1. Das & Ferbel, problem 10.1 (3 pts)

2. (4 pts) At high energy, $E_{\text{c.m.}} \gg m_\mu c^2$, the cross-section for $e^+e^- \to \mu^+\mu^-$ scattering due to electromagnetic interactions must have the form $\sigma = C\,\hbar^a\,c^b\,\alpha^p/s^q$, for some exponents a, b, p, q and overall dimensionless coefficient C, which turns out to equal $4\pi/3$. Here $\alpha \equiv e^2/(4\pi\epsilon_0\hbar c)$ is the fine structure constant, and $s \equiv -(p_{e^-}+p_{e^+})^2 = E^2_{\text{c.m.}}/c^2$. The power of α is the same as for elastic scattering due to Coulomb interactions. What is the exponent p? Justify your reasoning. Determine the other exponents using dimensional analysis. What is the value of the cross-section in barns (or nanobarns, femtobarns, *etc.*) when $E_{\text{c.m.}} = 1$ GeV?

3. (4 pts) High energy muon neutrinos can scatter inelastically off electrons and produce charged muons, $\nu_\mu e^- \to \nu_e \mu^-$. (a) In the electron rest frame, what is the minimum neutrino energy required for this reaction? (b) For center-of-mass energies large compared to the muon mass, the cross section for this reaction is $\sigma_{\nu_\mu e^- \to \nu_e \mu^-} = (G_F^2/\pi)\,s$, where $G_F \simeq 12\,\text{TeV}^{-2}$ and $s = -(p_{\nu_\mu} + p_{e^-})^2$. (Factors of $\hbar$ and c are omitted in this formula). Using your result from the previous problem, at what center-of-mass energy does this weak interaction cross section become comparable to the electromagnetic cross section for $e^+e^- \to \mu^+\mu^-$? What is the value of the cross section in barns (or picobarns, *etc.*) at this point? [The above formula ceases to be valid near or above this energy. The result of this comparison is an estimate of the energy scale where it no longer makes sense to treat weak and electromagnetic interactions as completely distinct.]

4. (12 pts) Recall that we briefly noted in Chapter 5 that the decays in the neutral kaon system are particularly interesting, and that the symmetry CP plays a special role. We explore these points in more detail here. The K_0 and $\bar{K}_0$ mesons are both neutral particles with zero baryon and lepton number, but with opposite strangeness. Let $|K_0\rangle$ denote a state containing a single K_0 at rest (and nothing else), and likewise let $|\bar{K}_0\rangle$ denote a single $\bar{K}_0$ at rest.

   (a) (2 pts) In a hypothetical world without weak interactions, these particles would be absolutely stable and degenerate in mass. Within the two-dimensional space of states spanned by $|K_0\rangle$ and $|\bar{K}_0\rangle$, the Hamiltonian would have the diagonal form $H = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix}$, where $A = m_{K_0}c^2$. Justify these assertions. What symmetry implies the equality of mass of these particles?

   (b) (2 pts) Weak interactions add terms to the Hamiltonian which do not respect all the symmetries of strong (and electromagnetic) interactions. Within this two-dimensional space of states, the effect of weak interactions is to add an odd-diagonal term, so that $H = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$, with B real. What are the eigenstates of this Hamiltonian? What are the corresponding rest-energies?

   (c) (2 pts) Given this (perturbed) Hamiltonian, what is the time-evolution of the state $|K_0\rangle$? In other words, if $|\Psi(0)\rangle = |K_0\rangle$, what is $|\Psi(t)\rangle$?

   (d) (2 pts) The phases of the states $|K_0\rangle$ and $|\bar{K}_0\rangle$ may be chosen so that charge conjugation interchanges the two states, $C|K_0\rangle = |\bar{K}_0\rangle$ and $C|\bar{K}_0\rangle = |K_0\rangle$. (Note that this choice of plus signs is *not* a uniformly followed convention. You will also find in the literature the alternative choice $C|K_0\rangle = -|\bar{K}_0\rangle$ and $C|\bar{K}_0\rangle = -|K_0\rangle$. The physics does not change, but the expressions look different.) What does a CP transformation do to these states? Show that the eigenstates of the perturbed Hamiltonian are also eigenstates of CP.

   (e) (2 pts) The above discussion ignores the fact that these states can decay. In reality, two types of neutral kaons are observed, with different decay modes and lifetimes and very slightly different masses. The K_S predominantly decays to two pions, with lifetime $\tau_{K_S} = 90$ ps. The

K_L predominantly decays to three pions, with lifetime $\tau_{K_L} = 52$ ns. Their fractional mass difference is less than a part in 10^{14}, $m_{K_L} - m_{K_S} = 3.5 \times 10^{-12}$ MeV. Explain why the two pions in the decay $K_S \to \pi^0\pi^0$ or $\pi^+\pi^-$ must have zero orbital angular momentum, and hence this final state has $CP = +1$. Similarly explain why a three π^0 or $\pi^+\pi^-\pi^0$ state, with no orbital angular momentum, will have $CP = -1$.

(f) (2 pts) Based on the above, which linear combination of $|K_0\rangle$ and $|\bar{K}_0\rangle$ should be identified with the K_S, and which linear combination makes a K_L?

Note: The above model of the Hamiltonian ignores CP violation (which is a part-in-10^3 effect), as well as the effect of finite lifetimes of the particles. For a more elaborate treatment which fixes these shortcomings, see chapter 12 in Das & Ferbel.