PbAf 527 – Quantitative Analysis I Winter 2005
Assignment 5

Part I

1. H_0: $\mu = 5$ H_1: μ ? 5

Critical points for μ=0.05 are t=±1.96

Test statistic: $t = \dfrac{2.3 - 5.0}{\dfrac{1.5}{\sqrt{120}}} = -19.7$

This is well below the critical value so we reject the null and conclude that changes are justified. This would have been our conclusion regardless of the significance level.

P-value =(2)*P($\bar{X}$ < 2.3)
=(2)*P(Z < -19.7)
=(2)*(0) This is two times approximately zero, p-value is close to zero.

Interpretation: There is essentially no chance of getting a sample mean as low as 2.3 if the actual population mean is 5.0.

2. H_0: p = 0.3 H_1: p ? 0.3

$\hat{p}$ = 852/2000 = 0.426

$$t = \frac{(.426 - .3)}{\sqrt{\frac{(.3)(.7)}{2000}}} = 12.3$$

Critical values of t at a= 0.05 are $\pm$1.96, at a= 0.01 are $\pm$2.576, and our calculated t is far outside in the rejection region at these and even stricter levels of significance.

We reject the null (that the proportion is 0.3) and conclude that more than 30% of small businesses are owned by women (due to the rejection occurring in the far right tail).

3. H_0: $\mu_1 - \mu_2 = 0$ H_1: $\mu_1 - \mu_2$? 0

$$t = \frac{(8.5 - 7.8) - 0}{\sqrt{\frac{2.1^2}{30} + \frac{1.8^2}{30}}} = \frac{.7}{\sqrt{.147 + .108}} = \frac{.7}{.505} = 1.39$$

This t is not significant at any reasonable level (0.05, 0.01, etc.) thus we cannot reject the null. We conclude that there is no difference between the two mainframes.

Part II

For illustration, look at the comparison of mean household wage earnings for households who did and did not make out of state purchases (a recoded variable from q8p8). I want to assess the potential effects of expanding the sale tax to out of state purchases.

The null hypothesis is that mean earnings are the same for both groups. The alternative hypothesis is that mean earnings of the two groups is different:

H_0: $\mu_1 - \mu_2 = 0$
H_1: $\mu_1 - \mu_2 \neq 0$

a. We can reject equal variance of earnings for the two samples given at a .05 level of significance. So, use the bottom row of the t-test table.

Independent Samples Test

Group Statistics

	at least 1 internet purchase out of state	N	Mean	Std. Deviation	Std. Error Mean
2003 TOTAL HOUSEHOLD WAGE EARNINGS	No Internet Purchase Out of State	1138	49145.65	79125.84	2345.564
	At least 1 Internet Purchase Out of State	2105	73917.48	66805.80	1456.090

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
2003 TOTAL HOUSEHOLD WAGE EARNINGS	Equal variances assumed	11.054	.001	-9.433	3241	.000	-24771.82	2626.0008	-29920.6	-19623.0
	Equal variances not assumed			-8.973	2020.075	.000	-24771.82	2760.7727	-30186.1	-19357.6

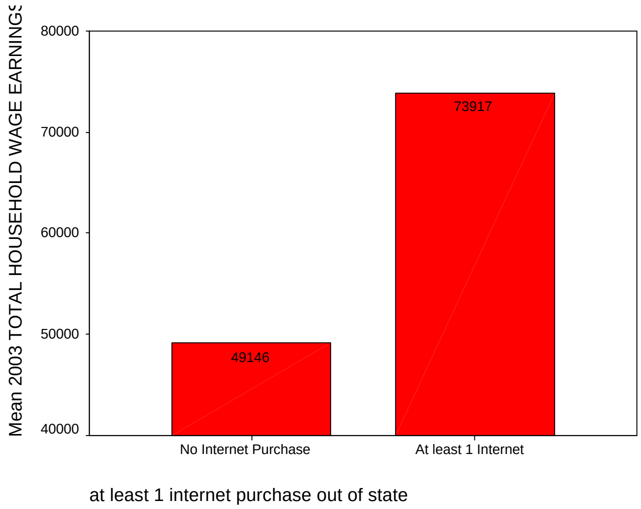
For α = .05 the critical value of the t (or the z since n is big for this example) is t = $\pm$1.96. To calculate the t-statistic for the difference of the means I use the formula:

$$t = \frac{\bar{x}_1 - \bar{x}_2 - 0}{s_{(\bar{x}_1 - \bar{x}_2)}} = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(49{,}146 - 73{,}917) - 0}{\sqrt{\frac{79126^2}{1138} + \frac{66806^2}{2105}}} = -8.97$$

I reject the null hypothesis because the t test statistic is above the critical value, i.e., the point estimate is more Standard Errors away from the hypothesized parameter than would be found in 95% of random samples. So there solid evidence that there is a difference in the mean earnings for the 2 groups. The difference of $24,882 is statistically significant and large as well.

b. To find the p-value, we need to look in the table for the area associated with z =8.97 and subtract it from .5 the multiply it by 2 (because we could see a value this far away on either side of the mean). But this Z is off the table so p<.001. The p-value is less than α (α = .05), and this is why we can reject the null hypothesis in part a above. This p-value shows that there less than a .1% chance that we would get a difference in earnings this large in a random sample if there were no difference in the population. Can you locate the p-value in the SPSS output above?

c. The graph below shows the mean income level for each purchase category and a scatter plot of income and the number of out of state purchases made. This might be an effective way to visually show the differences.



e. Using data from the 2004 Washington Population Survey, we have continued our research on the implications of requiring sales tax on out of state purchases by examining the relationship between personal wage earnings and out of state purchases. On average, households that made out of state purchases in 2003 had earnings higher by between $19,623 and $29,921.[1] Figure 1 shows the average earnings for people making out of state purchases ($73,917) and for those who made no purchases ($49,146). Overall, the relationship between out of state purchases and household earnings suggests that expanding the sales tax to these purchases is unlikely to increase the regressivity of Washington's tax system given higher average incomes for those making purchases.

[1] 95% Confidence Interval, P<.001