Chapter 0

Introduction

As we start this study of Particles and Symmetries it seems appropriate to try to provide an overview, *i.e.*, some version of the big picture goals for this course. As the title of the course implies, our goal is to provide an introduction to an area of physics that has seen dramatic progress in the last 50 years - particle physics. A central tool during this progress has been the exploitation of the underlying symmetries, the other subject in the title, of the interactions of these particles. The short version summary of this progress is encoded in the so-called Standard Model of Particle Physics (typically denoted the SM), which identifies the particles (degrees of freedom) and interactions between the particles relevant to the understanding of nearly all of the physical universe. When we include collective behavior (quarks bounds in nuclei, electrons bound in atoms, atoms bound in solid matter) plus classical gravity, we have a nearly complete explanation[1] for the physics of the very large, *e.g.*, the evolution of the universe from very early times, down to the physics at the shortest distances now observable at particle accelerators. To have full quantitative command of this fundamental understanding requires a tool not at our disposal - quantum field theory. However, we will be able to outline a "semi-classical" (building block) picture of particle physics using only special relativity, quantum mechanics and symmetries, which is remarkably complete and relatively quantitative. The most recent addition to the SM is the so-called Higgs boson (named after the British theoretical physicist Peter Higgs), whose initial discovery at the Large Hadron Collider (the LHC) at CERN (in Geneva, Switzerland) was announced on the 4th of July, 2012, and whose detailed properties have by now been largely confirmed to match those expected. Indeed the Nobel Prize in physics was awarded to Peter Higgs and Francois Englert last autumn for their work (50 years ago) that led to the prediction of this particle. Interestingly the existence and interactions of this particle were *predicted* based on theoretical (*i.e.*, mathematical) considerations, and the confirmation of the expected properties constitutes a major step forward in particle physics research.

From a pedagogical perspective, this study of particle physics will help us learn about two of the pillars of twentieth century physics, special relativity and quantum mechanics. The corresponding dimension-full constants c and $\hbar$ serve to set the "scale" for most of what we observe. The "uncertainty principle" of quantum mechanics, which says, for example, that the product the uncertainties

---

[1]The primary *missing* pieces are "dark matter" that serves to gravitationally binds galaxies but does not form stars, and "dark energy" that is apparently causing an *acceleration* in the expansion rate of the universe. The former category includes approximately 25% of the energy content of the universe, while the latter is about 70%. Stuff like us is a 5% effect.

in where we are (Δx) and where we are going (Δp, with p the momentum) is bounded below, $\Delta x \Delta p \geq \hbar/2$, guarantees that particles confined to small volumes must have large momenta. Thus, since the masses of individual particles (of the variety discussed in this course) are so small, they are very often (*i.e.*, in most reference frames of interest) moving with velocities approaching the speed of light. This situation will provide us with the opportunity to discuss special relativity in detail. The exercises will allow us to practice using special relativity to describe the kinematics of particle collisions at high energy, especially the role of 4-dimensional momentum conservation (which is itself associated with the symmetry associated with the invariance of physics under translations in space *and* time), and the speed of light as the *universal* speed limit. We will want to develop facility with 4-vector notation and the transformations (boosts) that take us between different inertial reference frames. Similarly, since the total angular momentum of an individual particle (*i.e.*, its "spin") is of order $\hbar$, the fact that angular momentum is quantized on this scale means that quantum mechanics will play a central role. We will make use of (and practice using) the uncertainty principle and the important role of the eigenstates of (commuting) operators. In particular, we will want to become efficient at using operator notation to relate different states within the degenerate multiplets that arise due to symmetries. You should have seen this structure in the context of states of *definite* total angular momentum, but *varying* angular momentum component along 1 spatial axis (*e.g.*, $|J, J_z\rangle$). Transformations between these states are accomplished using the (hopefully familiar) raising/lowering (ladder) operators, which are just a special form of a rotation (*i.e.*, a particle with its spin pointing "up" will look like its spin is pointing "down" if you stand on your head). Finally we will discuss how to use symmetries (and the underlying mathematics of group theory) to tie this all together and keep the mathematics simple. This approach will include the use of "approximate" symmetries - where there is no true (exact) invariance under certain transformations, but rather the transformations induce only numerically "small' changes. This will allow us to simplify complex computations in terms of *perturbative* expansions organized in terms of powers of these small changes (such expansions are an essential tool for your physics toolbox). All during the quarter we should be honing our skills for making order-of-magnitude estimates, *i.e.*, being able to estimate the numerical value of a given quantity even when we do not know (or do not understand) all of the details.

Do not be concerned if all of these concepts are not clear at the outset. Also, you should expect that initially portions of our discussions may seem more "abstract" than you are accustomed to. However, you should become concerned if clarity does not develop quickly over the next 10 weeks. Finally do not be surprised if our approach seems somewhat circular. We will try to introduce the relevant vocabulary and concepts in the early lectures, and then return to the same concepts in the context of a more complete formalism in the latter lectures.

We end this Introduction with a brief summary of the ideas, definitions, mathematical tools that you have already (hopefully) mastered in the prerequisite courses in mathematics and physics (especially Phys. 225, 227 and 228, although these classes do not always cover the same material with different instructors). If parts of the following do not seem familiar, you are encouraged to perform a more thorough review, and/or come chat with me.

From Phys. 227-8:

1. Power series expansion: We very often want to expand a mathematical expression in terms of some small parameter, which here we call $|\delta| \ll 1$ and note that δ is *necessarily* dimensionless. For this purpose it is useful to recall the simple power series expansion (really the usual Taylor series expansion),

$$(1 + \delta)^\alpha \approx 1 + \alpha\delta + (\alpha(\alpha - 1)/2!)\,\delta^2 + \mathcal{O}(\delta^3).$$

This expansion is valid independent of the signs of α and δ, but actually requires that the product $|\alpha\delta| \ll 1$ to be useful. However, the exponent α is typically of order unity.

2. Complex numbers:

   $z = x + iy = re^{i\phi} = r\cos\phi + ir\sin\phi$, $\mathrm{Re}\,z = x$, $\mathrm{Im}\,z = y$, $|z| = r = \sqrt{x^2 + y^2}$, $\phi = \arctan\frac{y}{x}$.

3. Exponential function e^z: Defined by

   $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = \frac{d}{dz}e^z$

4. Sinusoidal functions $\sin z, \cos z$: Defined by

   $\sin z = \frac{e^{iz} - e^{-iz}}{2i} = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!}$, $\frac{d}{dz}\sin z = \cos z$.

   $\cos z = \frac{e^{iz} + e^{-iz}}{2} = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$, $\frac{d}{dz}\cos z = -\sin z$.

   $\cos^2 z + \sin^2 z = 1$.

5. Hyperbolic functions $\sinh z, \cosh z$: Defined by

   $\sinh z = \frac{e^z - e^{-z}}{2} = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$, $\frac{d}{dz}\sinh z = \cosh z$.

   $\cosh z = \frac{e^z + e^{-z}}{2} = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}$, $\frac{d}{dz}\cosh z = \sinh z$.

   $\cosh^2 z - \sinh^2 z = 1$.

6. Vectors and Matrices: Facility with vector descriptions of both configuration space ("where the particle is") and momentum space ("where the particle is going") are central to this course. We will use vector notation in two dimensions (labeled 1 and 2 or x and y), three dimensions (labeled 123 or xyz) and four dimensions (labeled $0, 123$ or t, xyz), where the role of t will be distinguished from xyz. This facility with vector notation should include some familiarity with the representations of transformations like rotations as matrices. In particular, we would like to be able to think of transformations in four dimensions - Lorentz transformations - as generalizations (with some "funny" minus signs) of rotations in three dimensions.

7. Matrices, operators and commutation relations: A (hopefully) familiar feature of rotations in three dimensions is that, unlike rotations in two dimensions, 3D rotations do *not* commute in general. This essential property is explicitly represented by the corresponding rotation matrices, *i.e.*, $M_1 M_2 - M_2 M_1 \equiv [M_1, M_2] \neq 0$.

8. Another important set of functions of mathematical physics, which hopefully you were introduced to in Physics 227 or 228 (or somewhere that you learned about physics in 3 spatial dimensions), are the spherical harmonics, $Y_{\ell,m}(\theta, \phi)$. These functions form a *complete* (and orthonormal) set of functions on the surface of a sphere. This means that *any* function of θ and ϕ, $0 \leq \theta \leq \pi$ and $0 \leq \phi < 2\pi$, can be represented as a linear combination of the $Y_{\ell,m}(\theta, \phi)$ with ℓ integer valued and $-\ell \leq m \leq \ell$, also integer valued. We can interpret the $Y_{\ell,m}(\theta, \phi)$ as the eigenstates of the orbital angular momentum L and z projection L_z. The corresponding eigenvalues are $\ell\hbar$ and $m\hbar$.

   Quite generally we find it useful to have complete (and orthogonal) sets of functions of a certain class. For example, the functions $\sin(2\pi tn/T)$ and $\cos(2\pi tn/T)$ with n an integer comprise a complete (and orthogonal) set of functions of t that are periodic with period T ($f(t) = f(t + T)$). The $Y_{\ell,m}(\theta, \phi)$ above are such a set for functions that are well behaved (continuous) on the surface of a sphere. Such functions are almost always defined in terms of solving an eigenvalue/eigenfunction problem. This is especially true in the context of quantum mechanics.

From Phys. 225:

1. The uncertainty principle, $\Delta x \Delta p \geq \hbar/2$, requires that a small uncertainty in the location of a particle must be matched by a large uncertainty in the particle's momentum. We understand this point "physically" through that the fact that fine spatial resolution (*i.e.*, small Δx) corresponds to interactions with particles with short wavelengths (λ) and thus large momenta, where $p = h/\lambda$, with λ the de Broglie wavelength. This last point emphasizes the essential quantum mechanical point that real particles simultaneously exhibit *both* point-like (classical) and wave-like (quantum mechanical) behavior. As you may recall this feature is often used to motivate that fact that only certain energies and momenta are present for a particle confined to a box or an orbit, *i.e.*, the wavelength must be such as to "fit" in the box.

   The uncertainty principle encodes the fact that the processes corresponding to the measurement of location and the measurement of momentum "interact" with each other, *i.e.* the corresponding operators do *not* commute. Nonzero commutators play an essential role in quantum mechanics and in this course. Commutation relations (equations involving commutators) are at the heart of defining symmetries in terms of the underlying group theory.

2. The possible states of quantum mechanical (QM) systems are typically labeled in terms of the eigenvalues of a (complete) set of commuting operators. An example is the labeling of states in terms of the total energy and the total angular momentum J, or "spin" S for a single particle. Further, these quantities are "quantized" in the sense that only certain discrete (*not* continuous) values are allowed: $E_n = (n + 1/2)\hbar\omega$ (for quantized harmonic oscillators with n an integer), and $S = (n+1/2)\hbar$ (fermions) or $S = n\hbar$ (bosons). (We are unaware of this quantization in the context of "classical physics" simply because n is so large for classical systems, and a change by unity (*i.e.*, one unit of $\hbar$) is very difficult to detect. For example, recall that the number of atoms in a mole is of order 6×10^{23} and 10^{-23} is a *very* small number.)

   To the extent that the underlying physics is invariant under translations in time and rotations in space total energy and total angular momentum are constants of the motion. *i.e.*, are "conserved" quantities (a concept from Introductory physics).

   The simplest spin system is spin zero with the corresponding state vecor (or "bra") labeled as $|S, S_z\rangle = |0, 0\rangle$. This representation of the rotation group has only this *one* element and it follows that it *cannot* be changed by a rotation, *i.e.*, the spin structure is the same in all reference frames related by rotations and is typically referred to as the *singlet* representation. The next simplest case is spin 1/2 with two elements in the representation, $|S, S_3\rangle = |1/2, 1/2\rangle$ and $|S, S_3\rangle = |1/2, -1/2\rangle$ (the doublet representation). This fact leads to the *pair* of possible outcomes in the Stern-Gerlach experiment you discussed in Phys. 225. An appropriate rotation (or boost) can turn one of these states into the other. In general, a representation of spin S (with S an integer or half-integer) corresponds to $2S + 1$ elements (distinct states) labeled by $S_3 = S$, $S_z = S - 1$ ... $S_3 = -S$ (for a fixed choice of the $\hat{3}$ direction). Since the shift in S_3 at each step is unity (this is the quantization of spin in terms of $\hbar$), this arithmetic only works for S an integer or a half-integer, *i.e.*, for $2S + 1$ an integer. It is these states (same S but different S_3) that are transformed into each other by rotations (or the ladder-operator) and thus constitute a representation of the rotation group.

3. Quantum Mechanics as a description of "small" systems is characterized by "wave functions" for the energy eigenstates, which are *complex* valued solutions of differential equations, *e.g.*, Schödinger's Equation. The amplitudes for something to happen in QM are expressed in terms of "matrix elements" of operators, *i.e.*, an operator between a "bra" ($\langle|$) and a "ket" ($|\rangle$) representing the

"outgoing" and "incoming" states, respectively. The underlying arithmetic is greatly simplified by writing the wave functions or state vectors in terms of the eigenstates of the relevant operators, which typically means in terms of representations of the relevant symmetries. Hopefully the linear algebra describing the arithmetic of the corresponding state vectors and matrix elements (matrices) is a familiar concept for simple (low dimensionality) systems. (If not, we will work to make it familiar.) Finally note that probabilities, rates, cross sections are proportional to the absolute squares of amplitudes, *i.e.*, real numbers.

We will explicitly discuss some of the most useful of these topics in Chapters 1 and 5. The reader is also strongly encouraged to review the lecture notes from the last time I taught Phys. 227-228 (2008-2009), which are available here. The content of essentially all of the first ten lectures has application to our studies in Physics 226. Finally note that these 227/8 notes include worked examples and samples of how the computer program *Mathematica* can be used to both think about (make plots, *etc.*) and solve relevant exercises. That will remain true in Physics 226.