

Physics 226

Relativity

time dilation/Lorentz contraction: $\gamma = 1/\sqrt{1 - \vec{v}^2/c^2}$

4-coordinates:

$$x^\mu = (x^0, x^1, x^2, x^3), \quad x^0 = ct$$

Lorentz transformation matrix:

$$\|\Lambda^\mu{}_\nu\| = \begin{pmatrix} \gamma & \gamma(v/c) & 0 & 0 \\ \gamma(v/c) & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{for boost along } \hat{x}^1$$

Lorentz invariant dot product:

$$a \cdot b = a^0 b^0 - \vec{a} \cdot \vec{b} = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

invariant interval:

$$(\Delta x)^2 = (\Delta x^0)^2 - (\Delta x^1)^2 - (\Delta x^2)^2 - (\Delta x^3)^2$$

4-velocity:

$$u = \frac{dx(\tau)}{d\tau}, \quad u^\mu = (\gamma c, \gamma \vec{v}), \quad u^2 = c^2$$

4-momentum:

$$p = m u, \quad p^\mu = (E/c, \vec{p}) = (\gamma mc, \gamma m \vec{v})$$

$$p^2 = (E/c)^2 - \vec{p}^2 = m^2 c^2$$

4-force:

$$f = \frac{dp}{d\tau}, \quad p \cdot f = 0$$

4-acceleration:

$$a = \frac{du}{d\tau}, \quad u \cdot a = 0$$

constant acceleration:

$$u^0(\tau)/c = \cosh \frac{F\tau}{mc}, \quad u^1(\tau)/c = \sinh \frac{F\tau}{mc}$$

wave-vector:

$$k^\mu = (\omega/c, \vec{k})$$

observed frequency:

$$\omega_{\text{obs}} = u_{\text{obs}} \cdot k$$

E&M field strength:

$$\|F^\mu{}_\nu\| = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix}$$

Lorentz force:

$$f_{\text{Lorentz}}^\mu = \frac{q}{c} F^\mu{}_\nu u^\nu$$

$SU(2)$ Transformations

Pauli matrices: $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Transformation: $T_k(\alpha) = e^{i\sigma_k \alpha/2}$

Constants and Units

$$c = 2.997 \dots \times 10^8 \text{ m/s}$$

$$\hbar = 6.582 \dots \times 10^{-22} \text{ MeV s}$$

$$\hbar c = 197.3 \dots \text{ MeV fm}, \quad 1 \text{ fm} = 10^{-15} \text{ m}$$

$$(\hbar c)^2 = 0.389 \dots \text{ GeV}^2 \text{ mbarn}, \quad 1 \text{ barn} = 10^{-28} \text{ m}^2$$

$$\alpha = e^2/(4\pi\epsilon_0\hbar c) = 1/137.0 \dots$$

Nuclei

nucleus	symbol	rest energy	lifetime	spin*	decay type
neutron	n	939.6 MeV	15 min	1/2	β decay
hydrogen	${}_1^1\text{H}$	938.27 MeV	stable	1/2	—
deuterium	${}_1^2\text{H}$	1875.61 MeV	stable	1	—
tritium	${}_1^3\text{H}$	2808.92 MeV	17.8 yr	1/2	β decay
helium-3	${}_2^3\text{He}$	2808.39 MeV	stable	1/2	—
helium-4	${}_2^4\text{He}$	3727.38 MeV	stable	0	—
helium-6	${}_2^6\text{He}$	5605.5 MeV	1.16 s	0	β decay
lithium-6	${}_3^6\text{Li}$	5601.5 MeV	stable	1	—
lithium-7	${}_3^7\text{Li}$	6533.8 MeV	stable	3/2	—
beryllium-7	${}_4^7\text{Be}$	6534.2 MeV	77 day	3/2	e^- capture
beryllium-10	${}_4^{10}\text{Be}$	9325.5 MeV	2.2 Myr	0	β decay
boron-10	${}_5^{10}\text{B}$	9324.4 MeV	stable	3	—
boron-11	${}_5^{11}\text{B}$	10253 MeV	stable	3/2	—
boron-14	${}_5^{14}\text{B}$	13062 MeV	18 ms	2	β decay
carbon-11	${}_6^{11}\text{C}$	10254 MeV	29 m	3/2	e^+ emission
carbon-12	${}_6^{12}\text{C}$	11175 MeV	stable	0	—
carbon-14	${}_6^{14}\text{C}$	13041 MeV	5.7 Kyr	0	β decay
nitrogen-14	${}_7^{14}\text{N}$	13041 MeV	stable	1	—
calcium-41	${}_{20}^{41}\text{Ca}$	38146 MeV	1.5 Myr	7/2	e^- capture
bismuth-209	${}_{83}^{209}\text{Bi}$	194622 MeV	2×10^{19} yr	9/2	α decay

*In units of \hbar .

Leptons

particle	symbol	rest energy	lifetime	spin*	charge†	L
electron (anti)neutrino	$\nu_e(\bar{\nu}_e)$	< 2 eV	≈ stable	1/2	0	+1(−1)
muon (anti)neutrino	$\nu_\mu(\bar{\nu}_\mu)$	< 2 eV	≈ stable	1/2	0	+1(−1)
tau (anti)neutrino	$\nu_\tau(\bar{\nu}_\tau)$	< 2 eV	≈ stable	1/2	0	+1(−1)
electron(positron)	$e^-(e^+)$	0.511 MeV	stable	1/2	−1(+1)	+1(−1)
muon(antimuon)	$\mu^-(\mu^+)$	105.7 MeV	2 μs	1/2	−1(+1)	+1(−1)
tau(antitau)	$\tau^-(\tau^+)$	1777 MeV	0.3 ps	1/2	−1(+1)	+1(−1)

Hadrons

particle	symbol	rest energy	lifetime	spin	charge	B
pion	π^0	135.0 MeV	8×10^{-17} s	0	0	0
pion	π^+, π^-	139.57 MeV	26 ns	0	+1, −1	0
kaon	K^+, K^-	493.7 MeV	12 ns	0	+1, −1	0
kaon	K^0	497.7 MeV	90 ps/51 ns	0	0	0
eta	η	547.5 MeV	5×10^{-19} s	0	0	0
rho	ρ^+, ρ^0, ρ^-	775 MeV	4×10^{-24} s	1	+1, 0, −1	0
omega	ω	783 MeV	8×10^{-23} s	1	0	0
(anti)proton	$p(\bar{p})$	938 MeV	stable	1/2	+1(−1)	1(−1)
(anti)neutron	$n(\bar{n})$	940 MeV	886 s	1/2	0	1(−1)
eta-prime	η'	958 MeV	3×10^{-21} s	0	0	0
Lambda	Λ	1116 MeV	0.26 ns	1/2	0	1
Delta	$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	1232 MeV	6×10^{-24} s	3/2	+2, +1, 0, −1	1
Cascade	Ξ^0	1315 MeV	0.29 ns	1/2	0	1
Cascade	Ξ^-	1321 MeV	0.16 ns	1/2	−1	1
Omega	Ω^-	1672 MeV	0.08 ns	3/2	−1	1

†In units of $|e| = 1.602 \dots \times 10^{-19}$ C.