OCN/ATM/ESS 587
2009
Solutions

1. The heat stored per unit area in a column of air or water with heat capacity C, thickness H, density ρ, and temperature T (in °K) is approximately ρCTH. This is only approximately true because ρ and C are likely to be functions of T; unless H is very thin, ρ and C will vary through the column. Nonetheless, we can make an assessment of the statement in this problem by estimating both the atmospheric and oceanic heat content. Let the subscripts o and a refer to the ocean and atmosphere. Then the heat stored in columns of ocean and atmosphere per unit area are

Ocean heat per unit area = $\rho_o C_o T_o H_o$

= $(10^3 \text{ kg m}^{-3})(4\times10^3 \text{ joules kg}^{-1}\ °\text{K}^{-1})(3\times10^2\ °\text{K})(2 \text{ m}) = 2.4\times10^9 \text{ joules/m}^2$

Atmospheric heat per unit area = $\rho_a C_a T_a H_a$

= $(0.5 \text{ kg m}^{-3})(10^3 \text{ joules kg}^{-1}\ °\text{K}^{-1})(2.5\times10^2\ °\text{K})(2\times10^4 \text{ m}) = 2.5\times10^9 \text{ joules/m}^2$

Thus, to within the approximation that properties are constant in the atmosphere, this choice of parameters suggests that the statement is essentially true.

2. We can find the pressure by assuming that the ocean is in hydrostatic balance. The hydrostatic relation states that

$$\frac{\partial p}{\partial z} = -g\rho \qquad ,$$

where p is the pressure, z is the depth, g is the acceleration due to gravity, and ρ is the density. For the ocean, we can assume that the density is approximately constant and integrate directly (assuming that g is a constant) to find that the ocean pressure p_{oc} at any depth z below the surface is

$$p_{\text{oc}}(z) = -\rho g z + p(0)$$

where $p(0)$ is a constant of integration and represents the pressure at the sea surface. Taking this constant to be zero, then $p_{\text{oc}}(z) = -\rho g z$.

For the atmosphere, the density is not constant, and we must relate density to pressure in order to proceed. To do this, we note that a good, approximate equation of state for the atmosphere is given by the Ideal Gas Law,

$$p = \rho RT \quad \Rightarrow \quad \rho = \frac{p}{RT} \qquad ,$$

where T is the Kelvin temperature and R is the gas constant. Putting this relation for ρ into the hydrostatic relation, it is found that

$$\frac{\partial p_{\text{at}}}{\partial z} = -g\rho = -g\frac{p_{\text{at}}}{RT} \quad \Rightarrow \quad p_{\text{at}}(z) = p_{\text{at0}} e^{-\frac{gz}{RT}} \qquad ,$$

where p_{at0} is a reference pressure (a constant of integration) at $z = 0$, the bottom of the atmosphere.

We can evaluate each of these pressure functions at a distance of 5 km below the reference, since in both cases the pressure will increase downwards. For the ocean, at a depth of 5000 m it is found that

$$p_{oc} = -(1.027\times10^3 \text{ kg/m}^3)(9.8 \text{ m/sec}^2)(-5\times10^3 \text{ m}) = 5.032\times10^7 \text{ kg m}^{-2}\text{sec}^{-2}$$

$$= 5.032\times10^7 \text{ newton/m}^2 = 5.032\times10^7 \text{ pascals} = 5.032\times10^2 \text{ bar} = 5.032\times10^3 \text{ decibars.}$$

For the atmosphere, we can take p_{at} to be 500 millibars at a height of 5 km, a value typical of Earth's atmosphere; we note that 1 millibar = 10^{-3} bar = 10^{-2} decibar = 10^2 kg m^{-1}sec^{-2}, so that

$$p_{at0} = p_{at}(z)\mathrm{e}^{+\frac{gz}{RT}} \quad ,$$

and

$$p_{at0} = (5\times10^2\times10^2 \text{ kg m}^{-1}\text{sec}^{-2}) \exp[(9.8 \text{ m sec}^{-2})(5\times10^3 \text{ m})/((2.87\times10^2 \text{ m}^2\text{sec}^{-2}\ {}^\circ\text{K}^{-1})(2.78\times10^2 {}^\circ\text{K}))]$$

$$= (5\times10^4 \text{ kg m}^{-1}\text{sec}^{-2})(1.84) = 9.24\times10^4 \text{ kg m}^{-1}\text{sec}^{-2} = 9.24\times10^2 \text{ millibars} = 9.2 \text{ decibars .}$$

Thus, the hydrostatic pressure at the bottom of a 5 km ocean is roughly 500 times greater than the pressure at the bottom of a 5 km column of Earth's atmosphere.

3. The surface temperature of a planet will be a function of many things, but the most important factor will the nature of the planet's atmosphere. If the atmosphere is weak or nonexistent (example: the Moon), there will be a great deal of heat transfer from the ground into space via black body (outgoing longwave) radiation and sensible heat flux. If there is a denser atmosphere, some of the heat will be trapped at the ground via a greenhouse effect, and the temperature will remain more stable (constant) on the planet's surface. We can try to parameterize this effect by a diffusivity κ. For weak atmospheres, κ is relatively large and the heat at the ground will escape into space relatively quickly by "mixing" with the cold space above. For denser atmospheres, κ is relatively small, and the heat at the ground will be trapped for a longer period of time. Furthermore, the value of κ in this formulation for a given planet is probably related to the amount of atmospheric absorption of heat for the planet.

"Time" in this case can be measured by the rotation rate of the planet, Ω^{-1}. If distance is taken to be about equal to the equatorial circumference of the planet ($2\pi R$, where R is the planet's radius), the time T_D for the incident heat to diffuse completely around the planet is given very roughly by

$$T_D \sim \frac{(2\pi R)^2}{\kappa} = 4\pi^2\frac{R^2}{\kappa} \quad .$$

Suppose we define the parameter λ as

$$\lambda = \frac{T_D}{\Omega^{-1}} = \Omega T_D = 4\pi^2 R^2\Omega/\kappa \quad .$$

Thus, λ is the ratio of the diffusion time to the rotation time of the planet. For $\lambda >> 1$, T_D is long compared to the rotation time, and over a single "day" on the planet temperature gradients will develop between the day and night sides. On the other hand, for $\lambda << 1$, T_D is short compared to a day, and over a single day the gradients will completely diffuse away, leaving the temperature relatively uniform over the planet. We can examine the differences between Venus and Mars by looking at the parameter λ for the two planets; thus,

$$\frac{\lambda_M}{\lambda_V} \sim \left(\frac{\Omega_M}{\Omega_V}\right)\left(\frac{R_M}{R_V}\right)^2\left(\frac{\kappa_M}{\kappa_V}\right) \quad ,$$

where the subscripts M and V denote Mars and Venus. We can evaluate the various parts of this ratio as

$$\frac{\Omega_M}{\Omega_V} \sim \frac{244 \text{ days (Venus)}}{1 \text{ day (Mars)}} = 244$$

$$\left(\frac{R_M}{R_V}\right)^2 \sim \frac{(3395 \text{ km})^2}{(6070 \text{ km})^2} = 0.31$$

so that

$$\frac{\lambda_M}{\lambda_V} \sim (244)(0.31)\left(\frac{\kappa_M}{\kappa_V}\right) = 76\left(\frac{\kappa_M}{\kappa_V}\right) \quad .$$

The equivalent eddy diffusivities κ for the two planets are largely unknown, but it is known that Mars has a weak atmosphere while Venus has one of the densest atmospheres in the solar system; additionally, an analysis of temperature data from Venus suggests that there is a very strong greenhouse effect present, yielding very high surface temperatures. Thus, we might expect that $\kappa_M > \kappa_V$. None of these parameters is well-known, but this simple model can be used to attempt to diagnose the difference in the diurnal heating cycles of Mars and Venus. What has been shown here is that Mars is likely to lose much more heat over one daily cycle than will Venus, accounting for the differences in the diurnal heating on the two planets.

4. If Planet Z has no atmosphere but does have an internal source of heat, then in equilibrium the heat balance for Planet Z can be written as

$$Q_B - Q_S(1-\alpha) + Q_I = 0 \quad ,$$

where Q represents the various heat fluxes and the subscripts B, S, and I denote the black body radiation, the direct solar input of heat, and the flux of internal heat through the surface of the planet. The parameter α is the albedo. If the solar heating and internal heating are known, then we can write that

$$Q_S(1-\alpha) = Q_B + Q_I = \sigma T^4 \quad ,$$

where σ is the Stefan-Boltzmann constant (5.67×10^{-8} watts/m^2/°K^4) and T is the surface temperature of the planet. Since the planet is 60 AU from the sun, we expect that Q_S for Planet X is a factor of $(1/60)^2$

less than the analogous value for the Earth, which is about 342 watts/m^2. Thus, we estimate $(1-\alpha)Q_S$ as approximately $0.7\times9.5\times10^{-2}$ watts/m^2. Using this result, the heat balance for Planet X becomes

$$\sigma T^4 = (0.5+0.066) = 0.566 \Rightarrow T = \left(\frac{0.566}{\sigma}\right)^{1/4} = \left(\frac{0.566}{5.67\times10^{-8}}\right)^{1/4} = 56\,^{\circ}\mathrm{K} \quad .$$

(b) The estimated surface temperature of Planet Z, assuming that the planet has no atmosphere, is 56 °K, or −217 °C. If Planet Z has an atmosphere, and the atmosphere can absorb some of the incoming solar radiation via a greenhouse effect, then the surface of Planet Z should be warmer than 56 °K. This situation was outlined in the class notes.

At the top of the atmosphere, we have $\sigma T_{\mathrm{a}}^4 = 0.066$, where T_{a} is the temperature of the atmosphere. From this we find that

$$T_{\mathrm{a}} = \left[\frac{0.066}{5.67\times10^{-8}}\right]^{1/4} = 32.8\,^{\circ}\mathrm{K} \quad .$$

We can find the surface temperature T_{S} of the planet through the relation

$$(1-\alpha)Q_s + Q_I + \sigma T_{\mathrm{a}}^4 = \sigma T_{\mathrm{S}}^4 \quad .$$

Solving for T_{S}, we find that

$$T_{\mathrm{S}} = \left[\frac{(0.566+0.066)}{5.67\times10^{-8}}\right]^{1/4} = 57.8\,^{\circ}\mathrm{K} \quad .$$

5. The speed of an air parcel in the Jet Stream (U) is typically about 10 m/sec or 860 km/day. The length scale for eddies in the Jet Stream is roughly 1000 km. The Rossby number for a parcel in the Jet Stream is thus given by

$$Ro = \frac{860\,\mathrm{km/day}}{(2\pi/\mathrm{day})(10^3\mathrm{km})} \sim 0.14 \quad .$$

For a Gulf Stream eddy, a similar estimate yields

$$Ro = \frac{8.6\,\mathrm{km/day}}{(2\pi/\mathrm{day})(4\times10^2\mathrm{km})} \sim 0.003 \quad .$$

Thus, because the Gulf Stream has the smaller Rossby number, it is closer to being in geostrophic balance.

6. (a) The average rainfall over the Earth's surface is about 1 meter/year. If we call this the *flux* of rain, and hence the downward flux of freshwater over the Earth's surface, then in steady state the upward flux of freshwater due to evaporation (F_w) must be numerically the same as this downward flux in order to maintain a global steady state. Note that this value is consistent with the definition of a flux

given in class (an amount per unit area per unit time), as this is the amount of rain falling on any given area in one year. Thus, the upward flux of freshwater is also about 1 meter/year. (b) The latent heat of evaporation L for freshwater at 20 °C (the approximate global average temperature at the ground or the sea surface) is about 2.3×10^6 joules/kg. Thus, the energy flux E_w associated with the evaporative flux of freshwater is

$$E_w = \rho L F_w = \left(1\frac{\mathrm{g}}{\mathrm{cm}^3}\right)\left(\frac{2.3\times10^3\ \mathrm{joules}}{\mathrm{g}}\right)\left(10^2\ \frac{\mathrm{cm}}{\mathrm{yr}}\right) = \frac{2.3\times10^5\ \mathrm{joules}}{\mathrm{cm}^2\ \mathrm{yr}} = 7.5\times10^{-3}\ \mathrm{watts/cm}^2$$

$$= 75\ \mathrm{watts/m}^2$$

since there are about 3.14×10^7 seconds/yr.

(c) The incoming solar heat flux Qs is about 150 watts/m^2. So under these assumptions, the heat flux associated with evaporation is about 50% of the incoming solar heat flux (and opposite in sign).

(d) If we were to remove heat from the upper 100 m of the global ocean at a rate of E_w, the ocean would cool at a rate given approximately by

$$Q \sim \rho C h \frac{\Delta T}{\Delta t} = E_w \Rightarrow \frac{\Delta T}{\Delta t} = \frac{E_w}{\rho C h} = \frac{\left(75\ \mathrm{watts/m}^2\right)}{\left(1000\mathrm{kg/m}^2\right)\left(4.2\times10^3\ \mathrm{joules}/\left(\mathrm{kg}\,^\circ\mathrm{C}\right)\right)\left(100\ \mathrm{m}\right)}$$

$$= 1.8\times10^{-7}\,^\circ\mathrm{C/sec} \sim 5.4^\circ\mathrm{C/year}.$$