PBAF 527: Quantitative Methods **Assignment 3**
Answer Sheet

Distributions

Part I

4.4

a. The amount of flu vaccine in a syringe is measured on an interval, so this is a continuous random variable.
b. The heart rate (number of beats per minute) of an American male is countable, starting at whatever number of beats per minute is necessary for survival up to the maximum of which the heart is capable. That is, if *m* is the minimum number of beats necessary for survival, *x* can take on the values (*m*, *m* + 1, *m* + 2,…) and is a discrete random variable.
c. The time necessary to complete an exam is continuous as it can take on any value $0 \le x \le L$, where L = limit imposed by instructor (if any).
d. Barometric (atmospheric) pressure can take on any value within physical constraints, so it is a continuous random variable.
e. The number of registered voters who vote in a national election is countable and is therefore discrete.
f. An SAT score can take on only a countable number of outcomes, so it is discrete.

4.14

a. Yes. For all values of x, $0 \le p(x) \le 1$ and
$\sum p(x) = .01 + .02 + .03 + .05 + .08 + .09 + .11 + .13 + .12 + .10 + .08 + .06 + .05 + .03 + .02 + .01 + .01 = 1.00$

b. P(x = 16) = .06
c. $P(x \le 10) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10) = .01 + .02 + .03 + .05 + .08 + .09 = .28$
d.

$$P(5 \le x \le 15) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10) + p(11) + p(12) + p(13) + p(14) + p(15) =$$
$$.01 + .02 + .03 + .05 + .08 + .09 + .11 + .13 + .12 + .10 + .08 = .82$$

4.30

a. Let x = cost of treatment. The probability distribution for x is:

x	P(x)
$700	.40
$1,100	.10
$3,320	.45
$16,450	.05

b. μ=
$E(x) = \sum xp(x) = 700(.4) + 1110(.1) + 3320(.45) + 16450(.05) = 280 + 111 + 1.494 + 822.5 = 2707.5$

c. Let y = cost of either hepatitis or cirrhosis.
P(Hepatitis| hepatitis or cirrhosis) = .4/.5 = .8
P(Cirrhosis| hepatitis or cirrhosis) = .1/.5= .2

y	P(y)
$700	.80
$1,110	.20

d. μ = $E(x) = \sum xp(x) = 700(.8) + 1110(.2) = 560 + 222 = 782$

This is the expected or average cost of treatment for either hepatitis or cirrhosis. Over a large number of patients, the average cost of treatment is $782.

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5.16

Using Table IV, Appendix A:

a. $P(z > 1.46) = .5 - P(0 < z \le 1.46) = .5 - .4279 = .0721$
b. $P(z < -1.56) = .5 - P(-1.56 \le z \le 0) = .5 - .4406 = .0594$
c. $P(.67 \le z \le 2.41) = P(0 < z \le 2.41) - P(0 < z < .67) = .4920 - .2486 = .2434$
d. $P(-1.96 < z < -.33) = P(-.196 \le z < 0) - P(-.33 \le z < 0) = .4750 - .1293 = .3457$
e. $P(z \ge 0) = .5$
f. $P(-2.33 < z < 1.5) = P(-2.33 < z \le 0) + P(0 < z < 1.5) = .4901 + .4332 = .9233$
g. $P(z \ge -2.33) = P(-2.33 \le z \le 0) + P(z \ge 0) = .4901 + .5 = .9901$
h. $P(x < 2.33) = P(z \le 0) + P(0 \le z \le 2.33) = .5 + .4901 = .9901$

5.18

Using Table IV, Appendix A:

a. $P(-1 \le z \le 1) = P(-1 \le z \le 0) + P(0 < z \le 1) = .3413 + .3413 = .6826$
b. $P(-1.96 \le z \le 1.96) = P(-1.96 \le z < 0) + P(0 \le z \le 1.96) = .4750 + .4750 = .9500$
c. $P(-1.645 \le z \le 1.645) = P(-1.645 \le z < 0) = P(0 \le z < 1.645) = .45 + .45 = .9$ (using interpolation)
d. $P(-2 \le z \le 2) = P(-2 \le z < 0) + P(0 \le z \le 2) = .4772 + .4772 = .9544$

5.42

From Table IV, Appendix A, and σ=.4:

P(x > 6) = .01

P(x > 6) = $P\left(z > \dfrac{6 - m}{.4} \right) = P(z > z_0) = .01$

$A_1 = .5 - .01 = .4900$

From Table IV, $z_0 = 2.33$

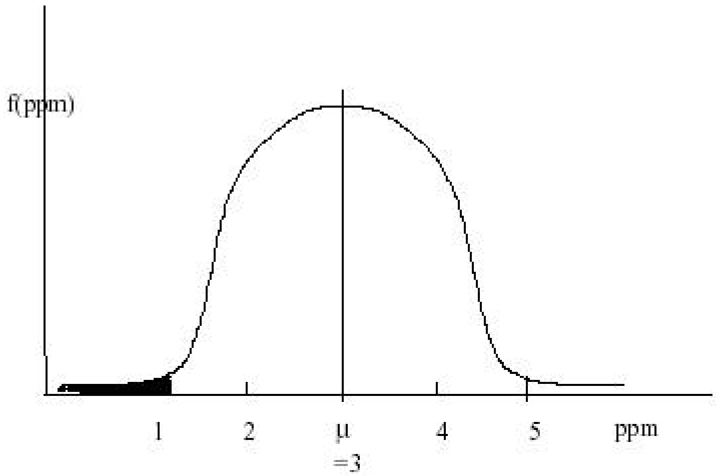
$$2.33 = \frac{6 - m}{.4} \Rightarrow m = 6 - 2.33(.4) = 5.068$$

5.48

Based only on the histogram provided, I would say that the data are probably not normally distributed. There are several observations that are very small compared to the rest of the data and some that are very large compared to the rest of the data.

Part II

1. We want to know what percentage of Washington water supplies has a fluoride concentration the falls below 1 ppm. First, draw a picture and shade in the area in which you are interested.



We know that the standard deviation is 1 and the mean is 3, or

σ=1
μ=3

We want to find out how many standard deviations 1 ppm is from the mean. We can look at the picture, or we can calculate this.

$$Z = \frac{X - \mu}{\sigma}$$
$$= \frac{1-3}{1} = -2$$

So, we know that 1ppm is –2 standard deviations from the mean. We find Z in the normal table (or use Excel), and the probability associated with –2 is .0228. That is, 2.3% of Washington water supplies have flouride levels below 1ppm.

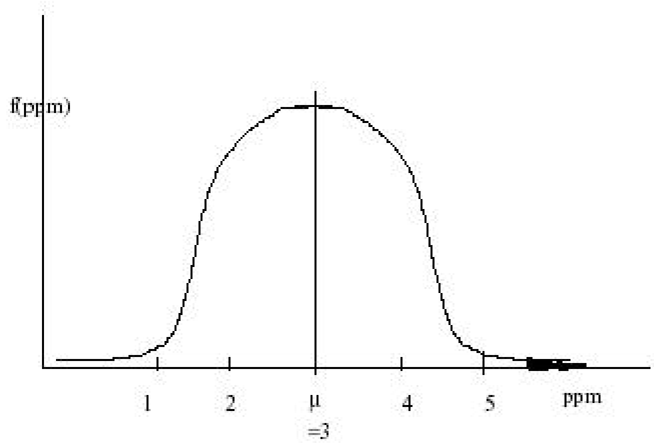
2. We want to know how much a program would cost the state. You can figure this out: (number water supplies in WA) x .0228 x $1000, or $22.8 for every water supply throughout the state. But, since we don't know how many water supplies in there are in Washington State, we can only say how we would calculate it. Sometimes you don't have all the information you need to give someone an answer.

3. We want to know the cut-off where 0.0013 of the water supplies are above.

We know we can calculate $X = \mu + Z\sigma$. Find the value of Z in the table where p=.0013. We look up in the table where p=1-.0013=.9987. Here Z=3.

$X = \mu + Z\sigma$
$X = 3 + (3)(1) = 6$

So, the cut off for high flouride is 6 ppm.



4. The great majority of water supplies in the State of Washington fall under the cut off 6ppm at which fluoride levels are dangerous. Less than one percent of water supplies are above this harmful cutoff, while 2.3% do not have fluoride that are up to a level that would be beneficial. The cost of bringing these water supplies up to a beneficial level is only $228 per water supply throughout the state.