OCN/ATM/ESS 587

The wind-driven ocean circulation….

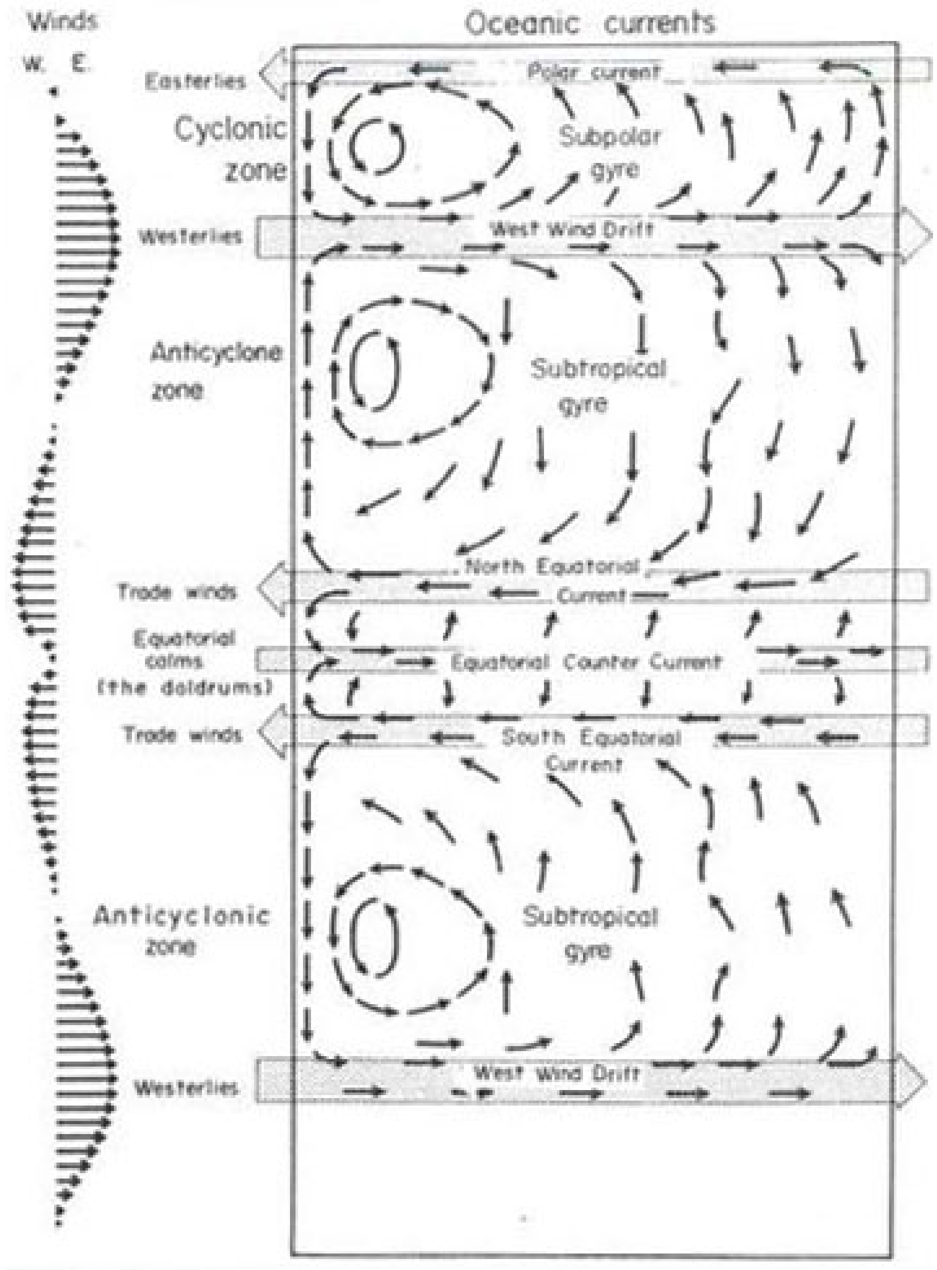
- **Friction and stress**
- **The Ekman layer, top and bottom**
- **Ekman pumping, Ekman suction**
- **Westward intensification**

The wind-driven ocean….

The major ocean gyres are strongly related to the strength and direction of the overlying winds.

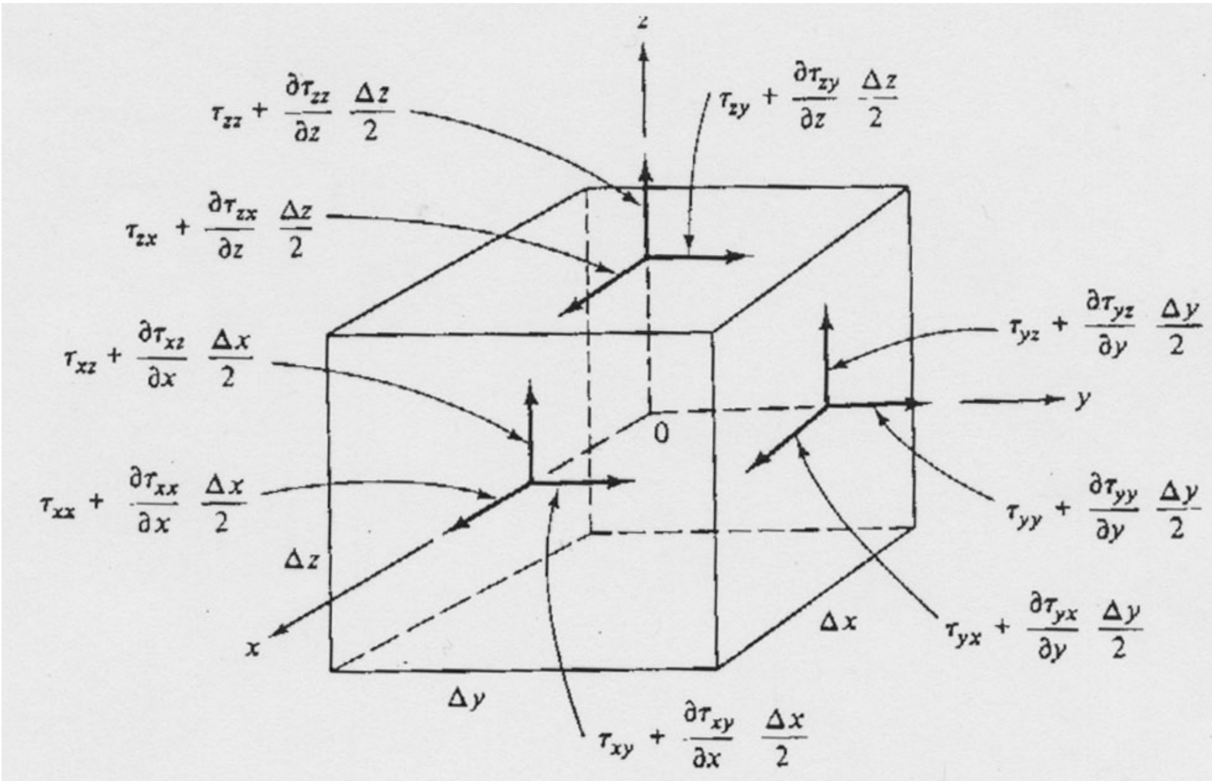
Clearly the wind is driving the ocean, but how is this accomplished?

[some friction is necessary]



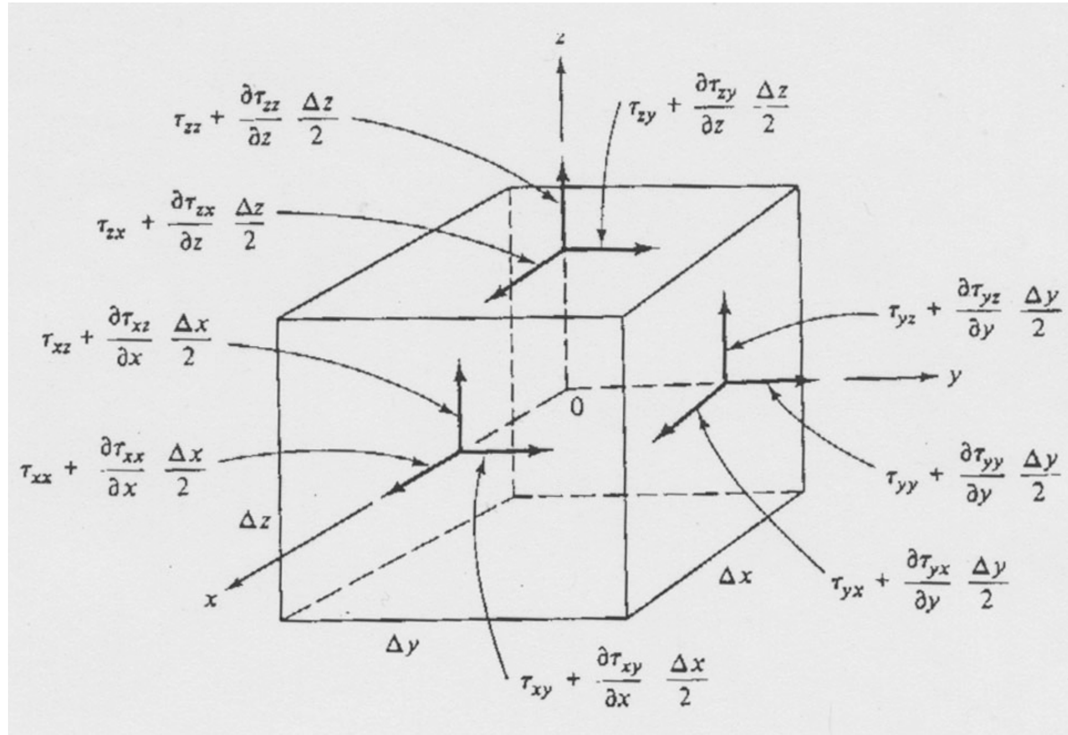
Recall: fluids cannot support a shearing force in equilibrium without deforming; a fluid element can only maintain its shape in the presence of normal forces (such as pressure).

But pressure is just one possible force/area on a fluid element. In general, we'll refer to a *stress* as a force/area acting on a fluid element, and in general the action of a stress will cause the element to deform.



τ_{ab} = the force/area in the plane normal to the *a* axis acting in the *b* direction

Stress is a *tensor* that has 9 components.



$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

[note: diagonal elements are the pressure]

How does fluid deform? [Recall, the distinguishing property of a fluid is that it cannot support a shearing stress without deforming.]

The shearing stresses (ie, the off-diagonal elements of the stress tensor) cause the fluid to deform.

It seems reasonable to postulate that the stress in some direction is directly proportional to the velocity in that direction relative to the opposite face of the cube, which is the *shear*.

Thus, we postulate that, for example,

$$\tau_{zx} \propto \frac{\partial u}{\partial z} \quad \Rightarrow \quad \tau_{zx} = \mu \frac{\partial u}{\partial z}$$

[Newton's Law of viscosity]

dynamic viscosity

$$\tau_{zx} \propto \frac{\partial u}{\partial z} \quad \Rightarrow \quad \tau_{zx} = \mu \frac{\partial u}{\partial z}$$

$$\tau \sim \left[\frac{mL}{T^2} / L^2 \right] = \mu \frac{(L/T)}{L} = \frac{\mu}{T} \Rightarrow \mu \sim \left[\frac{m}{LT} \right]$$

$$\upsilon = \frac{\mu}{\rho} \Rightarrow \left[\frac{L^2}{T} \right]$$

$\mu \approx$ **1.4×10^{-3} kg/(m-sec)**
$\nu \approx$ **1.4×10^{-6} m/sec^2**

kinematic viscosity

Stress and force....

Just as the pressure gradient represents a force acting on a fluid, the difference in stress between 2 opposing faces of the cube represents a net force.

$$\frac{\partial \tau_{zx}}{\partial z} = \text{a net force} = \frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z} \right) \approx \mu \frac{\partial^2 u}{\partial z^2}$$

Stress, viscosity, and force….

We can write and equation of motion, incorporating the concepts of stress and Newtonian friction, as

$$\underbrace{\frac{d\mathbf{u}}{dt}}_{\text{acceleration}} + \overbrace{f\mathbf{k}\times\mathbf{u}}^{\text{Coriolis}} = \overbrace{-\frac{1}{\rho}\nabla p}^{\text{pressure}} + \underbrace{\upsilon\nabla^2\mathbf{u}}_{\text{friction}} - \overbrace{g\mathbf{k}}^{\text{gravity}}$$

$$\rho\,\mathbf{a} = \Sigma F \quad \textbf{or} \quad \mathbf{a} = (1/\rho)\,\Sigma F$$

[Newtons' Law, per unit volume]

Boundary conditions….

Most of the stress (ie, friction) between 2 fluids will occur at the interface between them. The stress is related to the spatial change in velocity, so that near the interface,

$$\tau\,(\text{interface}) = \upsilon \left.\frac{\partial u}{\partial s}\right|_{\text{interface}}$$

Thus, for a wind with wind-stress τ_x blowing over the ocean (ie, in the eastward direction; vertical coordinate is *z*) the proper boundary condition at the air-sea interface is

$$\tau(z=0) = \upsilon \left.\frac{\partial u}{\partial z}\right|_{z=0}$$

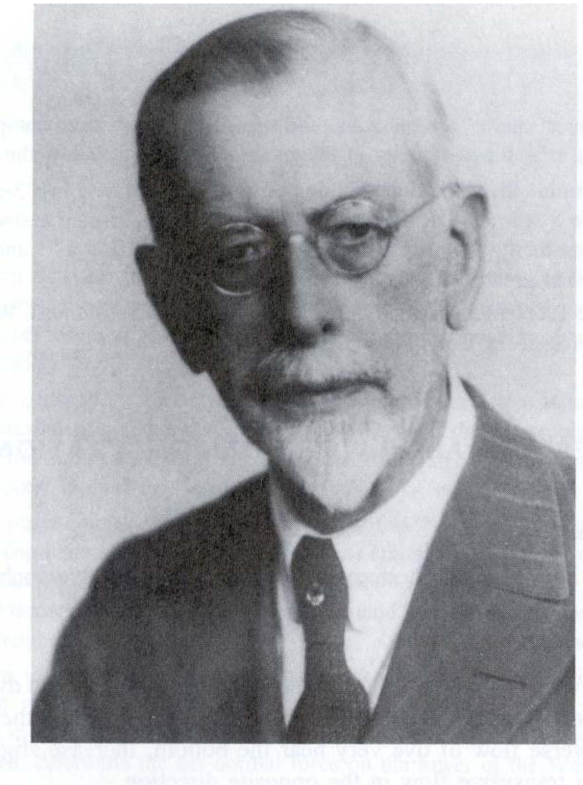
The Ekman layer….

Where and when is friction important?

- **The interior of the ocean is geostrophic (nearly frictionless)**

- **Friction (stress) is strongest where the spatial changes in speed are the largest (near top, bottom, and sidewall boundaries)**

- **The nature of these boundary layers was first examined by W. Ekman in the early part of the 20th century**



W. Ekman
1874-1954

The Ekman layer….

Let the total horizontal velocity field in the ocean be $\mathbf{u}_T$.

Let this velocity field be composed of geostrophic and nongeostrophic parts, so that $\mathbf{u}_T = \mathbf{u}_G + \mathbf{u}$.

Putting this definition of $\mathbf{u}_T$ into the equation of motion, we find that

$$\frac{d\mathbf{u}_\mathbf{T}}{dt} + f\mathbf{k}\times\mathbf{u}_\mathbf{T} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u}_\mathbf{T} - g\mathbf{k}$$

Removing the geostrophic part, we find that

$$\frac{\partial \mathbf{u}}{\partial t} + f\mathbf{k} \times \mathbf{u} = \upsilon \nabla^2 \mathbf{u}$$

Let's look for steady solutions (ie, $\partial/\partial t = 0$), so that

$$fv = \upsilon u_{zz}$$

$$-fu = \upsilon v_{zz}$$

The boundary conditions for $\tau = (\tau_x, 0)$ (a purely west wind) are

$$\left.\frac{\partial u}{\partial z}\right|_{z=0} = \frac{\tau_x}{\rho \upsilon}$$

$$u, v \to 0 \;\; as \;\; z \to -\infty$$

These equations can be combined to yield a single 2nd order ODE, and the solution can be found to be (see Pond and Pickard or Apel for the details)

$$u = \frac{\tau_x h}{\sqrt{2}\rho\upsilon} e^{z/h} \cos\left(\frac{z}{h} - \frac{\pi}{4}\right)$$

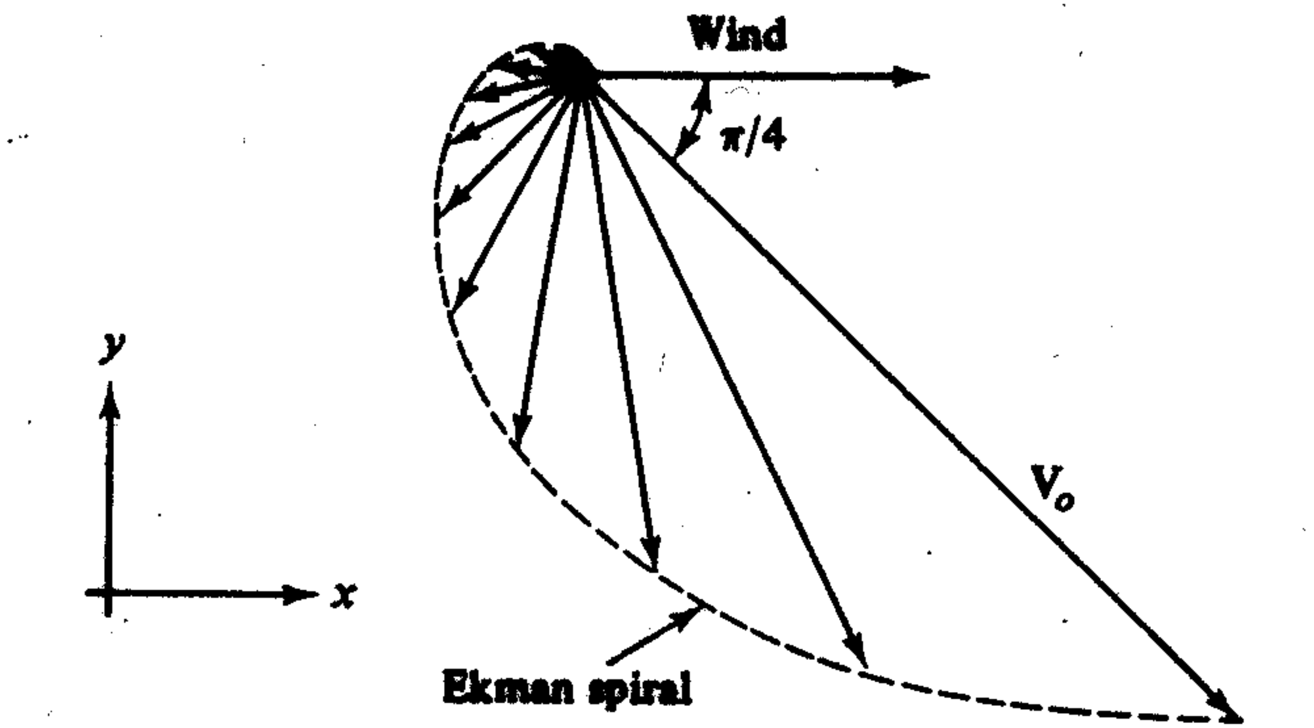
$$v = \frac{\tau_x h}{\sqrt{2}\rho\upsilon} e^{z/h} \sin\left(\frac{z}{h} - \frac{\pi}{4}\right)$$

at *z*=0, the velocity vector is in a direction 45° to the right of the wind; the vector decays with depth and spirals to the right

$$h = \sqrt{\frac{2\upsilon}{f}}$$

the spiral decays with a characteristic depth scale of *h*

The Ekman spiral at the sea surface (N. Hemisphere)….



[what is the physical interpretation of this?]

Transport in the Ekman layer….

$$\mathbf{T}_{Ek} = \int_{-\infty}^{0} \mathbf{u}\, dz = \left(0, -\frac{\tau_x}{f}\right)$$

[90° to the right of the wind; for a wind blowing to the *east*, the Ekman transport is *southward*]

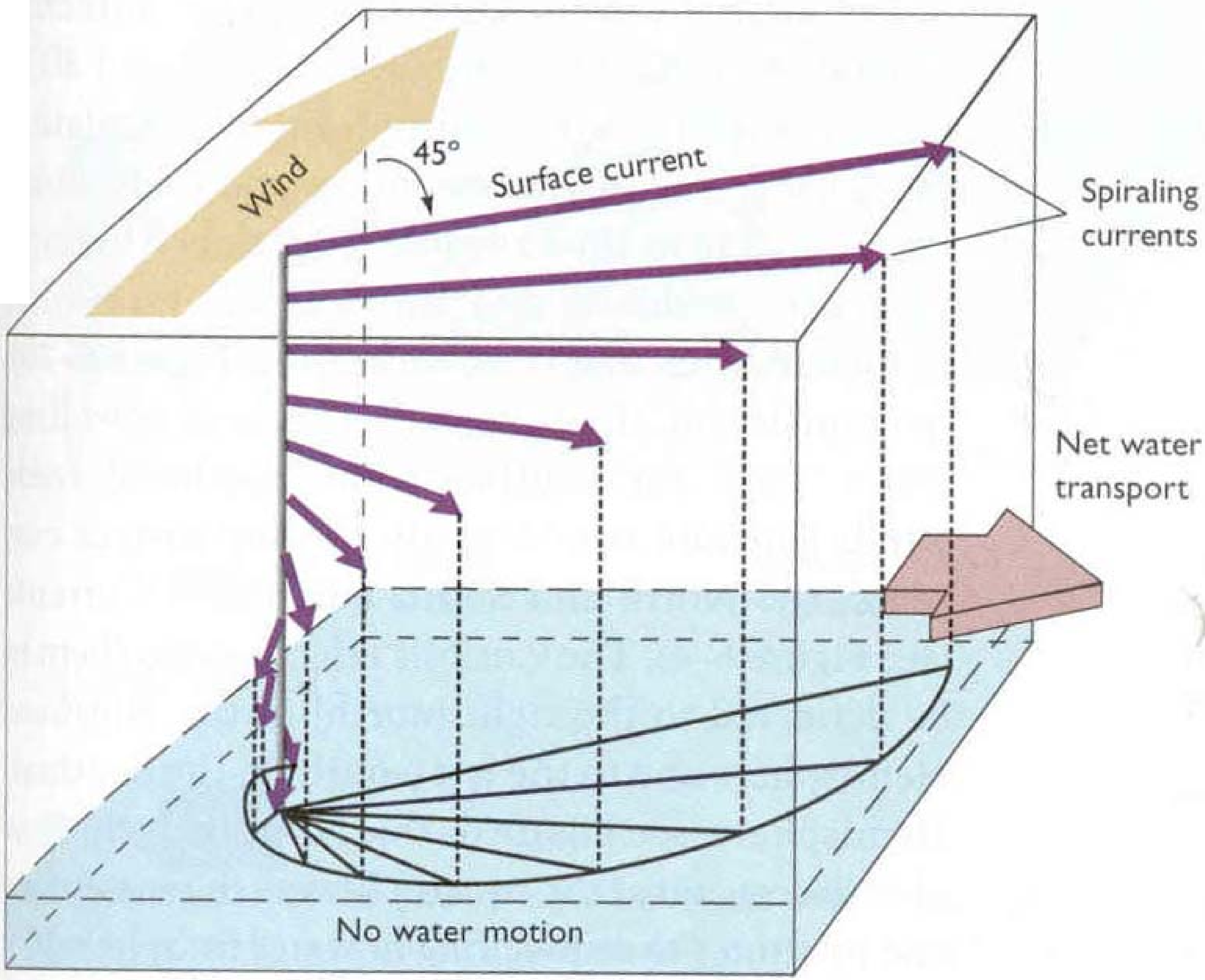
The Ekman depth….

$$h = \sqrt{\frac{2\upsilon}{f}} \sim \sqrt{\frac{(2)(1.4\times10^{-2}\ \text{cm}^2/\text{sec})}{10^{-4}\ \text{sec}^{-1}}} \sim 17\,\text{cm}$$

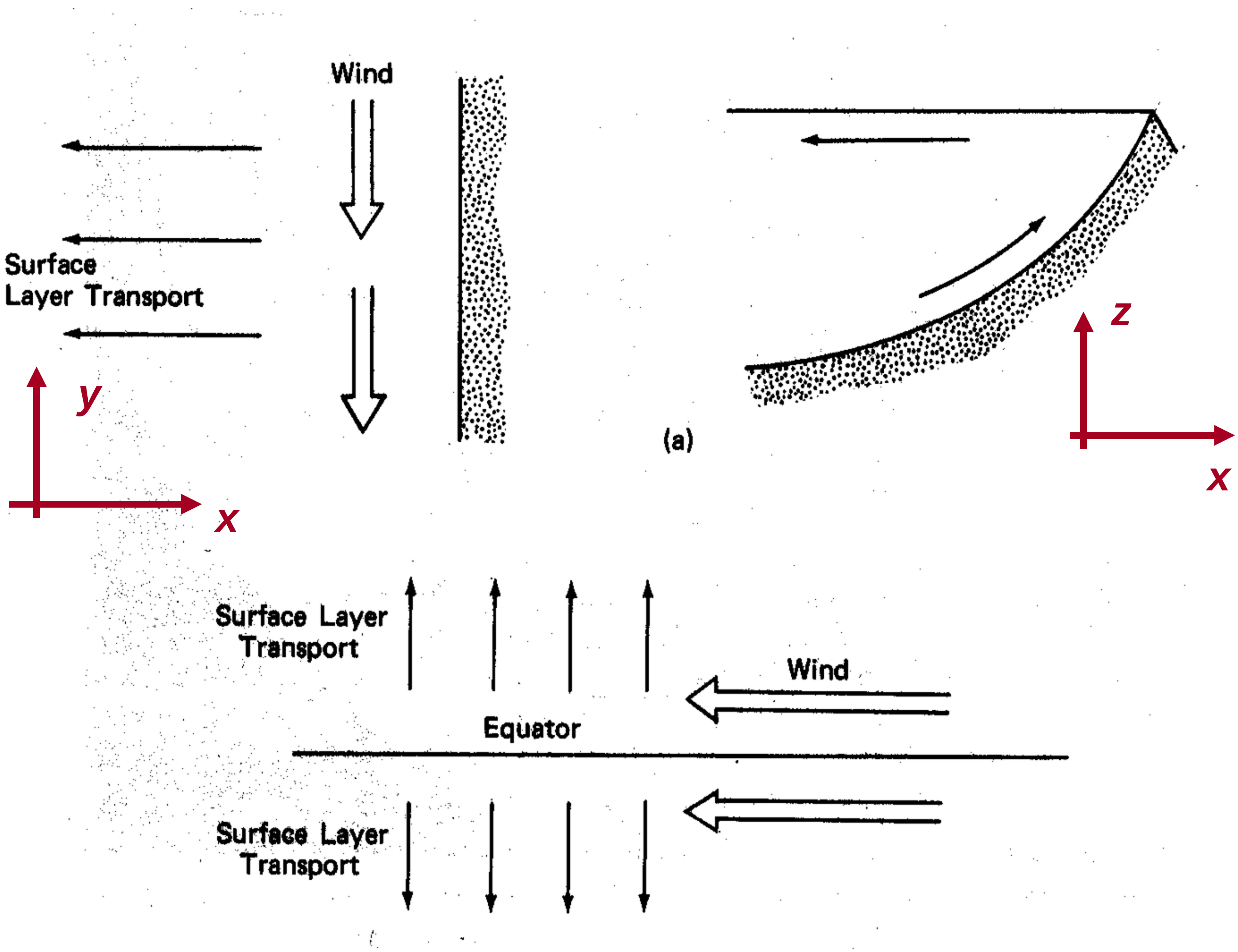
[too small]

Using an estimate of the turbulent viscosity instead ($\upsilon \sim 10^3$ cm^2/sec), we find

h ~ 50 meters [plausible]



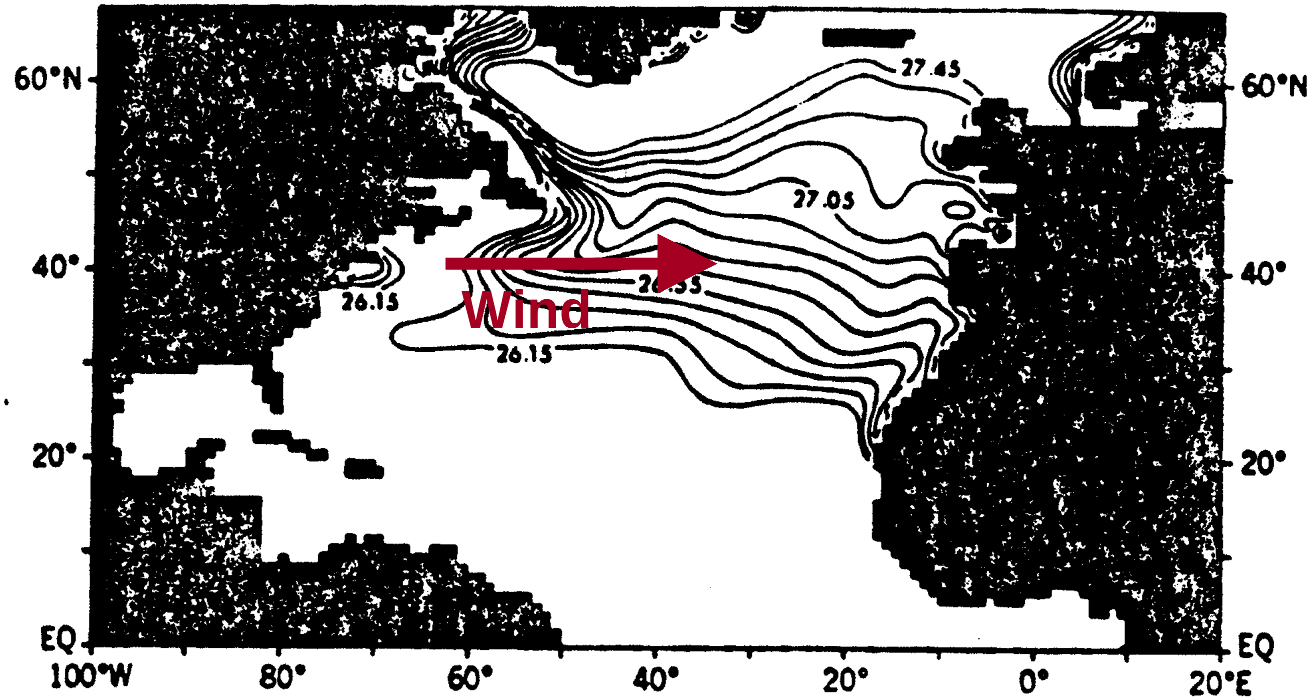
EKMAN SPIRAL IN THE NORTHERN HEMISPHERE



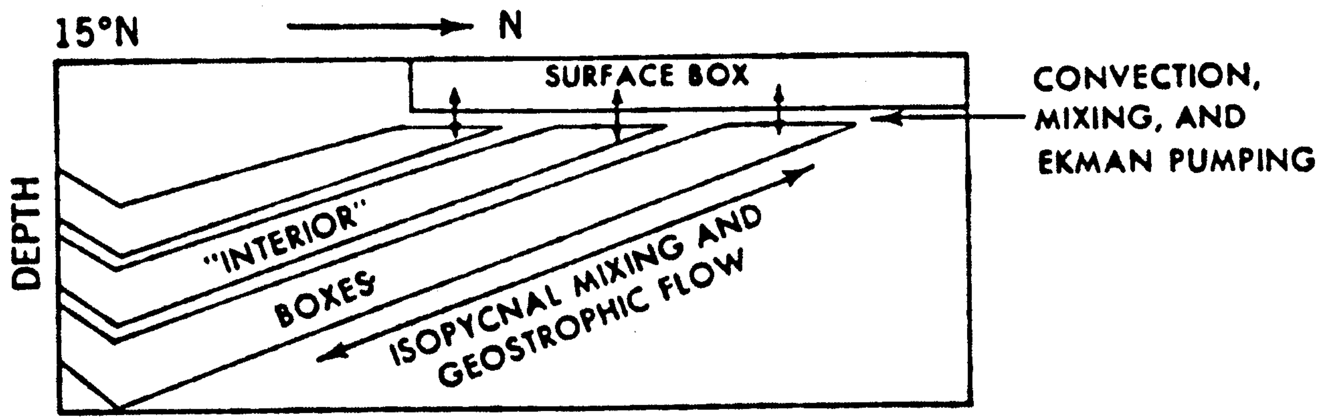
Upwelling at a coast or along the Equator driven by Ekman transport

Subduction….

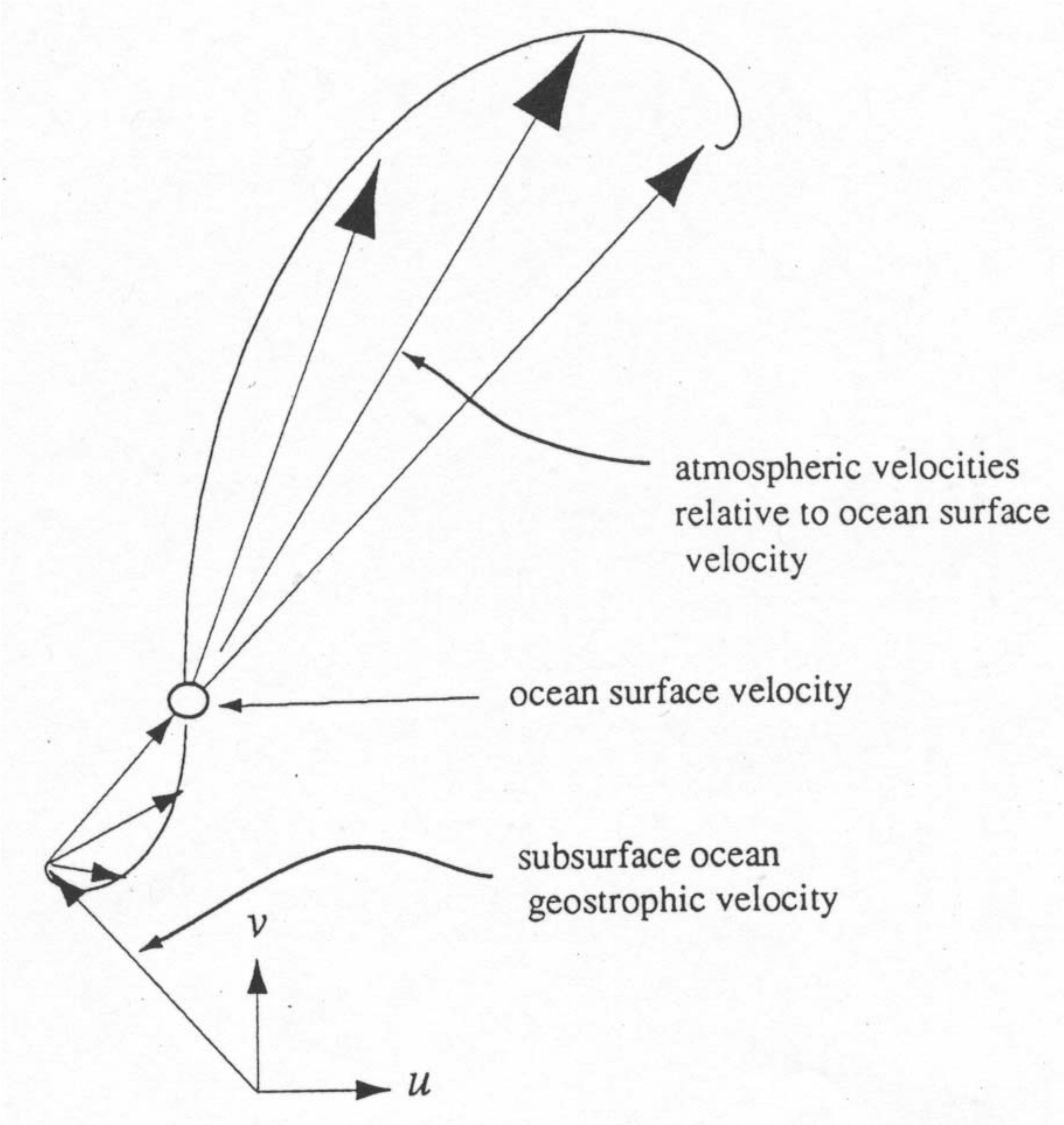
The surface mixed layer acts as a window between the atmosphere and deep ocean.



The wind pumps fluid and water properties down density surfaces



Ekman layers in the ocean and the atmosphere



Observations of Ekman layers….do they exist?

Problems with verification of the theory:

1. **Theory is steady state, but the ocean isn't.**

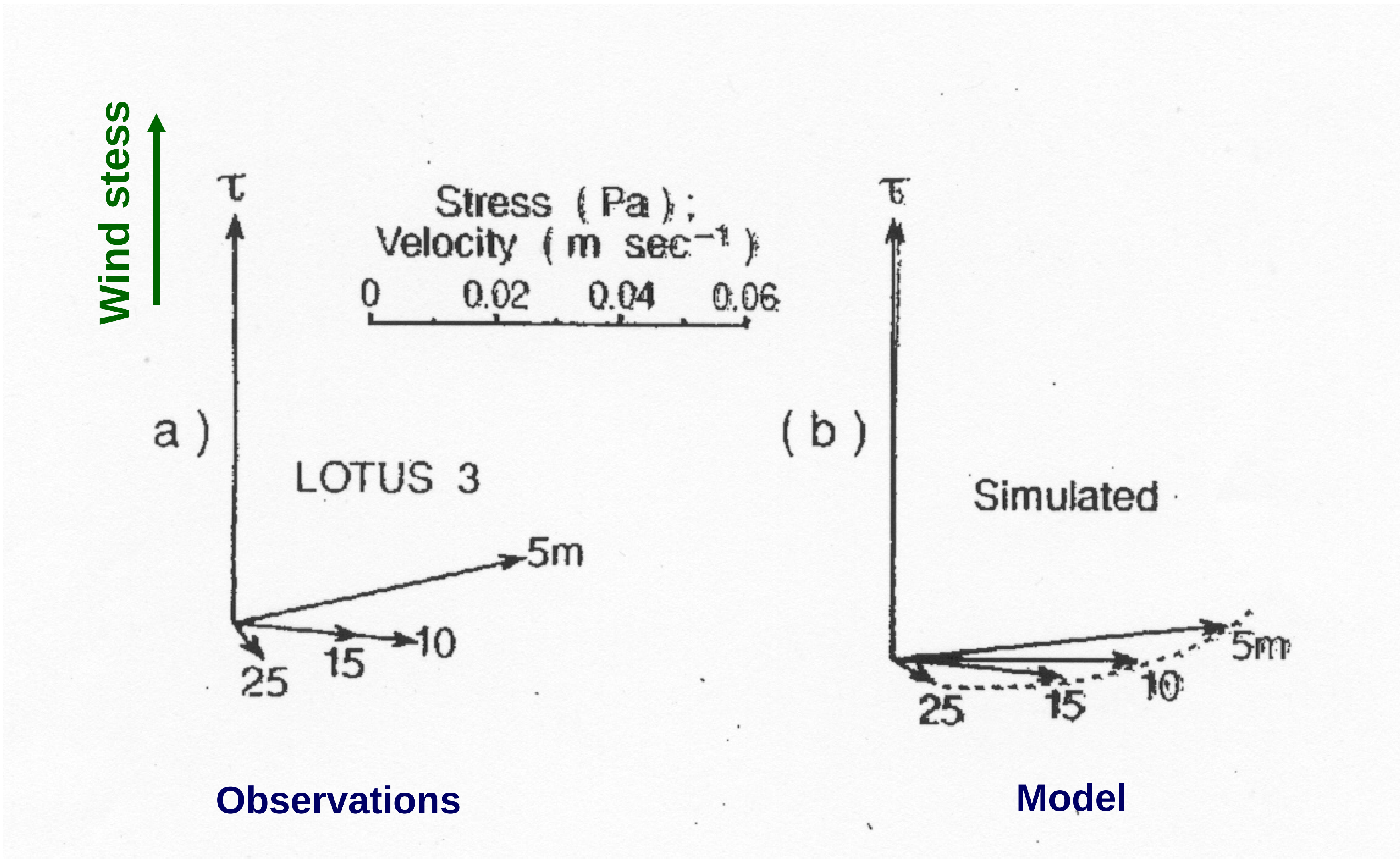
2. **It is difficult to observe velocity accurately in thin layers.**

3. **The ocean surface and bottom are complicated places.**

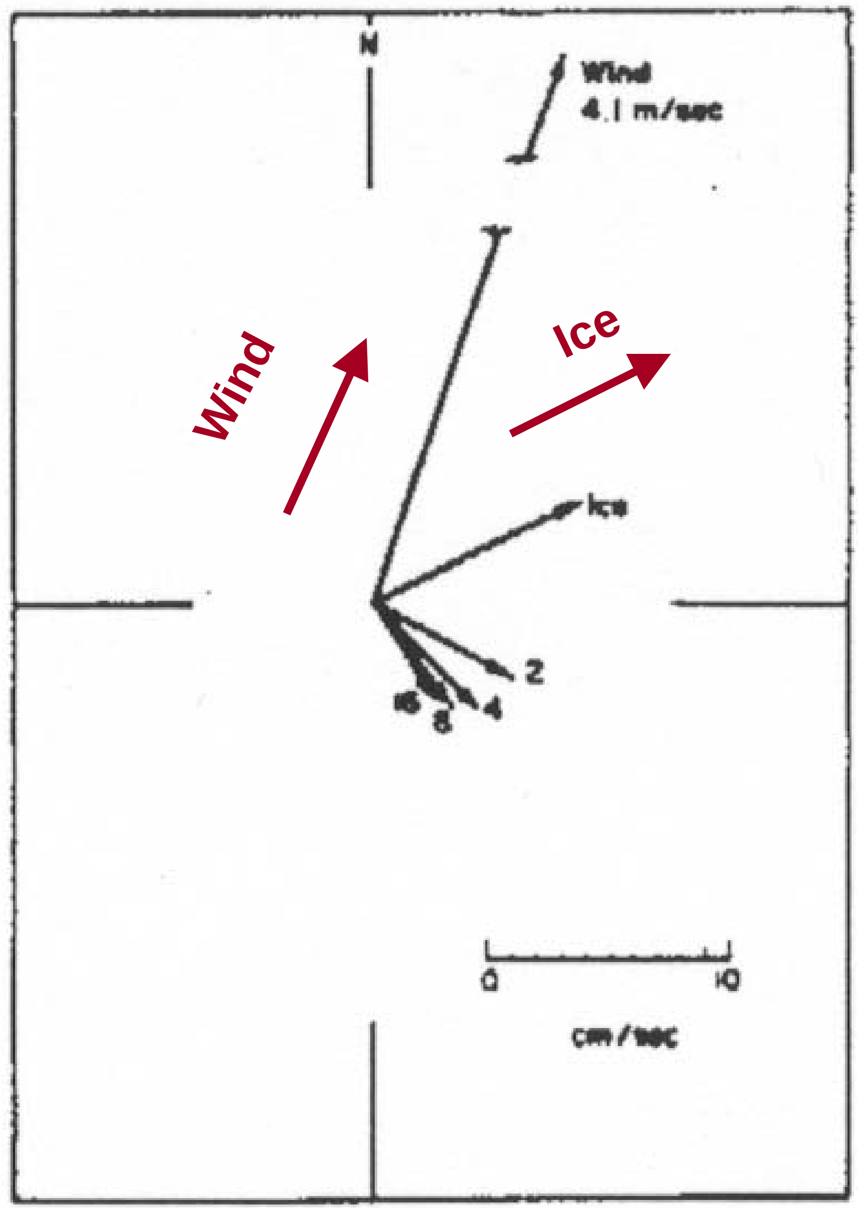
4. **Friction is more complicated than the parameterization in the theory.**

Attempts to observe Ekman layers….

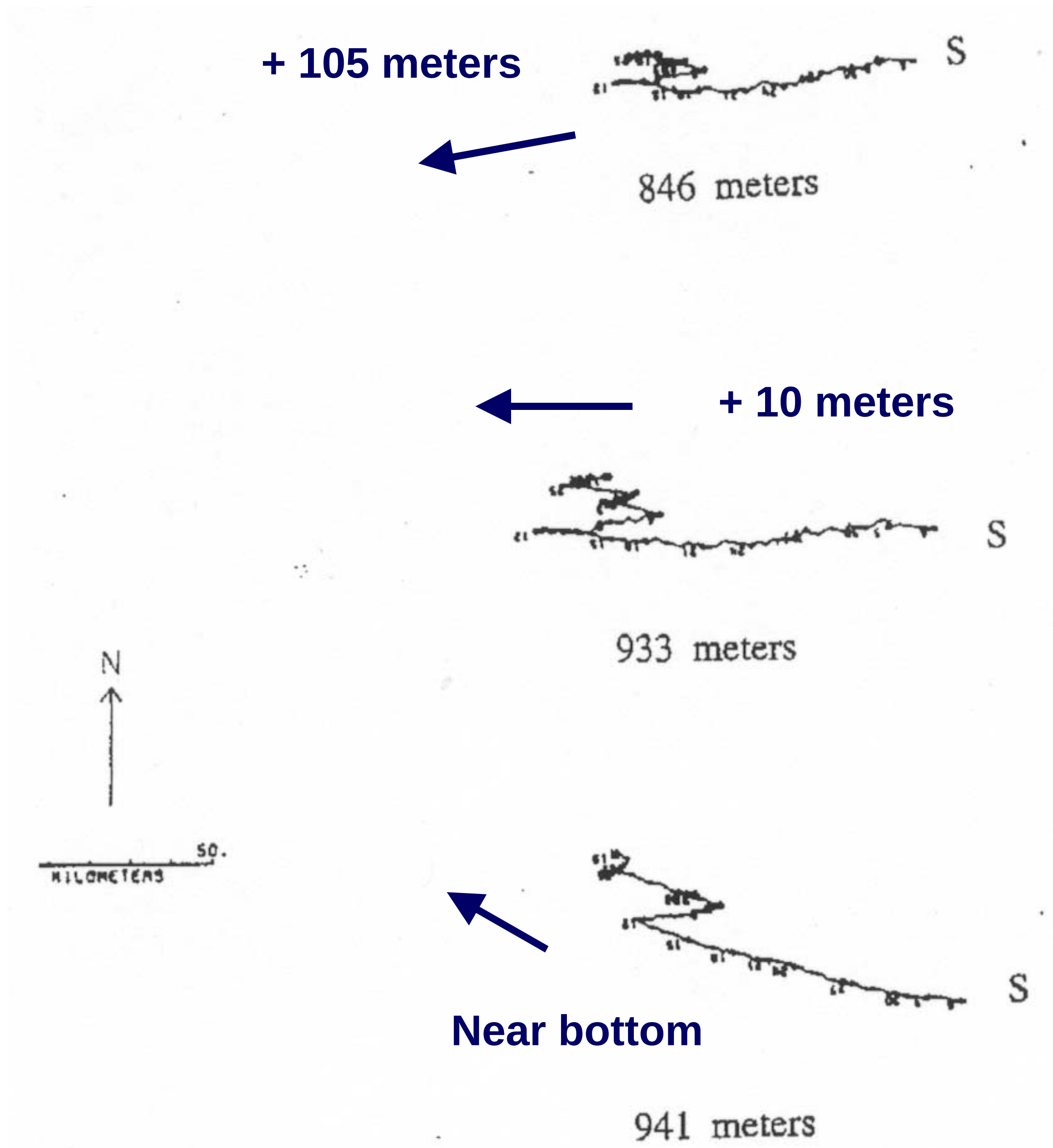
Price et al. (1987)….Ekman spirals in the western N. Atlantic…



Ekman spiral under Arctic ice (from Hunkins, 1974)

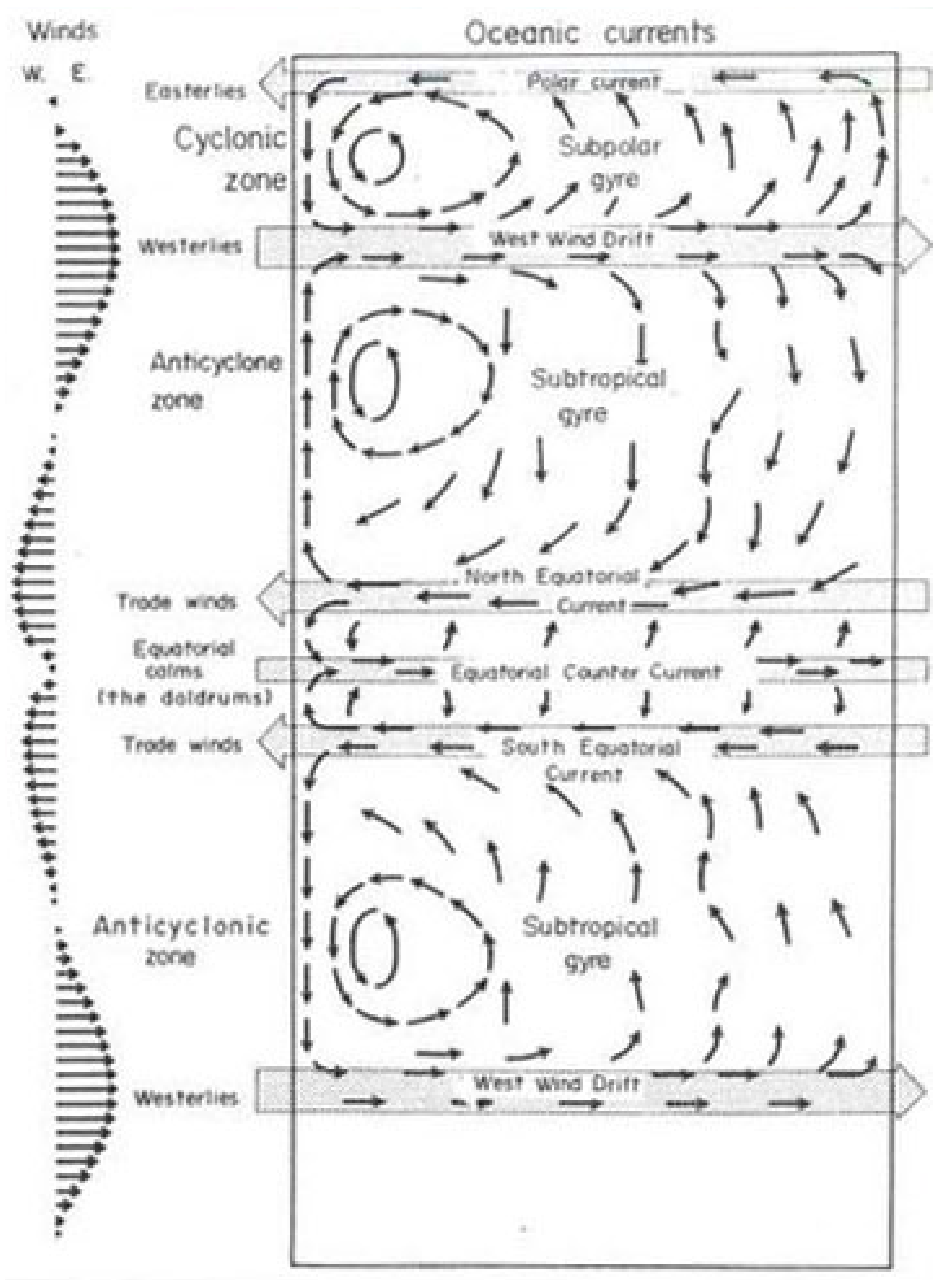


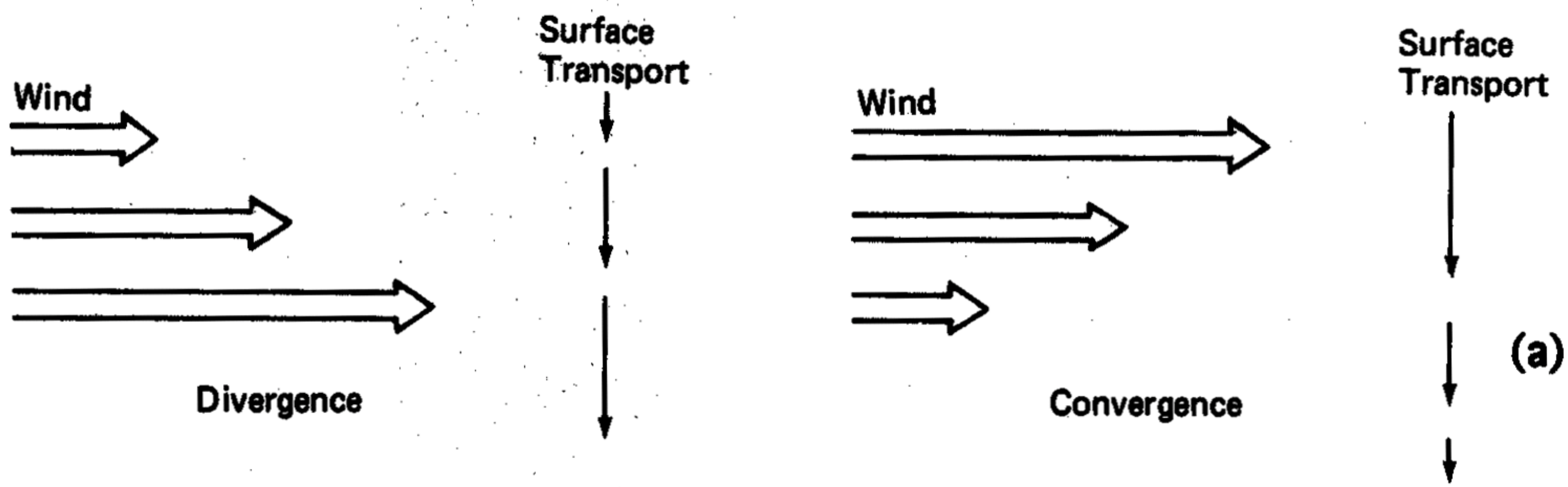
Bottom Ekman layer
(Wunch & Hendry, 1973)



The wind-driven ocean….

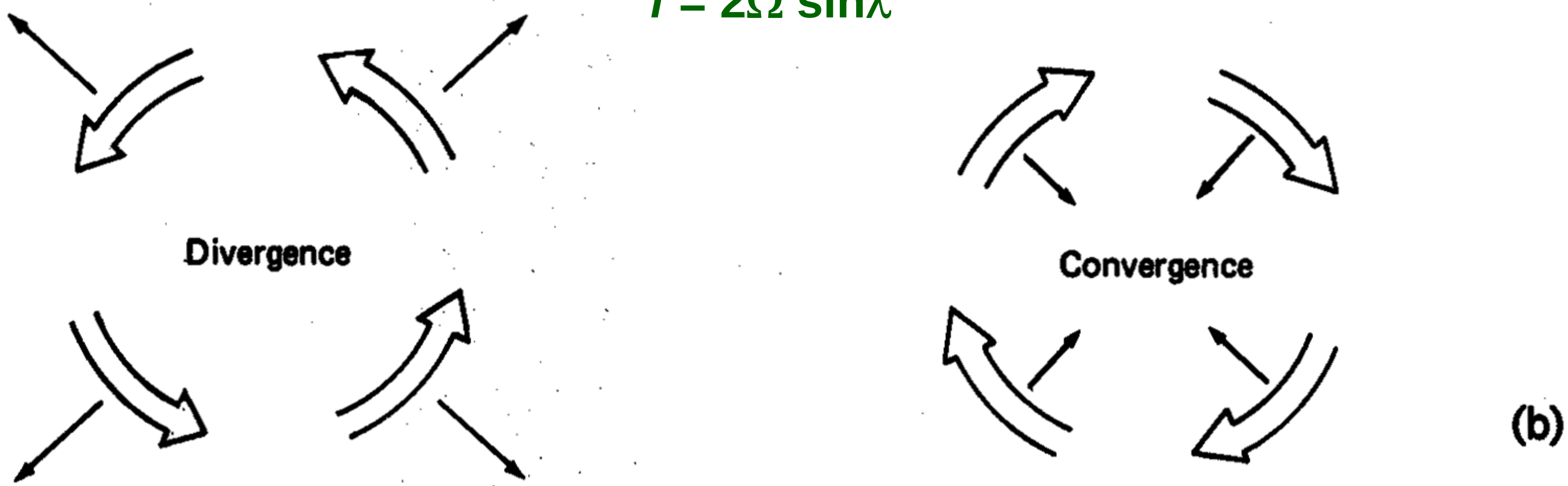
In reality, the wind fields over the world ocean have spatial structure; how does this affect Ekman's theory?



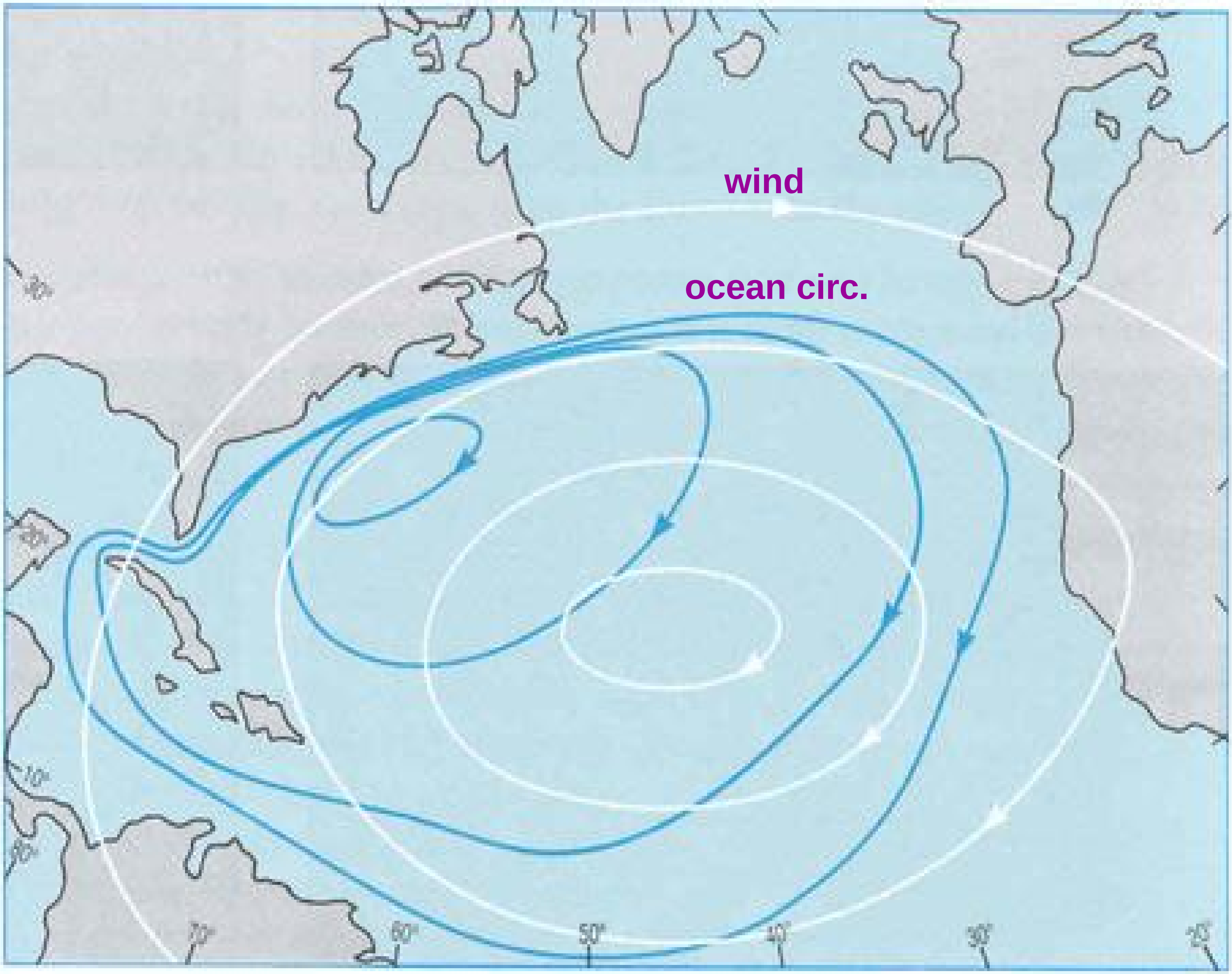


Recall: Ekman transport ~ $f^{-1/2}$

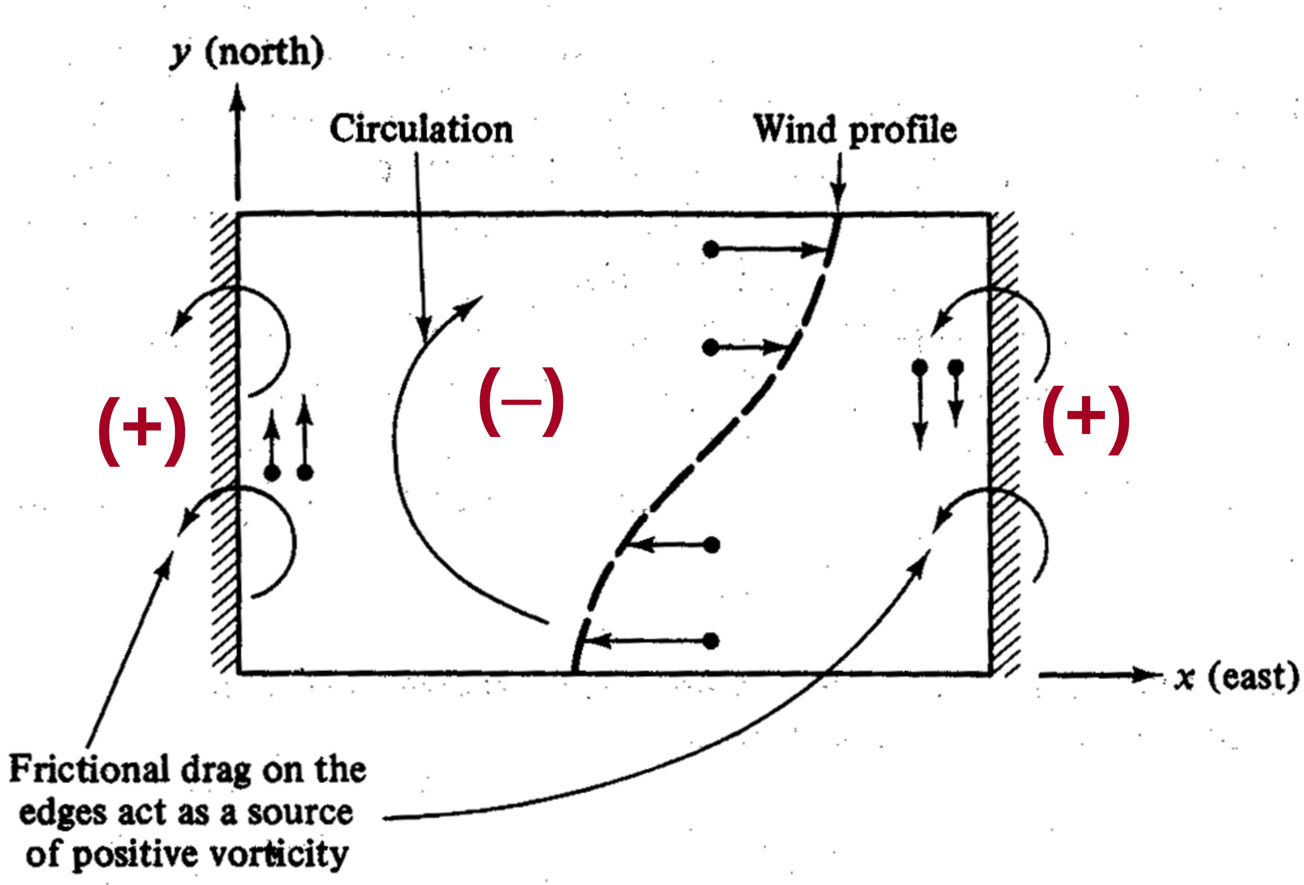
$f = 2\Omega \sin\lambda$



Spatial variations in the wind field cause Ekman convergence and divergence



Winds: symmetric; Ocean circulation: asymmetric….why?



Wind and friction introduce spin (or vorticity, or angular momentum) into the circulation, which must be balanced. But these effects are each symmetric, so their sum yields a symmetric circulation, which is not observed. What's missing?

What's missing?
Answer: rotation, spherical geometry

(1) Particles spinning around an axis perpendicular to the surface of the Earth at the Equator do not feel the rotation of the Earth, since their rotation axis is normal to the Earth's rotation axis.

(2) Particles spinning around an axis perpendicular to the surface of the Earth at high latitudes are significantly affected by the rotation of the Earth, since their rotation axis is nearly parallel to the Earth's rotation axis.

(3) It can be shown that the effect of Earth's rotation increases as sin λ .

(4) Particles must conserve angular momentum. Moving north, they acquire (+) planetary effects, so their local angular momentum must decrease (yielding an increase in clockwise spin). Moving south, they acquire (–) planetary effects, so their local angular momentum must increase (yielding an increase in counterclockwise spin).

Note: planetary vorticity effect is asymmetric.

Vorticity tendency in a symmetric ocean

Vorticity Tendency	**North-flowing currents on the western side of the ocean**	**South-flowing currents on the western side of the ocean**
Wind	-1.0	-1.0
Friction	+0.1	+0.1
Planetary	-1.0	+1.0
TOTAL	-1.9	+0.1

[note: nearly balanced in the east, far out of balance in the west]

[impossible]

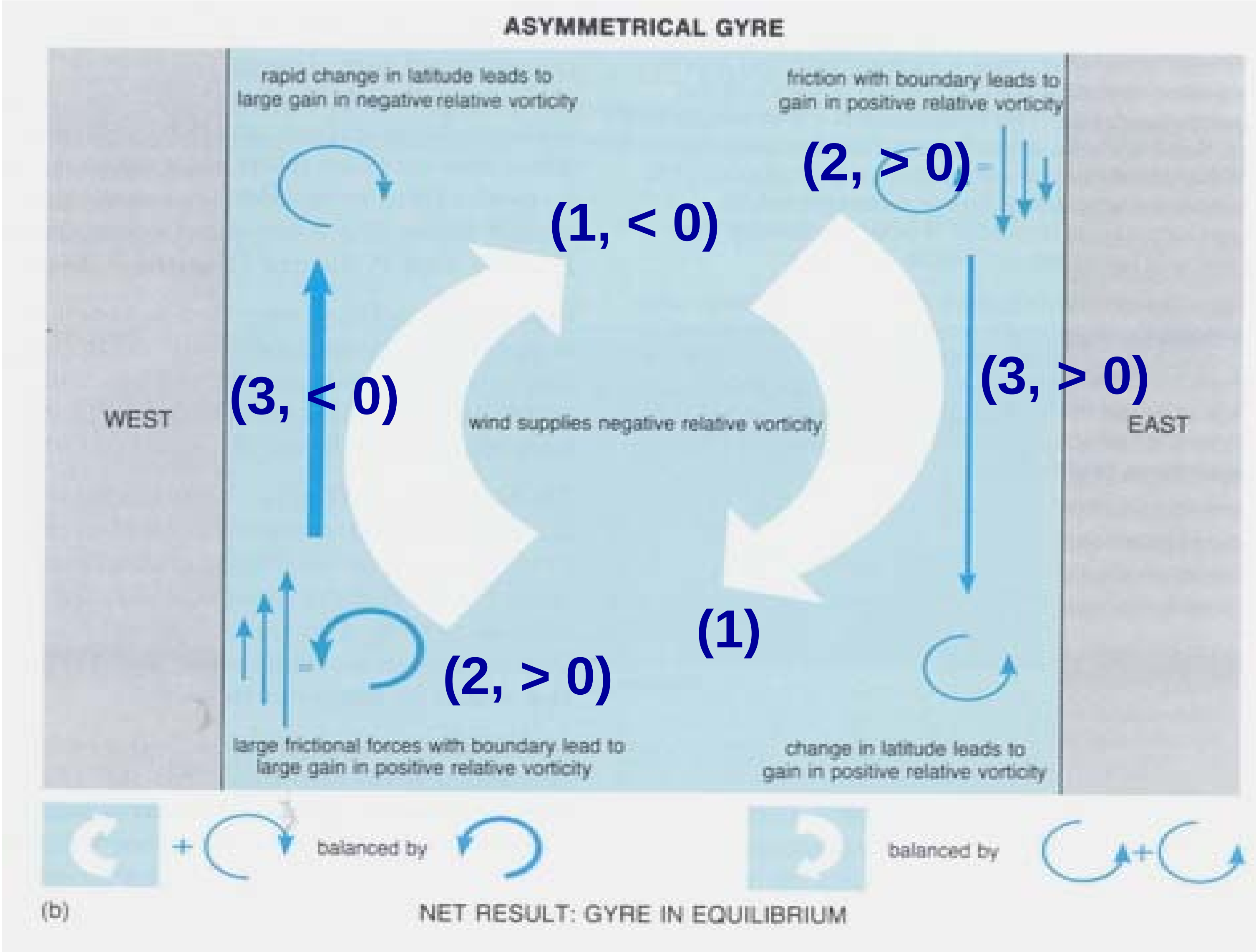
Vorticity tendency in an asymmetric ocean

Vorticity Tendency	**North-flowing currents on the western side of the ocean**	**South-flowing currents on the western side of the ocean**
Wind	-1.0	-1.0
Friction	+10.0	+0.1
Planetary	-9.0	+0.9
TOTAL	0.0	0.0

[note: balanced in both the east and west]

[possible]

3 effects:
(1) wind
(2) friction
(3) planetary

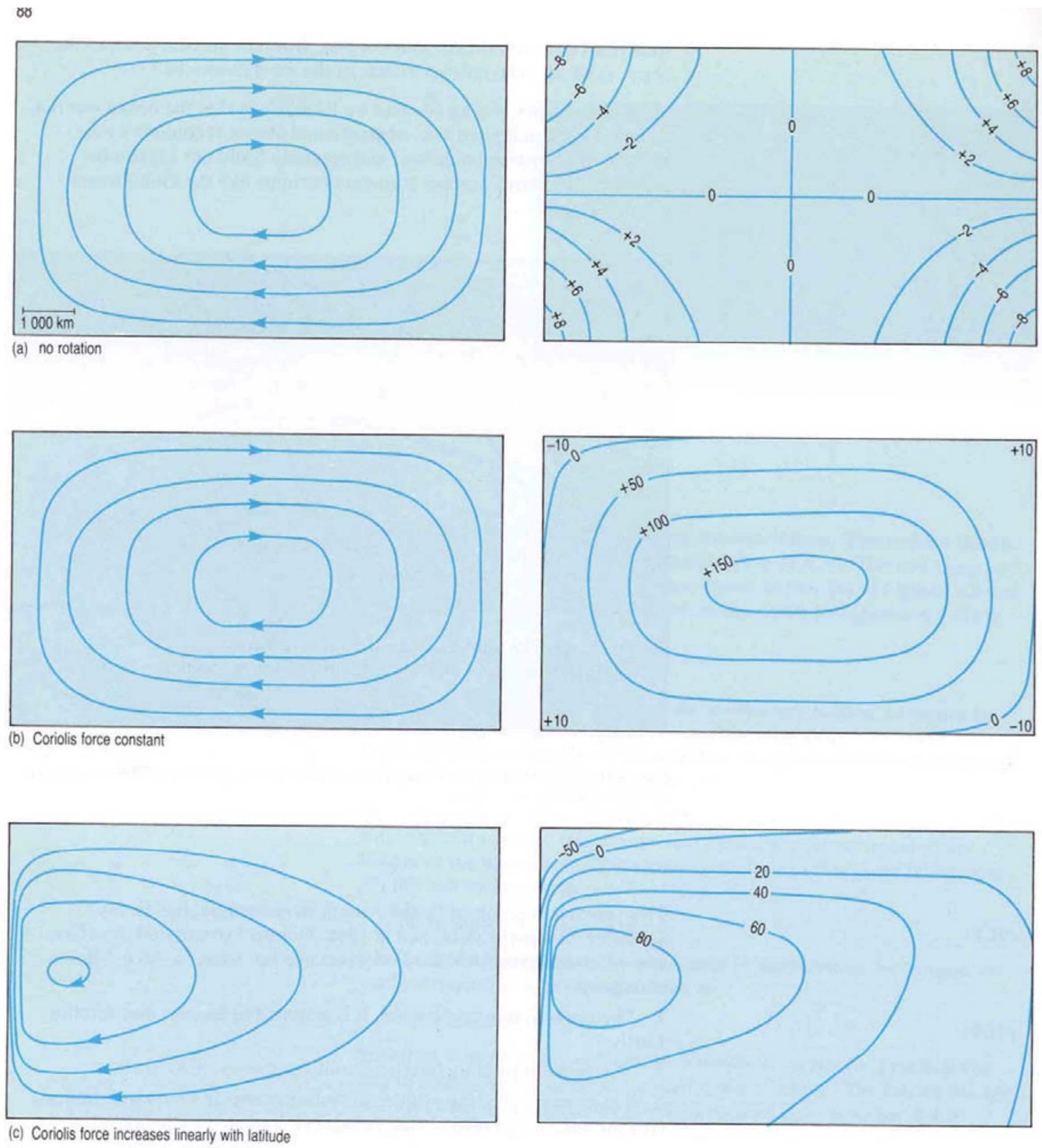


Asymmetric gyre: vorticity ("spin") can be balanced.

Symmetric (1)
(small lake)
(possible)

Symmetric (2)
(ocean basin)
(not possible)

Asymmetric
(ocean basin)
(possible)



Flow field **Sea level**