

**PBAF 527: Quantitative Methods Final Exam
Winter 2001 Answer Key**

1. Do women entrepreneurs rely more on personal debt for financing their businesses?¹ FleetBoston, a major bank, underwrote a study of 1,194 entrepreneurs¹—602 women and 592 men—who reported the use of different types of resources. 32 percent of the women used personal credit cards to finance their firms compared to 21 percent of men. Teri Cavanaugh, the Director of the Women’s Entrepreneurs’ Connection at FleetBoston Financial (part of their community outreach) has hired you as a summer intern. She’s asked you to summarize the results and listed out some questions (following) that she’d like you to answer for her. As you go through her questions and concerns, make sure to formally set up your hypotheses and clearly note the steps you take and the assumptions you make. After you’ve finished your analysis, you can write it up for her in non-technical language. For your analysis, use $\alpha=.05$ or 95% confidence intervals.

a. Teri thought that about 25 percent of all entrepreneurs (men and women combined) used personal debt to finance operations. Using the information provided above, perform a hypothesis test so that you can tell her whether that is the case. (10 points)

$$H_0: p_0 = .25 \quad H_a: p_0 \neq .25$$

Decision rule: $|z| > z_{\alpha/2}$ then reject the null
 $\alpha = .05$, $z_{\alpha/2} = 1.96$

$$\hat{p} = \frac{(.32)(602) + (.21)(592)}{1194} = .27$$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.27 - .25}{\sqrt{\frac{(.25)(.75)}{1194}}} = \frac{.02}{\sqrt{\frac{.2025}{1194}}} = \frac{.02}{.013023} = 1.53$$

Since $|z|$ is not greater than $z_{\alpha/2}$, we cannot reject the null hypothesis at a .05 level of significance. We cannot say that the actual proportion of entrepreneurs who use personal debt to finance operations is different from 25%.

¹ New Survey Shows Traits of Women Owners of Fast-Growth Businesses. Released February 27, 2001. Accessed March 2, 2001 at <http://www.nfwbo.org/>.

b. Teri would like an estimate based on the information gathered of the proportion of entrepreneurs who rely on credit card debt. Provide her with a range estimate. (5 points)

Create a 95% confidence interval to provide Teri with a range estimate.

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} = .27 \pm 1.96 \sqrt{\frac{(.27)(.73)}{1194}} = [0.24, 0.30]$$

We can be 95% confident that the true proportion of entrepreneurs who rely upon credit card debt to finance operations is between 24% and 30%.

c. Teri also wants to know what proportion of women entrepreneurs financed their businesses using personal debt. Provide her with a range estimate. (5 points)

Another 95% confidence interval, this time for women.

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} = .32 \pm 1.96 \sqrt{\frac{(.32)(.68)}{602}} = [0.28, 0.36]$$

We can be 95% confident that the true proportion of women entrepreneurs who rely upon credit card debt to finance operations is between 28% and 36%.

d. Teri would like to know if we have enough evidence to say that women entrepreneurs have a significantly greater reliance on personal debt than do male entrepreneurs. Perform a hypothesis test so that you can answer her question. What assumptions do you need to make? (10 points)

One-sided test, assumes that the distribution of the sample proportion are well approximated by the normal distribution. This is a right-tailed test because we are seeking extremely large values to disprove the null of the difference being 0 or less.

$$H_0: p_w - p_m \leq 0 \quad H_a: p_w - p_m > 0$$

decision rule: $z > z_\alpha$, $z_\alpha = 1.645$

$$z = \frac{(\hat{p}_w - \hat{p}_m) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_w} + \frac{1}{n_m}\right)}} \quad \hat{p} = .27 \text{ from (a.) above}$$

$$= \frac{(.32 - .21) - 0}{\sqrt{.27(1-.27)\left(\frac{1}{602} + \frac{1}{592}\right)}} = \frac{.11}{\sqrt{.1971(0.00335)}} = \frac{.11}{.025697} = 4.28$$

Since $z > z_\alpha$ (that is, $4.28 > 1.645$) we can reject the null hypothesis. Women entrepreneurs are more reliant on personal debt than are male entrepreneurs. This is statistically significant at a .05 level.

e. She'd also like an estimate of the difference in the use of personal debt for men and women entrepreneurs. Again use a range. (5 points)

Calculate a 95% confidence interval for the difference.

$$(\hat{p}_w - \hat{p}_m) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_w(1-\hat{p}_w)}{n_w} + \frac{\hat{p}_m(1-\hat{p}_m)}{n_m}} = .11 \pm 1.96 \sqrt{\frac{.2176}{602} + \frac{.1659}{592}}$$

$$= .11 \pm (1.96)0.025332 = [0.06, 0.16]$$

We are 95% sure that the proportion of women entrepreneurs financing their business operations with personal debt is between 6 and 16 percentage points more than the percentage of men doing so.

f. How strong is your evidence that there is a difference in the use of personal debt? What is the probability that you are wrong? (5 points)

What is the p-value associated with a z-score of 4.28? Well, we know the p-value is pretty small because the z-score is larger than 3.00 (we know that most of the probability for the difference should be within 3 se). The p-value associated with 4.28 is .000009. So, we have very strong evidence that there is a difference in the use of personal debt.

g. Teri would like to conduct another survey that would yield quality estimates of the proportion of both men and women entrepreneurs who use personal debt to finance their businesses. She'd like to be within 2 percentage points of the estimates. Based on the information provided in the current survey, tell her what size sample of each would she need. (You can assume that the response rate would be 100 percent if she were to conduct another survey.) (10 points)

Find the sample size for each subgroup.

$$n = \frac{z_{\alpha/2}^2 pq}{B^2} \quad B=.02 \text{ (how close Teri wants to be to the true value)}$$

$$\text{For women: } n = \frac{1.96^2 (.32)(.68)}{.02^2} = 2090$$

$$\text{For men: } n = \frac{1.96^2 (.21)(.79)}{.02^2} = 1594$$

h. Teri thinks that entrepreneur's reliance on personal debt may indicate a lack of access to capital or credit to expand and support their businesses. Summarize your findings for Teri in a paragraph or two. Make sure to answer her fundamental question of whether women entrepreneurs rely more upon personal debt, discuss ALL your findings, and address her concern about a lack of access to credit. In addition, talk about why do you think you see the results you do and what else it would be helpful for Terri to know in order to answer her question more fully. (20 points)

At your request, I have begun to analyze the information from your survey of entrepreneurs. Of the 1,194 respondents, 27% reported using credit cards to finance their business. From this, I estimate that, on average, between 24% and 30% of entrepreneurs rely on personal debt to finance their businesses.² A higher proportion--between 28% and 36%--of women entrepreneurs, though, rely on personal debt.³ Women are also more likely than men to use personal debt to finance their businesses--between 6% and 16% more women entrepreneurs do so.⁴ The evidence that women are more likely to finance their businesses with personal debt is very strong.⁵ If you wanted to conduct another survey, you would need to sample 2,090 women and 1,594 men entrepreneurs in order to come within 2% of the actual proportions of men and women entrepreneurs relying on personal debt. It is possible that women rely on personal debt more because they have less access to traditional credit sources, or it may be that traditional credit sources find women more risky. Adding questions to future surveys that ask about whether women and men entrepreneurs have sought bank financing and what the results were would help to assess this.

2. Are web grocers more expensive than grocery stores that are not entirely web-based? While they do provide a great deal of convenience, are customers paying a high price for that convenience (not taking into account any delivery charges)? Here is a selection of similar brand name products taken from Albertson's (which has both a web and a traditional grocery store presence) and WebVan, which recently bought our HomeGrocer.com and only has a web presence. Ten brand-name products were selected randomly and then searched out on both stores' web sites.

Product Number	WebVan	Albertson's
1	3.99	2.99
2	5.99	3.69
3	3.69	3.29

² 95% confidence interval.

³ 95% confidence interval.

⁴ Significant t-test at a .05 level; range estimate for the difference is a 95% confidence interval.

⁵ t-value is 4.28, with an associated p-value of <.001

4	1.49	1.39
5	3.99	4.19
6	2.09	2.49
7	0.79	0.79
8	1.69	1.79
9	1.39	1.69
10	0.99	0.99

a. Conduct a hypothesis test to see if the stores charge different prices for their groceries. Use a .05 level of significance. What assumptions do you need to make? (10 points)

This is a paired-sample, with the same product being found at two different stores. So, first we have to find the differences for each pair and calculate the mean difference, \bar{D} .

	webvan	albertson's	D
1	3.99	2.99	1.00
2	5.99	3.69	2.30
3	3.69	3.29	0.40
4	1.49	1.39	0.10
5	3.99	4.19	-0.20
6	2.09	2.49	-0.40
7	0.79	0.79	0.00
8	1.69	1.79	-0.10
9	1.39	1.69	-0.30
10	0.99	0.99	0.00
		$\bar{D} =$	0.28

$$H_0: \mu_{D0}=0 \quad H_a: \mu_{D0} \neq 0$$

decision rule: $|t| > t_{\alpha/2}$, $t_{\alpha/2} = 2.26$ at 9 df (n-1)

Assume that the population value for the difference approximates the normal distribution (small sample).

$$t = \frac{\bar{D} - m_{D0}}{\frac{s_D}{\sqrt{n}}} = \frac{.28}{\frac{.814862}{\sqrt{10}}} = 1.08$$

Because $|t| < t_{\alpha/2}$ we cannot reject the null hypothesis. So, we cannot say that WebVan costs more than Albertson's, or that web-based grocers are necessarily more expensive than one with a traditional store and a web presence.

b. Construct a 95 percent confidence interval for the range estimate for the difference in the prices at the two stores. What assumptions do you need to make? (10 points)

Using the t-distribution because of the small sample. Have to assume that the standard deviation of the sample is a good estimate of the standard deviation of the population.

$$\bar{D} \pm t_{\alpha/2} \frac{s_D}{\sqrt{n}} = .28 \pm 2.26 \left(.814862 / \sqrt{10} \right) = .28 \pm 2.26(.25) = [-0.30, 0.86]$$

We are 95% sure that prices at WebVan are between 30 cents less and 86 cents more than prices at Albertson's.

c. Write a summary of your results for the non-technical web consumer. Offer an explanation for the results you have. (10 points)

Although there have been complaints recently that web-based grocers charge more for groceries, a comparison of prices at WebVan (a web-based grocer) and Albertson's (a grocer with both a web presence and traditional stores) does not suggest this. Rather, WebVan prices are between 30 cents less and 86 cents more than Albertson's.⁶ Perhaps larger samples of other web-based grocers and traditional stores might suggest a difference, but right now we don't have enough evidence to say so.

⁶ 95% confidence interval.