

1. Das & Ferbel, problem 10.2 (4 pts) Proceed either as discussed in the context of Table 10.2 in the text or using the table of Clebsch-Gordan coefficients at the end of Chapter 5. Note that the  $I = 3/2$   $K^*$  meson postulated in part a) is not to be confused with the real  $I = 1/2$   $K^*$  meson, which could be the one in part b).
2. Das & Ferbel, problem 10.3 (4 pts) Again the table of C-G coefficients may be useful.
3. Das & Ferbel, problem 11.4 (3 pts)
4. Das & Ferbel, problem 11.5 (5 pts)
5. (8 pts) Consider the scattering of protons on protons. At a high energy collider, such as the now operating Large Hadron Collider (LHC) at Geneva, bunches of protons are accelerated and then caused to collide, essentially head-on, at the center of particle detectors. Suppose one bunch of protons, moving to the right at speed  $v_1$ , contains  $N_1$  protons, and the other bunch, moving to the left at speed  $v_2$ , contains  $N_2$  protons. In the frame in which this collision is being described, suppose that the right-moving bunch has length  $L_1$  and transverse area  $A$ , so that the density of protons in this bunch is  $n_1 = N_1/(L_1 A)$ . Assume, for simplicity, that the density of protons is uniform throughout the cylindrical volume of the bunch. Similarly, suppose that the left-moving bunch has length  $L_2$  and the same transverse area  $A$ .

The number of scattering events produced by the collision of these two bunches will obviously depend on the number of protons in the bunches, and on how tightly focused are the bunches (*i.e.*, decreasing the transverse area  $A$  should increase the probability of scatterings). The number of scattering events produced by the collision of these two bunches is related to the *cross-section*  $\sigma$  that characterizes the likelihood of a single proton-proton scattering via  $(\# \text{ events}) = (N_1 N_2 / A) \sigma$ .

- (a) (2 pts) The cross section, as just defined, is invariant under boosts along the direction of the colliding particles. Justify this.
- (b) (2 pts) Consider a classical picture in which the particles are regarded as little hard spheres with radius  $R$ . Explain how the above relation naturally emerges. What is the relation between the cross-section  $\sigma$  and  $R$ ?
- (c) (2 pts) Suppose  $L_2 \ll L_1$ . What is the time  $t_{\text{cross}}$  required for bunch 1 to sweep across bunch 2? Explain why the event rate  $\Gamma$ , defined as the number of scattering events per unit time, during the crossing of these two bunches, is given by  $\Gamma = v_{\text{rel}} n_1 N_2 \sigma$ , where  $v_{\text{rel}} = v_1 + v_2$  is the relative velocity of the two bunches.
- (d) (2 pts) At the LHC, there will be about  $10^{11}$  protons per bunch, with bunches spaced 25 ns apart. The proton energy (in the lab frame) will eventually be 7 TeV. The bunch length is about 8 cm, and the transverse size is about  $17 \mu\text{m}$ . At high energies, the proton-proton cross section is approximately given by  $\sigma \simeq A + B \ln^2(s/s_0)$ , with  $s \equiv -(p_{\text{tot}})^2$ ,  $A = 35 \text{ mb}$ ,  $B = 0.3 \text{ mb}$ , and  $s_0 = (5.4 \text{ GeV}/c)^2$ . How many scatterings should occur during a single bunch-crossing?