Student Number:_____ **February 11**
QUIZ 3

Ground rules:

- DO NOT WRITE YOUR NAME ON THE QUIZ. If you're worried about losing the first page, write your student number on each page.
- You can use your notes or books, but you may not communicate with other people about this exam nor the material covered by it.
- In order to receive as much credit as possible, please show all of your work. Showing that you understand the question and know how to set up the solution correctly is more important than arriving at the exact answer.
- Read each question carefully and answer all parts of each question.
- Good luck!

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1. Here is a probability distribution of number of children age 14 or younger living in a household in Washington State in 2002. Use this table to answer questions 1a and 1b below.

Number Children age 14 or younger X	P(X)
0	.693
1	.141
2	.118
3	.034
4	.011
5	.002
6	.001

1a. What is the probability that a household will have 4 or more children under the age of 14 living in it? Make sure to state your answers in terms of probability and to interpret your answer. (5 points)

P(X>=4)=.011+.002+.001=.014

The probability that a household in Washington State will have four or more children under the age of 14 is 1.4%, fairly small.

1b. From this distribution, calculate the average number of children under 14 in Washington Households and the standard deviation. Show your work and make sure to interpret your results. (10 points)

$E(X) = \sum xP(x)$

			Method 2 from notes		Method 1 from notes		
1	2	3	4	5	6	7	8
Number Children age 14 or younger X	P(X)	xP(x)	X^2	$X^2P(x)$	$(x-\mu)$	$(x-\mu)^2$	$(x-\mu)^2P(x)$
0	0.693	0	0	0	-0.539	0.290521	0.201331
1	0.141	0.141	1	0.141	0.461	0.212521	0.029965
2	0.118	0.236	4	0.472	1.461	2.134521	0.251873
3	0.034	0.102	9	0.306	2.461	6.056521	0.205922
4	0.011	0.044	16	0.176	3.461	11.97852	0.131764
5	0.002	0.01	25	0.05	4.461	19.90052	0.039801
6	0.001	0.006	36	0.036	5.461	29.82252	0.029823
	E(X)=	**0.539**	$E(X^2)=$	1.181		**V(X)=**	**0.890479**
						SD(X)=	**0.943652**

E(X)=0.539. So, the average household in Washington State has fewer than one child under the age of 14.

To find the standard deviation, need to use equation:

$s^2 = V(X) = E\left[(X-m)^2\right] = \sum (x-m)^2 P(x)$ (see columns 6-8 for this method)

OR

$s^2 = V(X) = E(X^2) - [E(X)]^2$ This is mathematically easier, so column 4 and 5 help us to calculate $E(X^2)$

Here are the pieces of the equation for method 2 from the notes:
E(X)=0.539, so $[E(X)]^2=0.539^2=.290521$ (from table, column 3)
$E(X^2)=1.181$ (from table, column 5)

V(X)=1.181-.290521=.89, $s = \sqrt{s^2} = SD(X)$, so SD(X)=.94

The standard deviation of children under 14 in households in Washington State is .94, so I would say that family size does not vary very much. Family size appears to be fairly closely clustered around the mean.

2. Recent new stories have talked about the problem of high levels of methylmercury (a form of mercury), which occurs naturally at low levels in most freshwater and saltwater fish. Swordfish have among the highest levels of mercury. The average mercury content in swordfish is 1.00 ppm (parts per million), with a standard deviation of 0.77 ppm.[1] Assume the mercury content in swordfish is normally distributed.

2a. In Canada, fish with mercury content of .5 ppm or more are considered unsafe. What proportion of swordfish contain unsafe levels of mercury according to officials in Canada? Make sure you show how you arrived at your answer. Interpret your results. (10 points)

$P(X>.5)=P\left(Z>\frac{x-m}{s}\right)=P\left(Z>\frac{.5-1}{.77}\right)$=P(Z>-.64935)=P(Z>-.65)

Look up z-score .64935 in table, so P(Z>-.65)=.5+.2422=.7422

About 74% of swordfish have levels of mercury at toxic levels by Canadian standards.

2b. In the U.S., the level of mercury contamination in fish that is considered unsafe is 1.0 ppm. What proportion of swordfish are unsafe according to U.S. officials? Make sure you show how you arrived at your answer. Interpret your results. (5 points)

Since we assume mercury levels in swordfish are normally distributed, then the mean=median. Since the mean=1.00 ppm, the half of all swordfish are contain toxic levels of mercury by U.S. standards.

P(X>1)=P(Z>0)=.50

[1] Source: U.S. Food and Drug Administration, Center for Food Safety and Applied Nutrition, Office of Seafood. May 2001. Mercury Levels in Seafood Species. Available at: http://www.cfsan.fda.gov/~frf/sea-mehg.html.

2c. In Canada, though, swordfish is exempt from the 0.5 guideline because it is known to generally have levels of mercury that fall between 0.5 and 1.5 ppm, allowing for occasional comsumption (say 1 meal per week). What proportion of swordfish are within this minimally acceptable level of mercury? Interpret your results. (5 points)

From (a) above, we see that the probability of measure of mercury .5 ppm away from the mean of 1 ppm is .2389. Since we want the range that is symmetric on 1 and the distribution is normal, we can add together the two areas: P(.5<X<1.5)=P(-.65<Z<.65)=.2422+.2422=.4844.

About 48% of swordfish have levels of mercury that Canada considers to be acceptable for occasional consumption.

3. Recap last quiz: Investing is a game of chance. Suppose there is a 40% chance that a risky stock investment will end up in a total loss of your investment. Because the rewards are so high, you decide to invest in four independent risky stocks. What is the probability that all four stocks end up in total losses? Make sure you state the probability rule you employ. State your work in terms of probability and in English. (5 points)

Each stock is independent and will fail with a probability of .40. Let F=failure for one stock. So P(F)=.40. We want P(failure for all stocks). To figure out the union of these independent events, we multiply the probability of each event failing (using the rule of unions for independent events):

$P(\text{failure for all})=P(F_1) \times P(F_2) \times P(F_3) \times P(F_4) = .4 \times .4 \times .4 \times .4 = .0256$

There is a 2.6% chance that all the stocks will fail.