Physics 226

Equation & Data Summary

Relativity

time dilation/Lorentz contraction: $\gamma = 1/\sqrt{1-\vec{v}^2/c^2}$

4-coordinates: $x^\mu = (x^0, x^1, x^2, x^3)\,, \quad x^0 = ct$

Lorentz transformation matrix: $\|\Lambda^\mu{}_\nu\| = \begin{pmatrix} \gamma & \gamma(v/c) & 0 & 0 \\ \gamma(v/c) & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ for boost along $\hat{x}^1$

Lorentz invariant dot product: $a \cdot b = a^0 b^0 - \vec{a}\cdot\vec{b} = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$

invariant interval: $(\Delta x)^2 = (\Delta x^0)^2 - (\Delta x^1)^2 - (\Delta x^2)^2 - (\Delta x^3)^2$

4-velocity: $u = \dfrac{dx(\tau)}{d\tau}\,, \qquad u^\mu = (\gamma c, \gamma\,\vec{v})\,, \qquad u^2 = c^2$

4-momentum: $p = m\,u\,, \qquad p^\mu = (E/c, \vec{p}) = (\gamma mc, \gamma m\vec{v})$

$p^2 = (E/c)^2 - \vec{p}^2 = m^2c^2$

Total Energy: $s = P^2_{\rm TOT} = (E_{\rm CM}/c)^2$

4-force: $f = \dfrac{dp}{d\tau}\,, \qquad p\cdot f = 0$

4-acceleration: $a = \dfrac{du}{d\tau}\,, \qquad u\cdot a = 0$

constant acceleration: $u^0(\tau)/c = \cosh\dfrac{F\tau}{mc}\,, \quad u^1(\tau)/c = \sinh\dfrac{F\tau}{mc}$

wave-vector (light) : $k^\mu = (\omega/c, \vec{k}) = (2\pi/\lambda)(1, \hat{k})$

wave-vector (particle) : $k^\mu = p^\mu/\hbar, \quad \lambda_{\rm deBroglie} = h/|\vec{p}|$

observed frequency: $\omega_{\rm obs} = u_{\rm obs}\cdot k$

E&M field strength: $\|F^\mu{}_\nu\| = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix}$

Lorentz force: $f^\mu_{\rm Lorentz} = \dfrac{q}{c}\,F^\mu{}_\nu\,u^\nu$

$SU(2)$ Transformations

Pauli matrices: $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Transformation: $T_k(\alpha) = e^{i\sigma_k \alpha/2}$

Quantum Mechanics & Symmetries

Hamiltonian:	$H = H^\dagger$
time evolution:	$i\hbar \frac{d}{dt}\|\psi(t)> = H\|\psi(t)>$ $\|\psi(t)> = U(t)\|\psi(0)>$ $U(t) = e^{-iHt}$
symmetry (time independent):	$T^\dagger = T^{-1}, \quad [T, H] = 0$
continuous symmetry:	$T(a) = e^{iQa}$ $Q^\dagger = Q, \quad [Q, H] = 0$
angular momentum addition:	$J^1 + J^2 \Rightarrow J^{\text{tot}} \in \{\|J^1\| + \|J^2\|, \cdots, \|J^1 - J^2\|\}$ $\frac{1}{2} + \frac{1}{2} \Rightarrow \begin{cases} J = 1: & \{\uparrow\uparrow, (\uparrow\downarrow + \downarrow\uparrow), \downarrow\downarrow\} \\ J = 0: & (\uparrow\downarrow - \downarrow\uparrow) \end{cases}$
ladder operators:	$J_\pm = J_1 \pm iJ_2,$ $J^2\|j, m\rangle = j(j+1)\hbar^2\|j, m\rangle,$ $J_3\|j, m\rangle = m\hbar\|j, m\rangle$ $J_\pm\|j, m\rangle = \hbar\sqrt{j(j+1) - m(m \pm 1)}\,\|j, m \pm 1\rangle$
parity (P):	$P\|\Psi_L^{A+B}\rangle = \eta_P^A \eta_P^B (-1)^L \|\Psi_L^{A+B}\rangle$
charge conjugation (C):	$C\|f\bar{f}\text{pair}, L, S\rangle = (-1)^{L+S}\|f\bar{f}\text{pair}, L, S\rangle$
CP:	$CP\|f\bar{f}\text{pair}, L, S\rangle = (-1)^{S+1}\|f\bar{f}\text{pair}, L, S\rangle$
G parity ($G = Ce^{i\pi I_2}$):	$G\|f\bar{f}\text{pair}, L, S, I\rangle = (-1)^{L+S+I}\|f\bar{f}\text{pair}, L, S, I\rangle$

Constants and Units

$\alpha = e^2/(4\pi\epsilon_0\hbar c) = 1/137.0\cdots$	$N_A = 6.022\cdots \times 10^{23}$ mol^{-1}
$c = 2.997\cdots \times 10^{8}$ m/s	1 (metric) ton $= 10^3$ kg
$\hbar = 6.582\cdots \times 10^{-22}$ MeV s	1 amu $= 1$ g$/(N_A$ mol$) = 931.5\cdots$ MeV$/c^2$
$= 1.055\cdots \times 10^{-34}$ J s	$= 1.661\cdots \times 10^{-27}$ kg
$\hbar c = 197.3\cdots$ MeV fm	1 fm $= 10^{-15}$ m
$(\hbar c)^2 = 0.389\cdots$ GeV^2 mbarn	1 barn $= 10^{-28}$ m^2

Gauge Bosons

particle	**symbol**	**rest energy**	**lifetime**	**spin***	**charge**†
photon	γ	0	stable	1	0
gluon	g	0	—	1	0
W-boson	$W^{\pm}$	80.4 GeV	3×10^{-25} s	1	± 1
Z-boson	Z	91.2 GeV	3×10^{-25} s	1	0

Quarks

flavor	**symbol**	**mass**	**spin***	**charge**†
up	u	≈ 2 MeV$/c^2$	1/2	2/3
down	d	≈ 5 MeV$/c^2$	1/2	$-1/3$
strange	s	≈ 95 MeV$/c^2$	1/2	$-1/3$
charm	c	1.2 GeV$/c^2$	1/2	2/3
bottom	b	4.2 GeV$/c^2$	1/2	$-1/3$
top	t	173 GeV$/c^2$	1/2	2/3

Leptons

particle	**symbol**	**rest energy**	**lifetime**	**spin***	**charge**†	***L***
electron (anti)neutrino	$\nu_e(\bar{\nu}_e)$	< 2 eV	$\approx$ stable	1/2	0	$+1(-1)$
muon (anti)neutrino	$\nu_\mu(\bar{\nu}_\mu)$	< 2 eV	$\approx$ stable	1/2	0	$+1(-1)$
tau (anti)neutrino	$\nu_\tau(\bar{\nu}_\tau)$	< 2 eV	$\approx$ stable	1/2	0	$+1(-1)$
electron(positron)	$e^-(e^+)$	0.511 MeV	stable	1/2	$-1(+1)$	$+1(-1)$
muon(antimuon)	$\mu^-(\mu^+)$	105.7 MeV	2 μs	1/2	$-1(+1)$	$+1(-1)$
tau(antitau)	$\tau^-(\tau^+)$	1777 MeV	0.3 ps	1/2	$-1(+1)$	$+1(-1)$

*In units of $\hbar$.

†In units of $|e| = 1.602\cdots \times 10^{-19}$ C.

Nuclei

nucleus	symbol	rest energy	lifetime	spin*	decay type
neutron	n	939.6 MeV	15 min	1/2	β decay
hydrogen	${}^{1}_{1}\mathrm{H}$	938.27 MeV	stable	1/2	—
deuterium	${}^{2}_{1}\mathrm{H}$	1875.61 MeV	stable	1	—
tritium	${}^{3}_{1}\mathrm{H}$	2808.92 MeV	17.8 yr	1/2	β decay
helium-3	${}^{3}_{2}\mathrm{He}$	2808.39 MeV	stable	1/2	—
helium-4	${}^{4}_{2}\mathrm{He}$	3727.38 MeV	stable	0	—
helium-6	${}^{6}_{2}\mathrm{He}$	5605.5 MeV	1.16 s	0	β decay
lithium-6	${}^{6}_{3}\mathrm{Li}$	5601.5 MeV	stable	1	—
lithium-7	${}^{7}_{3}\mathrm{Li}$	6533.8 MeV	stable	3/2	—
beryllium-7	${}^{7}_{4}\mathrm{Be}$	6534.2 MeV	77 day	3/2	e^- capture
beryllium-10	${}^{10}_{4}\mathrm{Be}$	9325.5 MeV	2.2 Myr	0	β decay
boron-10	${}^{10}_{5}\mathrm{B}$	9324.4 MeV	stable	3	—
boron-11	${}^{11}_{5}\mathrm{B}$	10253 MeV	stable	3/2	—
boron-14	${}^{14}_{5}\mathrm{B}$	13062 MeV	18 ms	2	β decay
carbon-11	${}^{11}_{6}\mathrm{C}$	10254 MeV	29 m	3/2	e^+ emission
carbon-12	${}^{12}_{6}\mathrm{C}$	11175 MeV	stable	0	—
carbon-14	${}^{14}_{6}\mathrm{C}$	13041 MeV	5.7 Kyr	0	β decay
nitrogen-14	${}^{14}_{7}\mathrm{N}$	13041 MeV	stable	1	—
oxygen-16	${}^{16}_{8}\mathrm{O}$	14899 MeV	stable	0	—
calcium-41	${}^{41}_{20}\mathrm{Ca}$	38146 MeV	1.5 Myr	7/2	e^- capture
bismuth-209	${}^{209}_{83}\mathrm{Bi}$	194622 MeV	2×10^{19} yr	9/2	α decay

*In units of $\hbar$.
†In units of $|e| = 1.602\cdots \times 10^{-19}$ C.

Mesons

particle	symbol	rest energy	lifetime	spin$^{*\text{Parity}}$	charge†	strangeness
pion	π^0	135.0 MeV	8.4×10^{-17} s	0^-	0	0
pion	π^+, π^-	139.6 MeV	2.6×10^{-8} s	0^-	$+1, -1$	0
kaon	K^+, K^-	493.7 MeV	1.2×10^{-8} s	0^-	$+1, -1$	$+1, -1$
kaon	$K^0, \bar{K}^0$	497.6 MeV	90 ps/51 ns	0^-	0	$+1, -1$
eta	η	547.5 MeV	5×10^{-19} s	0^-	0	0
rho	ρ^+, ρ^0, ρ^-	775 MeV	4×10^{-24} s	1^-	$+1, 0, -1$	0
omega	ω	783 MeV	8×10^{-23} s	1^-	0	0
K-star	K^{*+}, K^{*-}	891.7 MeV	10^{-23} s	1^-	$+1, -1$	$+1, -1$
K-star	$K^{*0}, \bar{K}^{*0}$	896.0 MeV	10^{-23} s	1^-	0	$+1, -1$
eta-prime	η'	958 MeV	3×10^{-21} s	0^-	0	0
phi	ϕ	1020 MeV	2×10^{-22} s	1^-	0	0

Baryons

particle	symbol	rest energy	lifetime	spin$^{*\text{Parity}}$	charge†	strangeness
proton	p	938 MeV	stable	$\frac{1}{2}^+$	$+1$	0
neutron	n	940 MeV	882 s	$\frac{1}{2}^+$	0	0
Lambda	Λ	1116 MeV	2.6×10^{-10} s	$\frac{1}{2}^+$	0	-1
Sigma	Σ^+	1189 MeV	8.0×10^{-11} s	$\frac{1}{2}^+$	$+1$	-1
Sigma	Σ^0	1193 MeV	7.4×10^{-20} s	$\frac{1}{2}^+$	0	-1
Sigma	Σ^-	1197 MeV	1.5×10^{-10} s	$\frac{1}{2}^+$	-1	-1
Delta	Δ^{++}, Δ^+	1232 MeV	6×10^{-24} s	$\frac{3}{2}^+$	$+2, +1$	0
	Δ^0, Δ^-				$0, -1$	
Cascade	Ξ^0	1315 MeV	2.9×10^{-10} s	$\frac{1}{2}^+$	0	-2
Cascade	Ξ^-	1322 MeV	1.6×10^{-10} s	$\frac{1}{2}^+$	-1	-2
Sigma-star	$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	1385 MeV	2×10^{-23} s	$\frac{3}{2}^+$	$+1, 0, -1$	-1
Cascade-star	Ξ^{*0}, Ξ^{*-}	1530 MeV	7×10^{-23} s	$\frac{3}{2}^+$	$0, -1$	-2
Omega	Ω^-	1672 MeV	8.2×10^{-11} s	$\frac{3}{2}^+$	-1	-3

*In units of $\hbar$.

†In units of $|e| = 1.602\cdots \times 10^{-19}$ C.

40. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

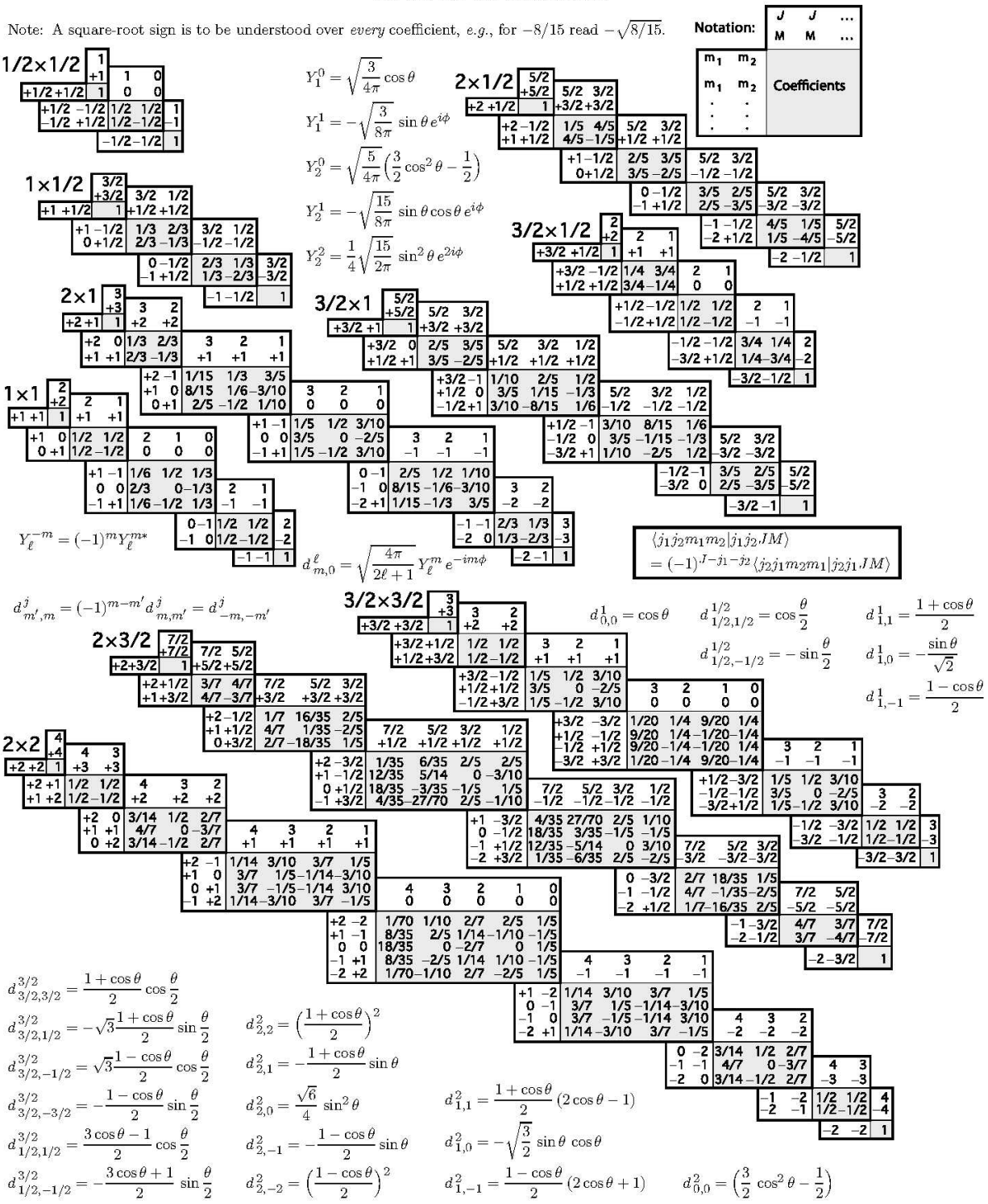

Figure 40.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).