Small Sample t-test Example 1 (1)

Most water treatment facilities monitor the quality of their drinking water on hourly basis. One variable monitored it is pH, which measures the degree of alkalinity or acidity in the water. A pH below 7.0 is acidic, one above 7.0 is alkaline, and a pH of 7.0 is neutral. One water treatment plant has a target pH of 8.5 (most try to maintain a slightly alkaline level). The mean and standard deviation of 1 hour's test results, based on 17 water samples at this plant are:

$$\bar{x} = 8.24 \qquad s = .16$$

Does this sample provide sufficient evidence that the mean pH level in the water differs from 8.5?

Small Sample t-test Example 1 (2)

1. Establish hypotheses H_0: $\mu=8.5$ H_a: $\mu\neq 8.5$
2. Set the decision rule for the test:

   if $|t|>t_\alpha$ at n-1 df then reject the null hypothesis

pick α α=.05 (for two-sided test this is .025 in each tail)

find t_α at n-1 df t_α=2.12 with 16 degrees of freedom

3. Find test statistic

$$t=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}=\frac{8.42-8.5}{\frac{.16}{\sqrt{17}}}=\frac{-.08}{.039}=-2.05$$

4. Compare test statistic to critical value.

   Since $|t|< t_\alpha$ we cannot the null hypothesis at a 5% level. We cannot conclude that that the mean pH differs from the target based on the sample evidence.

Small Sample t-test Example 2 (1)

A major car manufacturer wants to test a new engine to determine whether it meets new air-pollution standards. The mean emission μ for all engines of this type must be less than 20 parts per million of carbon. 10 engines are manufactured for testing purposes, and the emission level for each is determined. The mean and standard deviation for the tests are:

$\bar{x} = 17.17$ s=2.98

Do the data supply enough evidence to allow the manufacturer to conclude that this type of engine meets the pollution standard? Assume the manufacturer is willing to risk a Type I error with probability α=.01.

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Small Sample t-test Example 2 (2)

1. Establish hypotheses $H_0: \mu = 20$ $H_a: \mu < 20$
2. Set the decision rule for the test:

if $t < t_\alpha$ then reject the null hypothesis

pick α $\alpha = .01$ (for one-sided test this is .01 in the tail)

find t_α $t_\alpha = -2.821$ with 9 degrees of freedom

3. Find test statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{17.17 - 20}{\frac{2.98}{\sqrt{10}}} = -3.00$$

4. Compare test statistic to critical value.

We can reject the null. The actual value is less than 20 ppm, and the new engine type meets the pollution standard.

Large Sample Test for the Population Proportion

When the sample size is large (np and nq are greater than 5)

- Assume $\hat{p}$ is distributed normally with mean p and standard deviation $\sqrt{\frac{pq}{n}}$ where q=1-p
- Test statistic: $$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$$
- 2- or 1-tailed tests

Large Sample Tests for Proportion Example (1)

In screening women for breast cancer, doctors use a method that fails to detect cancer in 20% of the women who actually have the disease. Suppose a new method has been developed that researchers hope will detect cancer more accurately. This new method was used to screen a random sample of 140 women known to have breast cancer. Of these, the new method failed to detect cancer in 12 women.

Does this sample provide evidence that the failure rate of the new method differs from the one currently in use?

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Large Sample Tests for Proportion Example (2)

1. Establish hypotheses $H_0: p = .2$ $H_a: p ? .2$
2. Set the decision rule for the test:

   if $|z| > z_\alpha$ then reject the null hypothesis

pick α $\alpha = .05$ (for two-sided test this is .025 in each tail)

find t_α $z_\alpha = 1.96$

3. Find test statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} = \frac{.086 - .2}{\sqrt{(.2)(.8)/140}} = \frac{-.114}{-.034} = -3.36$$

4. Compare test statistic to critical value.

Since the test statistic falls in the rejection region, we can reject the null. The rate of detection for the new test differs from the old at a .05 level of significance.