Public Affairs 527 **Hypothesis testing example solutions Part I**

1. The research design compares an experimental group (program participants) with a control group (non-participants). The outcome variable is the recidivism rate and the main explanatory variable is program participation. The other variables (opportunity days, age, high risk) serve as control variables that could also affect the outcome. Any other factors that might be correlated with program participation AND the outcome of recidivism could confound our results. If participation in the program is correlated with factors that affect recidivism (such as motivation to succeed or community ties), then outcomes for the groups could differ even IN THE ABSENCE of the program!

2. Here we need a 90% confidence interval around the proportion re-convicted for the program participants (.235).

Because it is a proportion, we use the proportion to calculate the SE.

P = .235 $SE = \sqrt{p(1-p)/n} = \sqrt{.235(1-.235)/187} = .031$

Use Z score (because it is a proportion and large sample) for 5% in each tail, so the confidence interval is:

.235 ± 1.65(.031) = **[.184, .286]**

If there are 187 participants, then this translates to 34 to 54 **re-convictions** expected by the end of the evaluation (187*.184 and 187*.286). Note that we know exactly how many there were--but with a different group of participants the number could have been different.

3. We want to test the hypothesis that the population mean for participants is .25 (the alternative is that it is not .25).

The decision rule for significant at .10 is reject if the test statistic is greater in absolute value than **1.65**.

Our test statistic is: $z = \frac{\hat{p} - p_{null}}{\sqrt{\frac{p_{null}(1-p_{null})}{n}}} = \frac{.235 - .25}{\sqrt{\frac{.25(1-.25)}{187}}} = -.47$

So, we **can't reject** the null hypothesis that the program sample has the same recidivism rate as the prison average.

To get the p value, we can look up the test statistic in the z table to find the probability associated with a sample mean being at least .47 standard errors away from the population mean.

The probability that Z is between 0 and 47 is .1808, so we take .5 -.1808=.3192 and multiple it by 2 to get the 2 sided **p value of .64**. So if the true (long term) recidivism rate were .25 for the program participants, then we'd get a value this far from .25 more than half the time (64 percent of the samples). This shows quite a bit of support for the idea that the program participants are NOT different than other inmates.

To test our hypothesis at the **5%** significance level we use the same test statistic, but a critical value of 1.96. Since **|-.47| < 1.96 we can't reject at the 5% level**. At the **1%** significance level our critical value is 2.58 and we can't reject because **|-.47| < 2.58.** Be sure that you understand why we will ALWAYS not be able to reject at a lower significance level (e.g., 5% vs. 10%) if we can't reject at the higher level. Lower significance levels are the complement of higher confidence levels which are higher standards to meet. We couldn't reject until the significance level got to 64% (the p value)!