Public Affairs 527 **Hypothesis testing example solutions**
PART II

4. Here we need to test the difference in proportions for 2 independent samples. Our point estimate is .235 - .257 = -.022 So the re-conviction rate is lower for participants by about 2 percentage points.

$$\hat{P} = \frac{P_1(n_1) + P_2(n_2)}{n_1 + n_2} = \frac{.235(187) + .257(292)}{187 + 292} = .248$$

$$SE = \sqrt{\hat{P}(1-\hat{P})(\frac{1}{n_1} + \frac{1}{n_2})} = \sqrt{.248(1-.248)(\frac{1}{187} + \frac{1}{292})} = .040$$

For a proportion, we need to calculate the SE by using an average P for the sample:
Our decision rule will be to reject the null hypothesis of no difference in proportions (diff = 0) if our Z value is far enough away from 0 (|Z| > 1.65).

$$Z = \frac{(P_1 - P_1) - 0}{SE} = -.022/.040 = -.55$$

So, **we cannot reject** the null hypothesis at the 10% significance level because our point estimate is less than 1.65 standard errors from the null difference of 0.

We need to look up the p value in the Z table. Look up .55 and subtract from .5. The probability associated with .55 is .2088 so the 2-tailed p value is 2(.2912) = **.5824** This shows relatively high support for the null hypothesis of no difference in the rates of re-conviction for the 2 groups.

The 10% confidence interval is:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = (.235 - .257) \pm 1.65\sqrt{\frac{.235(1-.235)}{187} + \frac{.257(1-.257)}{292}} = -.09 \text{ to } .04$$

5. Each of these factors could affect reconvictions and, if these factors differ for participants and non-participants, then we might not get a good measure of the effects of the program by the simple comparison.

On average, program participants had a longer period over which to be re-convicted (higher average opportunity days). [The difference of 41.2 days is not statistically significant, but is fairly large.]
$\hat{SE} = \sqrt{S_2^2/n_2 + S_1^2/n_1} = \sqrt{355^2/187 + 375^2/292} = 34.0$ $t = 41.2/34.0 = 1.21$ More opportunity days could raise the re-conviction rate and make a significant program impact less likely.

The average ages differ only slightly (2.2 years), but here the difference is statistically significant.
$\hat{SE} = \sqrt{6.9^2/187 + 8.4^2/292} = .70$ $t = -2.2/.7 = -3.14$ We don=t know anything about relationship of age to reconviction, but if younger releasees are more likely to be reconvicted, this too could create an underestimate in the effect of the program.

The proportion at high risk of reconviction differs by 4 percentage points, which is not statistically significant but could nonetheless affect our comparison. Because FEWER of the participants are high risk, our comparison might OVERESTIMATE the effect of the program. To check this we would need the data separately for low and high risk for participants and nonparticipants. [This is a check on how good their risk assessment tool is.]

$$\hat{p} = (p_1 n_1 + p_2 n_2)/(n_1 + n_2) = (.34(187) + .38(292))/(187 + 292) = .36$$

$$\hat{SE} = \sqrt{\hat{p}(1 - \hat{p})(1/n_2 + 1/n_1)} = \sqrt{.36(1 - .36)(1/187 + 1/292)} = .045$$

$$t = .04/.045 = .89$$

So, it is possible that the longer evaluation period (opportunity days) and younger age among the participants could result in an underestimate of the effects of the program on reconviction by our simple comparison. But more high risk among non-participants could create an overestimate of the program effect.

6. A simple back-of-the envelope answer to this question we can just divide the budget cost by the expected number of re-convictions prevented by the program. The program did not prevent all re-convictions, but did lower the rate by 2.2 percentage points.

Expected number of prevented convictions is:
(# of participants)x(difference in re-conviction rate) = 187(.022) = 4.11 re-conviction.

Break-even cost of reconviction is:
(Annual Budget cost of program)/(prevented re-conviction) = 150,000/4.11 = **$36,496**
So, if the cost to the public of a re-conviction is greater than $36,496, then the program will break even.

In a more complete analysis:

- X Use a confidence interval around the proportion to show the sensitivity to uncertainty
- X Use information on the timing of costs and benefits (expected re-convictions in each year come later than costs which come in first year) and discount rate.
- X Include some assessment of costs (or benefits) associated with use of volunteer labor [It is a cost of labor for volunteers, but they (and system) may benefit in other ways.]
- X Include the value of other possible effects of program on behavior of inmates, self-esteem, or quality of life.
- X Use more precise estimate of effect of program on re-convictions by using multivariate analysis to control for other differences (see 1C).

7. We have completed the initial evaluation of the life-skills program. We used information on reconvictions, time since release, age, and high-risk assessment for program 187 participants and, for comparison, a group of 292 non-participants.

We found a slightly lower rate of reconviction among participants (23.5 percent) than for non-participants (25.7 percent), but the difference is not large enough to statistically verify the effectiveness of the program.[1] However, program participants were somewhat younger and had longer time periods in which to be reconvicted (808 days vs. 766 days for non-participants).[2] These differences could lead to an underestimate of the effect of the program on reconvictions. On the other hand, the program effect might be overestimated because fewer participants were at high risk for reconviction (34% vs. 38% for non-participants). Further analysis of subgroups of high-risk inmates, younger inmates, and inmates with similar opportunity days could facilitate understanding of these effects.

Time and data limitations did not allow a complete benefit-cost of the program. However, we estimated that given the current annual budget of $150,000 and the decrease of 2.2 percentage points in re-conviction rates, each prevented re-conviction would need to save $36,496 in order to make this program break-even.

These data suggest that the effects of the program on reconvictions are likely to be small, but may be significant over a longer time period and if the public costs of reconviction are large. We stand ready to do more complete multivariate analysis of these data and an expanded benefit-cost analysis if the committee deems it to be worthwhile.

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[1] Z=.55; p= .58

[2] These differences were not statistically significant, but may still lead to a underestimate of the program effect on reconvictions.