of the Hamiltonian can be linear combinations of basis states which have simple quark content. The form of this mixing will be discussed in more detail later, but the important conclusion here is that the basic description of mesons as bound states of quarks works.

5.4 Baryons

One can go through a similar exercise for baryons. Instead of dealing with two constituents, one is now dealing with three. Combining three spin 1/2 objects can yield either spin 1/2 or 3/2. The lightest observed baryons are, in fact, either spin 1/2 or spin 3/2. Tables 5.5 and 5.6 list the lightest spin 1/2 and spin 3/2 baryons, respectively. As Table 5.6 shows, the ten light $J = 3/2$ baryons precisely match the ten possible choices of triples of quark flavors. From the masses listed in Table 5.6 one sees that the Σ^* baryons, which contain one strange quark, are heavier than the Δ baryons, which contain only u and d quarks, by about 150 MeV. The Ξ^* baryons, which contain two strange quarks are heavier than the Σ^* by an additional ≈ 150 MeV, and the Ω^- baryon, containing three strange quarks, is yet heavier by about the same increment. This is consistent with our expectations that substituting heavier quarks for lighter quarks should increase the mass of bound states, as the binding dynamics due to the color interactions are independent of the quark flavors involved. The ten baryons shown in Table 5.6 are referred to as the spin 3/2 baryon *decuplet.*

As Table 5.5 shows, there are only eight light $J = 1/2$ baryons, which are referred to as the spin 1/2 baryon *octet.* This difference between $J = 3/2$ and $J = 1/2$ bound states may be understood as a consequence of the Pauli exclusion principle. Quarks, being fermions, must obey the Pauli exclusion principle. Wavefunctions of multi-quark states must be antisymmetric under the interchange of identical quarks. To see how this leads to the difference between the $J = 3/2$ decuplet and the $J = 1/2$ octet, we must examine baryon wavefunctions in more detail.

[5] Tables 5.3 and 5.4 list parity odd mesons, as this is the parity of quark-antiquark bound states with no orbital excitation. We will discuss parity assignments in the next chapter. Note that the mass values listed in these and subsequent tables should really have units of MeV/c^2 (or the column should be labeled "rest energy" instead of "mass"). We will become increasingly sloppy about this distinction, since one can always insert a factor of c^2, as needed, to convert mass to energy or vice-versa.

$(\#s)+(\#\bar{s})$	$Q=1$	$Q=0$	$Q=-1$
0	$u\bar{d}$	$u\bar{u}$, $d\bar{d}$	$d\bar{u}$
1	$u\bar{s}$	$s\bar{d}$, $d\bar{s}$	$s\bar{u}$
2		$s\bar{s}$	

Table 5.2: Possible light quark-antiquark combinations

meson	**mass**	**lifetime**	**dominant decays**	**quark content**
π^0	135.0 MeV	8×10^{-17} s	$\gamma\gamma$	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$
π^+	139.6 MeV	2.6×10^{-8} s	$\mu^+\nu_\mu$	$u\bar{d}$
π^-	139.6 MeV	2.6×10^{-8} s	$\mu^-\bar{\nu}_\mu$	$d\bar{u}$
K^+	493.7 MeV	1.2×10^{-8} s	$\mu^+\nu_\mu$, $\pi^+\pi^0$	$u\bar{s}$
K^-	493.7 MeV	1.2×10^{-8} s	$\mu^-\bar{\nu}_\mu$, $\pi^-\pi^0$	$s\bar{u}$
$K^0_{\rm S}$	497.7 MeV	8.9×10^{-11} s	$\pi^+\pi^-,\pi^0\pi^0$	$\frac{1}{\sqrt{2}}(d\bar{s}-s\bar{d})$
$K^0_{\rm L}$	497.7 MeV	5.1×10^{-8} s	$\pi^\pm e^\mp\nu_e$, $\pi^\pm\mu^\mp\nu_\mu$	$\frac{1}{\sqrt{2}}(d\bar{s}+s\bar{d})$
η	547.5 MeV	5×10^{-19} s	$\gamma\gamma$, $\pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$	$\approx\frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s})$
η'	957.8 MeV	3×10^{-21} s	$\pi^+\pi^-\eta$, $\rho^0\gamma$, $\pi^0\pi^0\eta$	$\approx\frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s})$

Table 5.3: Light spin zero, parity odd mesons.

meson	**mass**	**lifetime**	**dominant decays**	**quark content**
ρ^+,ρ^0,ρ^-	775.5 MeV	4×10^{-24} s	$\pi\pi$	$u\bar{d}$, $\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}),d\bar{u}$
ω	782.7 MeV	8×10^{-23} s	$\pi^+\pi^-\pi^0$	$\approx\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$
K^{*+},K^{*-}	891.7 MeV	1.3×10^{-23} s	$K\pi$	$u\bar{s}$, $s\bar{u}$
$K^{*0},\bar{K}^{*0}$	896.0 MeV	1.3×10^{-23} s	$K\pi$	$d\bar{s}$, $s\bar{d}$
ϕ	1019.5 MeV	2×10^{-22} s	K^+K^-, $K^0_{\rm L}K^0_{\rm S}$, $\pi\pi\pi$	$\approx s\bar{s}$

Table 5.4: Light spin one, parity odd mesons.

baryon	**mass**	**lifetime**	**dominant decays**	**quark content**
p	938.3 MeV	stable	—	uud
n	939.6 MeV	9×10^2 s	$pe^-\bar{\nu}_e$	udd
Λ	1116 MeV	2.6×10^{-10} s	$p\pi^-, n\pi^0$	uds
Σ^+	1189 MeV	0.8×10^{-10} s	$p\pi^0, n\pi^+$	uus
Σ^0	1193 MeV	7×10^{-20} s	$\Lambda\gamma$	uds
Σ^-	1197 MeV	1.5×10^{-10} s	$n\pi^-$	dds
Ξ^0	1315 MeV	2.9×10^{-10} s	$\Lambda\pi^0$	uss
Ξ^-	1321 MeV	1.6×10^{-10} s	$\Lambda\pi^-$	dss

Table 5.5: Light spin 1/2, parity even baryons.

baryon	**mass**	**lifetime**	**dominant decays**	**quark content**
$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	1232 MeV	6×10^{-24} s	$p\pi$, $n\pi$	uuu, uud, udd, ddd
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	1385 MeV	2×10^{-23} s	$\Lambda\pi$	uus, uds, dds
Ξ^{*0}, Ξ^{*-}	1530 MeV	7×10^{-23} s	$\Xi\pi$	uss, dss
Ω^-	1672 MeV	0.8×10^{-10} s	ΛK^-, $\Xi\pi$	sss

Table 5.6: Light spin 3/2, parity even baryons.

5.5 Baryon wavefunctions

To understand how the fermionic nature of quarks produces the observed pattern of spin and flavor for baryons, we must first return to the earlier assertion that only colorless bound states of quarks exist. What does this really mean? Just as you can think of a spin 1/2 particle as having a wavefunction which is a two-component complex vector,

$$\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \begin{pmatrix} \langle\uparrow|\psi\rangle \\ \langle\downarrow|\psi\rangle \end{pmatrix}, \tag{5.5.1}$$

the wavefunction of a quark (of definite flavor and spin) is a three-component vector in "color" space,

$$\vec{\Psi} \equiv \begin{pmatrix} \psi_\mathrm{r} \\ \psi_\mathrm{g} \\ \psi_\mathrm{b} \end{pmatrix} = \begin{pmatrix} \langle\mathrm{red}|\psi\rangle \\ \langle\mathrm{green}|\psi\rangle \\ \langle\mathrm{blue}|\psi\rangle \end{pmatrix}. \tag{5.5.2}$$

The component ψ_r gives the amplitude for the quark color to be red, ψ_g is the amplitude to be green, etc. The assertion that hadrons must be "colorless" really means that the multi-quark wavefunction must not depend on the choice of basis in three-dimensional color space. Since the quark (color) wavefunction is a three-component vector, to build a colorless state from three quarks, A, B and C, one must combine the three color vectors describing the individual quarks, $\vec{\psi}_A$, $\vec{\psi}_B$, and $\vec{\psi}_C$, in such a way that the result is basis independent.

This may sound peculiar, but the mathematical problem is the same as asking how to build a rotationally-invariant scalar from three spatial vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, in such a way that the result is a linear function of each of the vectors. You already know the (essentially unique) answer, namely the triple-product of the three vectors, $\vec{A} \cdot (\vec{B} \times \vec{C})$. This triple product may be expressed in several equivalent ways involving the determinant of components,

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \det \begin{vmatrix} A_1 \, B_1 \, C_1 \\ A_2 \, B_2 \, C_2 \\ A_3 \, B_3 \, C_3 \end{vmatrix} = \epsilon_{ijk} \, A_i \, B_j \, C_k \, . \tag{5.5.3}$$

In the last form, ϵ_{ijl} is the totally antisymmetric tensor which equals +1 when (ijk) is any cyclic permutation of (123), −1 when (ijk) is any cyclic permutation of (321), and zero otherwise.[6] Recall that a determinant changes sign if any two columns (or rows) are interchanged; this is also encoded in the antisymmetry of the ϵ_{ijk} symbol. Consequently, the triple product is antisymmetric under any interchange of two of the vectors. The triple product (5.5.3) defines a rotationally invariant scalar, independent of the basis used to define vector components — and this is exactly the property we need when combining color wavefunctions of three quarks to produce a colorless result. Thus, the color part of the wavefunction for three quarks has the form

$$\Psi_{\text{color}} = \epsilon_{ijk} \, \psi_i^{q_1} \, \psi_j^{q_2} \, \psi_k^{q_3} \, , \tag{5.5.4}$$

where $\psi_i^{q_1}$ is the i'th component of the color wavefunction (5.5.2) for quark 1, etc.

The complete wavefunction describing three quarks in a bound state must characterize not only the color of the quarks, but also their flavor, spin, and spatial location. To a good approximation, the wavefunction will be a product of a spatial wavefunction (depending only on the quark positions), a color wavefunction (depending only on the color vectors), and a flavor & spin wavefunction,

$$\Psi = \Psi_{\text{space}} \times \Psi_{\text{color}} \times \Psi_{\text{spin+flavor}} \, . \tag{5.5.5}$$

The essential point of the above discussion about triple products is that the color wavefunction for three quarks is antisymmetric under any interchange of the color vectors of any two quarks. The lightest hadrons which can be built from a given set of quark flavors will have a spatial wavefunction which is symmetric under interchange of quark positions. If this is not true then the wavefunction will have nodes across which it changes sign, and this increases the kinetic energy of the state. (In particular, this means the lightest hadrons will have no orbital angular momentum.) Because quarks are fermions, the *total* wavefunction must be antisymmetric under interchange of any two quarks — which means simultaneous interchange of the positions, spins, flavors, and colors of the two quarks. Since the color part of the wavefunction must be antisymmetric, while the spatial part should be symmetric, this means that the flavor and spin part must also be *symmetric* under permutations.[7]

For a spin 3/2 baryon, such as the Δ^{++}, the flavor structure of the wavefunction is trivial, and totally symmetric, since all three quarks are the same type, namely uuu. For the $S_z = 3/2$ state,

[6] The geometric definition of a cross product only makes sense for real vectors, but the expressions (5.5.3) involving components of the vectors are equally well-defined for complex vectors.

[7] This was a significant puzzle in the early days of the quark model (circa 1970), when quarks had been postulated as constitutents of baryons, but the role of a color quantum number was not yet understood. The spatial, flavor and spin wavefunctions that matched observed baryons were clearly symmetric, but quarks had spin 1/2 and hence were fermions. Was Pauli wrong? The additional attribute of color saved the spin-statistics theorem, and paved the way to the formulation of a microscopic theory of strong interactions, namely QCD.

the spin structure is also trivial, and totally symmetric, since all three quarks must individually have $S_z = 1/2$ if the total spin projection is 3/2.[8] Therefore the combined spin+flavor part of the wavefunction,

$$\Psi^{\Delta^{++}}_{\text{spin+flavor}} \sim (uuu) \times (\uparrow\uparrow\uparrow) \,, \tag{5.5.6}$$

satisfies the above condition of symmetry under permutation of quark spins and flavors. Analogous spin+flavor wavefunctions may be constructed for all baryons (with any spin projection) in the spin 3/2 decuplet. Just like the example (5.5.6), these wavefunctions are simple products of a flavor part and a spin part, and are independently symmetric under permutations of quark spins, or permutations of quark flavors.

For $J = 1/2$ baryons, the situation is more complicated. To produce a spin 1/2 state from three spin 1/2 constituents, the spins of two constituents can be combined in a manner which produces spin zero (not spin one), so that adding the third constituent spin results in total spin 1/2. But building a spin zero state out of two spin 1/2 objects involves a spin wavefunction, $\uparrow\downarrow - \downarrow\uparrow$, which is antisymmetric under interchange of the two spins. Consequently, the spin wavefunction (alone) for a spin 1/2 baryon cannot be totally symmetric under permutations of the spins. If we are to build a spin+flavor wavefunction which is symmetric under combined permutations of flavors and spins of quarks, then the flavor part of the wavefunction must also not be totally symmetric, and must compensate for the antisymmetry in the spin part of the wavefunction.

If all three quarks have the same flavor, this is not possible — like it or not, a flavor wavefunction such as uuu is totally symmetric. This explains why there are no light spin 1/2 baryons composed of three up (or three down, or three strange) quarks, in contrast to the case for spin 3/2 baryons. But if there are at least two distinct quark flavors involved, then it is possible to build a flavor wavefunction with the required symmetry. As an example, let us build a spin+flavor wavefunction for the proton. We need two u quarks and one d quark. A spin wavefunction of the form $(\uparrow\downarrow - \downarrow\uparrow)\uparrow$ describes a state in which the first two quarks have their spins combined to form an $S = 0$ state, so that adding the third spin yields a total spin of 1/2, as desired. Since this spin wavefunction is antisymmetric under interchange of the first two spins, we need a flavor wavefunction which is also antisymmetric under interchange of the first two quark flavors, namely $(ud - du)u$. If we multiply these, we have a spin+flavor wavefunction,

$$[\,(ud - du)u\,] \times [\,(\uparrow\downarrow - \downarrow\uparrow)\uparrow\,] = (udu - duu)\,(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\,, \tag{5.5.7}$$

which is symmetric under combined spin and flavor exchange of the first two quarks. But we need a wavefunction which is symmetric under interchange of any pair of quark spins and flavors. This can be accomplished by adding terms which are related to the above by cyclic permutations (or in another words by repeating the above construction when it is the second and third, or first and third quarks which are combined to form spin zero). The result for a spin-up proton, which is unique up to an overall normalization factor, is

$$\Psi^{\text{proton}}_{\text{spin+flavor}} = (udu - duu)\,(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + (uud - udu)\,(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow) + (uud - duu)\,(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)\,. \tag{5.5.8}$$

[8]This ignores the possibility of further constituents in the baryon in addition to the three up quarks. Using an improved description of the structure of baryons does not change the essential conclusions of the following discussion.

One may write this wavefunction more explicitly with the various terms multiplied out,

$$\begin{aligned}\Psi^{\rm proton}_{\rm spin+flavor} = \big[&2\,u\uparrow u\uparrow d\downarrow\; - u\downarrow u\uparrow d\uparrow\; - u\uparrow u\downarrow d\uparrow \\ &+ 2\,u\uparrow d\downarrow u\uparrow\; - u\downarrow d\uparrow u\uparrow\; - u\uparrow d\uparrow u\downarrow \\ &+ 2\,d\downarrow u\uparrow u\uparrow\; - d\uparrow u\downarrow u\uparrow\; - d\uparrow u\uparrow u\downarrow\,\big]/\sqrt{18}\,,\end{aligned} \tag{5.5.9}$$

(where we have also normalized the result). You can, and should, check that this does satisfy the required condition of symmetry under interchange of spins and flavors of any pair of quarks. Similar constructions can be performed for all the other members of the spin 1/2 baryon octet.

One notable feature of the set of octet baryons, shown in Table 5.5 , is the presence of two different baryons whose quark content is *uds*, specifically the Λ and the Σ^0. This is not inconsistent. When three distinct flavors are involved, instead of just two, there are more possibilities for constructing a spin+flavor wavefunction with the required symmetry. A careful examination (left as a problem) shows that there are two independent possibilities, completely consistent with the observed list of spin 1/2 baryons.

The mass values in Table 5.5 show that for spin 1/2 baryons, just as for spin 3/2 baryons, baryons with strange quarks are heavier than those with only up and down quarks; each substitution of a strange quark for an up or down raises the energy of the baryon by roughly 120–170 MeV.

5.6 Baryon number

Baryon number, denoted B, is defined as the total number of baryons minus the number of antibaryons, similar to the definition (4.7.1) of lepton number L. Since baryons are bound states of three quarks, and antibaryons are bound states of three antiquarks, baryon number is the same as the number of quarks minus antiquarks, up to a factor of three,

$$B = (\#\text{ baryons}) - (\#\text{ antibaryons}) = \tfrac{1}{3}\left[(\#\text{ quarks}) - (\#\text{ antiquarks})\right]. \tag{5.6.1}$$

All known interactions conserve baryon number.[9] High energy scattering processes can change the number of baryons, and the number of antibaryons, but not the net baryon number. For example, in proton-proton scattering, the reaction $p+p \to p+p+n+\bar{n}$ can occur, but not $p+p \to p+p+n+n$.

5.7 Hadronic decays

Turning to the decays of the various hadrons listed in Tables 5.3–5.6 , it is remarkable how much can be explained using a basic understanding of the quark content of the different hadrons together with considerations of energy and momentum conservation. As an example, consider the baryons in the spin 3/2 decuplet. The rest energy of the Δ baryons is larger than that of a nucleon by nearly 300 MeV. This is more than the ≈ 140 MeV rest energy of pions, which are the lightest mesons. Consequently, a Δ baryon can decay to a nucleon plus a pion via strong interactions, which do not change the number of quarks minus antiquarks of each quark flavor. (Specifically, a Δ^{++} can decay

[9] This is not quite true. As with lepton number, the current theory of weak interactions predicts that there are processes which can change baryon number (while conserving $B-L$). The rate of these processes is so small that baryon number violation is (so far) completely unobservable.

to $p\pi^+$, a Δ^+ can decay to either $p\pi^0$ or $n\pi^+$, a Δ^0 can decay to $p\pi^-$ or $n\pi^0$, and a Δ^- can decay to $n\pi^-$.) These are the (overwhelmingly) dominant decay modes observed. The short lifetime of Δ baryons, $\tau \simeq 6 \times 10^{-24}$ s or $c\tau \simeq 1.8$ fm, is also indicative of a decay via strong interactions. (You can think of "strong" interactions as meaning rapid interactions.) This lifetime corresponds to a decay (or resonance) width $\Gamma_\Delta = \hbar/\tau \approx 120$ MeV, which is 10% of the rest energy of a Δ. Since light takes about 3×10^{-24} s to travel one fermi, a Δ baryon barely has time to "figure out" that it exists before it decays.[10]

The Σ^* and Ξ^* baryons also have very short lifetimes, on the order of a few times 10^{-23} s. The Σ^* contains one strange quark. The Σ^* mass of 1385 MeV/c^2 is larger than the 1116 MeV/c^2 mass of the Λ, the lightest baryon containing a strange quark, by more than the mass of a pion. So strong interactions can cause a Σ^* to decay to a Λ plus a pion, which is the dominant observed decay. Similarly, Ξ^* baryons, containing two strange quarks, can decay via strong interactions to a Ξ (the lightest doubly strange baryon) plus a pion.

The final member of the $J = 3/2$ decuplet, the Ω^- baryon, cannot decay via strong interactions to a lighter baryon plus a pion, because there are no lighter baryons containing three strange quarks (and strong interactions preserve the net number of strange quarks). It could, in principle, decay via strong interactions to a Ξ baryon (containing two strange quarks) and a K meson (containing one strange quark) — but it doesn't have enough energy. Its mass of 1672 MeV/c^2 is less than the sum of Ξ plus K masses. In fact, the Ω^- baryon cannot decay via *any* strong interaction process. Nor can it decay via electromagnetic processes, which also preserve the net quark flavor. But weak interactions are distinguished by the fact that they can *change* quarks of one flavor into a different flavor. Consequently, the Ω^- baryon can decay via weak interactions to a lighter baryon plus a meson. The dominant decays involve the conversion of one strange quark into an up or down quark, leading to final states consisting of a Λ baryon plus K^- meson, a Ξ^0 baryon plus π^0, or a Ξ^- plus π^+. The Ω^- was first seen in bubble chamber photographs, and was relatively easy to discover due to its distinctive "cascade" decay,

$$\begin{array}{l} \Omega^- \longrightarrow \Xi^0 + \pi^- \\ \qquad\qquad \hookrightarrow \Lambda^0 + \pi^0 \\ \qquad\qquad\qquad \hookrightarrow p + \pi^- \, . \end{array}$$

So the overall process is $\Omega^- \to p\,\pi^-\,\pi^-\,\pi^0$ (with the pions eventually decaying to leptons and photons). Note that all final states of Ω^- decay conserve baryon number and electric charge, and are allowed by energy conservation. The 10^{-10} s lifetime of the Ω^- is much longer than a strong interaction decay, and is indicative of a weak interaction process.

Similar reasoning can be applied to the $J = 1/2$ baryons. The proton is stable (so far as we know), while all the other members of the octet decay via weak interactions — except for the Σ^0 which can decay to a Λ plus a photon via electromagnetic interactions. Note that the 7×10^{-20} s lifetime of the Σ^0 is much shorter than a weak interaction lifetime, but is longer than typical strong interaction

[10]One might ask, "how long must some 'particle' live to justify calling it a particle?" With a lifetime under 10^{-23} s, a Δ baryon produced in some particle collision will never fly away from the interaction point and reach a particle detector, located some macroscopic distance away, before decaying. For such unstable particles, what is eventually detected are the decay products of the Δ baryon. Measuring the interaction *rate* as a function of energy in pion-nucleon scattering experiments, for example, one finds a resonance peak at the energy corresponding to production of Δ baryons. Because the Δ width is only 10% of its energy, this resonance peak is very recognizable.

lifetimes. (In a very real sense, electromagnetic interactions are stronger than weak interactions, but weaker than strong interactions.) The lifetimes of Λ, Ξ, and $\Sigma^{\pm}$ baryons are all around 10^{-10} s, typical of weak interaction decays. The 900 second lifetime of the neutron is vastly longer than a normal weak interaction lifetime. This reflects the fact that neutron decay is just barely allowed by energy conservation. The mass of the final proton plus electron (and antineutrino) is so close to the mass of the neutron that only about 8 MeV, or less than 0.1% of the rest energy of neutron, is available to be converted into kinetic energy of the decay products.

Similar observations apply to the decays of mesons. Just as most spin 3/2 baryons have strong interaction (*i.e.*, fast) decays into a spin 1/2 baryon plus a pion, the spin 1 (or "vector") mesons of Table 5.4 all have strong decays into spin 0 ("scalar") mesons with the same strange quark content, plus a pion. Lifetimes of these vector mesons are short, 10^{-22} – 10^{-24} s, indicative of strong interactions. Some aspects of these decays invite questions which we will consider in the next chapter. For example, the ρ^0 and ω are both neutral vector mesons with no strange quarks; why does the ω decay to three pions while the ρ^0 decays to just two pions? And in decays of ρ^0, the actual two pion final state is $\pi^+\pi^-$, not $\pi^0\pi^0$. Why is that?

For the scalar mesons of Table 5.3 , energy conservation rules out any strong interaction decays. The neutral π^0, η and η' mesons all have electromagnetic decays with photons in the final state and lifetimes of order 10^{-20} s (similar to the Σ^0 lifetime). The K mesons (or "kaons") can only decay via weak processes which turn a strange quark into an up or down quark (or $\bar{s}$ into $\bar{u}$ or $\bar{d}$). And the charged pions can only decay, via weak interactions, into leptons. All these weak decay lifetimes are in the 10^{-8} – 10^{-10} s range (a little slower or comparable to the Ω^- lifetime).

You are encouraged to look at the much more extensive listing of information about known mesons and baryons at the Particle Data Group website . Pick a few particles which have not been discussed above, and see if you can predict the dominant decay modes.

5.8 Example problems

5.8.1 Neutron spin+flavor

Q: Find the spin+flavor wavefunction for a neutron.

A: This is most easily done by looking at the answer (5.5.8) for a proton, and just interchanging u and d quarks. So for a spin up neutron,

$$\Psi^{\text{neutron}}_{\text{spin+flavor}} = (dud - udd)\,(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + (ddu - dud)\,(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow) + (ddu - udd)\,(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)\,.$$

This, as required for baryons, is symmetric under the combined interchange of spin and flavor of any pair of quarks.

5.8.2 Λ and Σ^0 spin+flavor

Q: Find two (mutually orthogonal) spin+flavor wavefunctions for $J = 1/2$ baryons with quark content uds. Can you deduce (or guess) which wavefunction represents the Σ^0, and which represents the Λ?

A: One $J = 1/2$ uds wavefunction can be found using exactly the same logic which works for the proton. Start with two quarks, say u and d, in the state $(ud - du) \times (\uparrow\downarrow - \downarrow\uparrow)$ which is symmetric under combined interchange of spins and flavors, and has the spins combined to form

$S = 0$. Add the strange quark with spin, say, chosen to be up, to obtain the $J = 1/2$ three quark state $[(ud - du)s] \times [(\uparrow\downarrow - \downarrow\uparrow)\uparrow] = (uds - dus)(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$. To make this fully symmetric under combined interchange of spin and flavor of any pair of quarks, we need to add analogous terms related by cyclic permutations of the three quarks. This produces our first answer,

$$\Psi^{\rm A}_{\rm spin+flavor} = (uds - dus)(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + (sud - sdu)(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow) + (usd - dsu)(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)\,.$$

To find a second orthogonal answer, one can start with a ud two quark in which the quark spins are combined to form $S = 1$ (instead of $S = 0$), and then add the third s quark in such a way that the resulting final spin is $S = 1/2$, not $S = 3/2$. This is a little bit trickier. The two spin wavefunction $\uparrow\uparrow$ has spin projection $S_z = 1$ and (necessarily) total spin $S = 1$. The two spin wavefunction $\uparrow\downarrow + \downarrow\uparrow$ has spin projection $S_z = 0$ but also has total spin $S = 1$. Either of these spin wavefunctions can be combined with the flavor wavefunction $ud + du$ to obtain a two quark spin+flavor wavefunction which is symmetric under combined interchange of spin and flavor. To each of these two quark states one may now add a strange quark, with its spin chosen in such a way that the resulting three quark state has $S_z = 1/2$, yielding $(uds + dus) \times (\uparrow\uparrow\downarrow)$ or $(uds + dus) \times (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$. By construction, each of these states has total $S_z = +1/2$, but neither state has definite total spin — each wavefunction describes a mixture of $S = 3/2$ and $S = 1/2$ states. We need to find the linear combination which describes a pure $S = 1/2$ state. The correct answer must be orthogonal to the wavefunction which describes three spins combined to form $S = 3/2$ with $S_z = 1/2$. That spin wavefunction is $\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow$, and is completely symmetric under interchange of spins (just like the $\uparrow\uparrow\uparrow$ spin state describing $S = 3/2$, $S_z = 3/2$). The linear combination of $\uparrow\uparrow\downarrow$ and $(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$ which is orthogonal to this $S = 3/2$, $S_z = 3/2$ state is $2\uparrow\uparrow\downarrow - (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$, up to an arbitrary overall normalization factor.

Therefore, the three quark uds spin+flavor wavefunction which is symmetric under combined interchange of spin and flavor of the first two quarks, has those quark spins combined to form $S = 1$, but then has all three quark spins combined in such a way that the result is pure $S = 1/2$, $S_z = 1/2$, is

$$(uds + dus) \times [\,2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\,]\,.$$

To make the result symmetric under combined interchange of spin and flavor of any pair of quarks we must, as before, add circular permutations. The result is our second answer,

$$\begin{aligned}\Psi^{\rm B}_{\rm spin+flavor} &= (uds + dus)\,[\,2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\,] \\ &\quad + (sud + sdu)\,[\,2\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow\,] \\ &\quad + (dsu + usd)\,[\,2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\,]\,.\end{aligned}$$

The state $\Psi^{\rm A}_{\rm spin+flavor}$ desribes the spin+flavor structure of the Λ, while state $\Psi^{\rm B}_{\rm spin+flavor}$ desribes the spin+flavor structure of the Σ^0. To see (or at least motivate) why, notice that under interchange of just the flavors of the two non-strange quarks, $\Psi^{\rm A}_{\rm spin+flavor}$ is *antisymmetric* while $\Psi^{\rm B}_{\rm spin_flavor}$ is *symmetric*. Contrast these behaviors with that of the wavefunction of a Σ^+ baryon: which looks just like the result (5.5.8) for the proton but with d everywhere replaced by s. The Σ^+ wavefunction, having two u quarks, is necessarily (and trivially) *symmetric* under interchange of just the flavors of the non-strange quarks. The Σ^0 differs from the Σ^+ just by replacing one up quark with a down quark. Up and down quarks have nearly identical masses and (reflecting this) so do the Σ^0 and Σ^+. Therefore, it must be $\Psi^{\rm B}_{\rm spin+flavor}$ which describes the Σ^0, as this state has the non-strange flavor interchange symmetry which matches the result for Σ^+.