PBAF 527: Quantitative Methods | **Midterm Exam**
Winter 2003 | **Due 10 am, Friday Feb 6**

Please write your student number here:____________________________________

Ground rules:

- Do not write your name on the exam. If you're worried about losing the first page, write your student number on each page of the exam.
- If you use pages other than this exam to do your work, STAPLE them to the exam, clearly identify each question with a number, write your student number on those pages and hand them in.
- You can use your notes or books, but you may not communicate with other people about this exam or the material covered by it.
- If you have questions about the exam as you work through it, please email the listserv pb_af527u_wi03@u.washington.edu so that we all can "hear" the question and the answer.
- In order to receive as much credit as possible, please show all of your work. Showing that you understand the question and know how to set up the solution correctly is more important than arriving at the exact answer.
- Feel free to use Excel or another program for your calculations, if necessary. Please attach a print out of your spreadsheet.
- Please be neat -- you can't get credit if I can't read your answers.
- To turn in the exam, you can email it as an attachment (just your student number on the attachment), put it in my box in Par 208, or bring by my office. If I'm not there put it under my door (209F Parrington). If you end up putting it under my door rather than handing it to me, send me an email to let me know you did so.
- Good luck!

1. There were 35 million seniors—people aged 65 and older—living in the U.S. in the year 2000.[1] Interest in the senior population has been growing, as estimates suggest that between 2000 and 2030 seniors will have risen from 12.4% of the population to be 20% of the U.S. population. Concerns about poverty among seniors, about increasing numbers living alone without any support drive much of the policy discussion about the quality of life of aging Americans. Suppose 5 people are selected at random from the U.S. population.

a. What is the probability that at least one of them will be a senior in 2000? How about in 2030? (10 points)

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One can think of this as following the binomial distribution.  A
success is being a senior, and the number of trials is 5.
in 2000 p=.125
in 2030 p=.20
n=5

in 2000 P(x>=1)=1-P(x=0)=1-.516=.484
in 2030 P(x>=1)=1-P(x=0)=1-.328=.672

So, in 2000 the probability that at least one of the 5 people is a
senior is 52% and in 2030 the probability is 67%.

Alternatively, you could think of this as being 5 independent draws
(this is a big population), and so could use the law of unions of
independent events to figure out the probabilities.
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P(at least 1 a senior in 2000)=1-P(1 is not a senior)^5
=1-(.876)^5=1-.5158464=.48415
P(at least 1 a senior in 2030)=1-P(1 is not a senior)^5
=1-(.8)^5=1-.32768=.67232

b. What is the probability that only one of them will be a senior in 2000? How about 2030? (10 points)

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Use binomial distribution: in 2000 p=.125, in 2030 p=.20, n=5

in 2000 P(x=1)=.365 (from excel), in 2030 P(x=1)=.410 (from excel)

So, in the year 2000, the probability that 1 of the 5 will be 65 years
of age or older is 37%.  In 2030, the probability will be 41%.

Or, a couple of people conceptualized this as a random variable
problem. Each of the 5 people drawn from the population is independent
from the others because one is drawing from a large population.  So, we
can multiply the probabilities for each combination that would lead to
only one senior.

Here’s the sample space:  SNNNN NSNNN NNSNN NNNSN NNNNS
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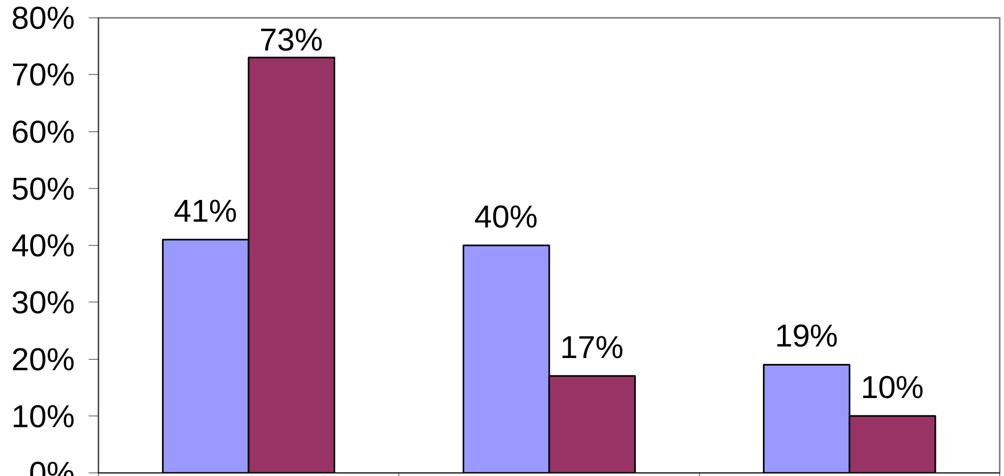
[1] *A Profile of Older Americans: 2002*. Administration on Aging, U.S. Department of Health and Human Services.

P(only 1 senior in 2000)=sum of all the probabilities of all the ways to see only 1 senior when one chooses 5
$=(.124)(.876)^4+(.124)(.876)^4+(.124)(.876)^4+(.124)(.876)^4+(.124)(.876)^4$
=.073*5=.36509=.37

P(only 1 senior in 2030)=sum of all the probabilities of all the ways to see only 1 senior when one chooses 5
$=(.20)(.80)^4+(.20)(.80)^4+(.20)(.80)^4+(.20)(.80)^4+(.20)(.80)^4$
=.0819*5=.4096=.41

The following graph and table presents information on the living arrangements of seniors in the United States in 2000. About 41% of seniors are men.

Living Arrangements of Persons 65+ in 2000



	living with spouse	living alone	other
women	41%	40%	19%
men	73%	17%	10%

c. What is the probability that randomly selected senior from the U.S. population is a woman living alone? How about a man living alone? (5 points)

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P(woman and living alone)
=P(living alone|woman)*P(woman)
=.41*.59=.239

The probability that a randomly selected senior is a woman
who lives alone is 24%.

P(man and living alone)
=P(living alone|man)*P(man)
=.17*.41=.0697

The probability that a randomly selected senior is a man
who lives alone is 7%.
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d. Given that a randomly selected senior is lives with a spouse, what is the probability that the senior is a man? A woman? What explanation can you offer for this? (15 points)

Let M=Man, W=Woman, L=Lives with Spouse
P(M)=.41, P(W)=.59
Use Bayes Theorem:

$$P(\mathrm{M}\mid\mathrm{L})$$

$$= \frac{\mathrm{P(L\mid M)P(M)}}{\mathrm{P(L\mid M)P(M)+P(L\mid M^c)P(M^c)}}$$

$$= \frac{\mathrm{P(L\mid M)P(M)}}{\mathrm{P(L\mid M)P(M)+P(L\mid W)P(W)}}$$

$$= \frac{(.73)(.41)}{(.73)(.41)+(.41)(.59)}$$

$$= \frac{.29993}{.29993+.2129} = .55$$

P(W|L)=1-P(M|L)=.45

Although in the general population, one would expect that half of people who are married are men and half are women, among seniors this isn't the case—55% of men 65 and over are married compared to 45% of women age 65 and older. Likely this is because these senior men are married to women under the age of 65.

e. Are housing arrangements and gender independent for seniors in the U.S.? Show how you arrive at your answer. (10 points)

If events are independent, then the following will be true:
P(A|B)=P(A)
P(B|A)=P(B)
$P(A\cap B)=P(A)P(B)$

For example: Let A=Lives with Spouse and B=Man
P(A|B)=73% $\neq$ P(B)=41%
So, we know that housing arrangements and gender are not independent.

f. Write a paragraph interpreting at least two columns of the graph as well as your findings in a-f for a non-technical policy audience. Discuss the policy implications of your findings. (20 points)

Seniors currently make up 12.4% of the U.S. population. in 2000, if you randomly selected 5 people to talk to, there was a 48% chance that at least one would turn out to be a senior. In 2030, that chance would be 67%. This increase makes it important to look at the living arrangements of seniors. In fact, there is a 37% chance that only 1 of the 5 people selected would be a senior in 2000, but in 2030, the probability increases to 41%.

In 2000, 41% of the senior population was male, while 59% was female. It is important to remember that the averate female lives longer than the average male. Thus, it is nor surprising that, if you are a senior woman, you have similar chance of living with a spouse (41%) or living alone (40%), while men who are senior are much more likely to be living with a spouse than any other living arrangement—73% of senior men live with a spouse compared to 17% who live alone. Furthermore, 55% of married seniors are men, while 45% are women, likely due to older men marrying women who are not seniors. Of all seniors, nearly a quarter are women who live alone, while only 7% are men who live alone.

The findings here indicate that gender and housing arrangments are dependent on each other. Policy makers could compare outcomes for women who live with spouses compared to those who live alone or in other living arrangements, looking at health, poverty, or the need for services. It may be that over time, policy makers may need to think about alternatives to living alone, especially for older women.

2. The housing costs of seniors in the U.S. vary. The census bureau just released its most recent summary information on housing costs for the elderly in 2001. They report information collected in a large sample survey of 50,000 housing units, the American Housing Survey. They report that half of elderly householders in the U.S. have housing costs that exceed $367 dollars a month.

a. What measure of central tendency are they reporting? (5 points)

The median

b. Are they using inferential or descriptive statistics in reporting their results? How do you know? (5 points)

They are using inferential statistics. They use data from a sample survey of 50,000 housing units to generalize about all elderly householders in the U.S. (A descriptive task would be to describe the 50,000 housing units).

c. The Census Bureau reports that on average seniors pay $434 dollars per month for housing, and that their housing costs have a standard deviation of $318. Is the cost of housing for elders normally distributed? (10 points)

One could go through the 4 ways of determining whether data approximate the normal distribution (p. 237 in your book), but we really don't have enough information to do that. However, the first of these is to graph the data. While we don't have the data to graph, one reason to graph is to look for symmetry. That is, we know that for data to be normally distributed, the mean=median=mode. We know that the mean=$434 $\neq$ median=$367. So, no the data are not normally distributed.

3. As you know, over the past 10 years, concerns have grown over increasing traffic in the Puget Sound. One way the Washington State Department of Transportation (WDOT) measures traffic congestion is by looking at travel time. To help people plan their travel, WDOT estimates that average trip from Seattle to Bellevue at 5:40 pm (at peak) takes 17 minutes and, 95% the time, the trip will not exceed 30 minutes. They also believe this travel time approximates a normal distribution.

a. What is the standard deviation of peak travel time for the trip from Seattle to Bellevue? (10 points)

From the question, we know that 95 times in 100, the trip will take 30 minutes or less. So, the entire area to the left 30 is 95%. To use the normal table to determine z, subtract .5 from the probability, and you are left with 45%, corresponding to a z-score of 1.645.

$$z = \frac{x - m}{s}$$

$$s = \frac{x - m}{z}$$

$$s = \frac{30 - 17}{1.645} = 7.903$$

So, the standard deviation is about 8 (7.9) minutes.

b. Using your answer in 3a, what proportion of drive times fall between 20 and 30 minutes? (10 points)

We are looking for P(20<z<30).
P(20<z<30)=P(z<30)-P(z<20)=.95-.6480=.302 or 30% of drive times fall between 20 and 30 minutes.

c. Write a paragraph explaining your results to a non-technical policy audience.

The average trip from Seattle to Bellevue at peak travel time, 5:40 pm, takes about 17 minutes and most of the time (19/20 times) it takes less than 30 minutes. However, 68% of the time, it will be between about 9 (9.1) and 25 (24.9) minutes. In 3 out of 10 trips, the trip will take between 20 and 30 minutes.

Extra Credit (10 Points):

On a busy holiday weekend, a national airline has many requests for standby flights at half of the usual one-way fare. However, past experience has shown that these passengers have only a 1 in 5 chance of getting on the standby flight. When they fail to get on the standby flight, their only other choice is to fly first class on the next flight out. Suppose the usual one-way fare to a certain city is $80 and the cost of flying first class is $120. Should a passenger who wishes to fly to this city opt to fly as a standby?

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p=.20
$40 for standby
$120 for first class
$80 for regular fare

Probability distribution for x:
P(x=$40)=.20
P(x=$120)=.80

Find the expected value of x:
40*.20+120*.80=$104

No, one should not opt for standby because the expected
cost ($104) is greater than the usual one-way fare ($80).
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