Energy and radiation
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Review of the electromagnetic spectrum

Different parts of the electromagnetic spectrum

Name	Wavelength range
Gamma rays	$< 10^{-10}$ m
X-rays	10^{-10} m to 10^{-9} m
Ultra violet	10^{-9} m to 10^{-7} m
Visible	400 to 700 nm
Infrared	10^{-6} to 10^{-3} m
Microwaves	10^{-3} m to 10^{0} m
Radio	$> 10^{0}$ m

The electromagnetic spectrum is a continuum of electromagnetic waves, ranging from gamma rays at the short wavelength (high frequency) to radio waves at the long wavelength (low frequency). The range of e-m waves found in nature spans many orders of magnitude in wavelength, and visible light occupies only a tiny fraction of that spectrum. In quantum mechanics, light and other e-m waves are quantized into packets of energy, photons. Whether light is really a wave or a particle is a question for philosophers. Sometimes it behaves like a wave, sometimes a particle. For this course, we will treat it as a wave. The measurement of the electromagnetic radiation emitted or absorbed by an object tells us a great deal about it.

Basic properties of electromagnetic radiation

Although the concept of a blackbody is a little abstract, it is actually extremely important in physics. The radiation coming from a blackbody is a function *only of its temperature and the wavelength of the radiation*. It is therefore a very useful standard against which to compare and understand the radiation emitted by more realistic objects.

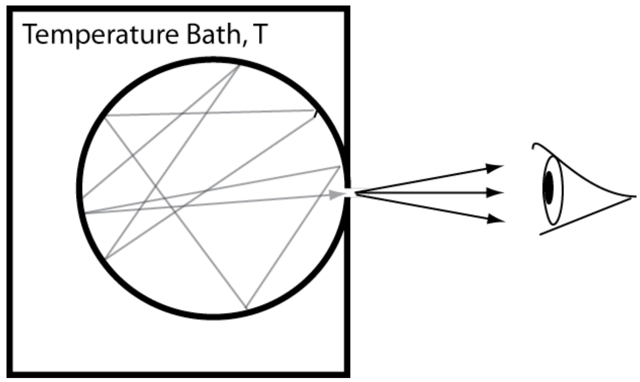


Figure shows a schematic illustration of an experimental set-up to measure blackbody radiation. A cavity (with matte inside walls) is set in a temperature bath to ensure it is in thermodynamic equilibrium. Many interactions of the radiation with the sidewalls occur, ensuring a good equilibrium, before it escapes through a small hole. An observer (i.e., a spectrometer) measures the intensity as a function of wavelength.

Blackbody radiation is the radiation emitted by a perfectly absorbing body that is in thermodynamic equilibrium. It is idealized concept, although in practice some simple experimental set-ups can come close (see diagram).

The reason for this rather precise definition is that, as a result, blackbody radiation is a function only of the temperature and the wavelength (and not for example, what the body is made out of.

Many objects in nature emit a spectrum of radiation that can be approximated (to a lesser or greater degree) by that of a blackbody. We will see several examples of this.

Properties of blackbody radiation

1. Planck's law of blackbody radiation

This law, one of the earliest and greatest achievements of quantum physics, describes the energy emitted by a blackbody as a function of the wavelength of the radiation and the temperature of the blackbody.

$$E_\lambda = \frac{c_1}{\lambda^5(\exp(c_2/\lambda T)-1)}. \qquad (1)$$

where E_λ is the radiative energy flux per unit wavelength [W m^{-2} per m], and T is the temperature in Kelvin.

c_1 is a constant 3.74 x 10^{-16} W m^2,
c_2 is a constant = 1.44 x 10^{-2} m K,

Note E_λ is a function of T and λ only. See figure in the handout for the shape of this function. Note that the total energy emitted by a blackbody at this temperature is the area under the curve.

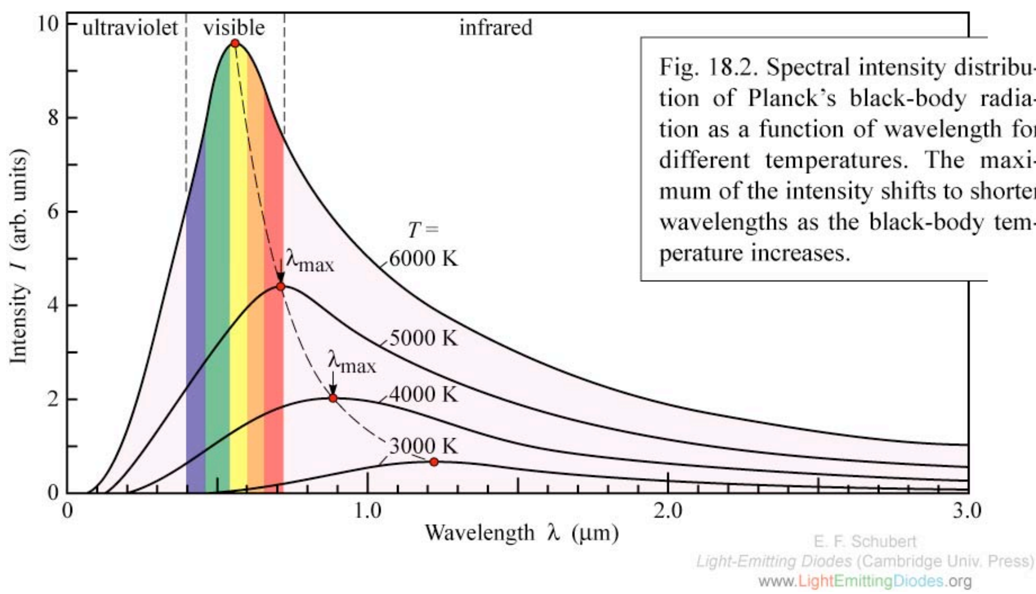


Figure shows several blackbody radiation curves at different temperatures. Also shown is the region of wavelengths comprising the visible (to us) part of the electromagnetic spectrum.

2. Wiens law.

From Planck's law it can be shown that for a blackbody at temperature T, the wavelength at which the radiation peaks, λ_m, obeys a particularly simple relationship:

$$\lambda_m T = 2877\ \mu\text{m K} \tag{2}$$

Note the strange units on the right hand side. The above figure shows blackbody radiation curves at different temperatures, and you can see that the warmer the temperature the more the peak of the curve is shifted to lower wavelengths (which is the same thing as higher, more energetic frequencies).

3. *Stefan-Boltzmann law*
This expression gives the total energy flux (energy flux equals energy per unit area per unit time) emitted by a blackbody at temperature *T*. It is equal to the area under the blackbody radiation curve in above figure:

$$E = \int_0^\infty E_\lambda d\lambda = \sigma T^4 \qquad (3)$$

Where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}. So the total radiation emitted goes as the fourth power of the temperature, and is thus a sensitive function of the temperature. You can also see from the blackbody curve that the greater the area under the curve increases quickly as the temperature gets hotter.

Case study: the sun

From an optical thermometer (e.g., a pyrometer), the surface temperature of the sun can be measured as approximately 6000 K. So total radiation emitted by the sun, I_0, can be calculated. It is equal to the radiation per unit area multiplied by the surface area of the sun, R_s. In other words

$$I_0 = \sigma T_s^4 \times 4\pi R_s^4 \qquad (5)$$

Since $R_s = 700 \times 10^6$ m, this gives

$I_0 \approx 3.9 \times 10^{26}$ J s^{-1},

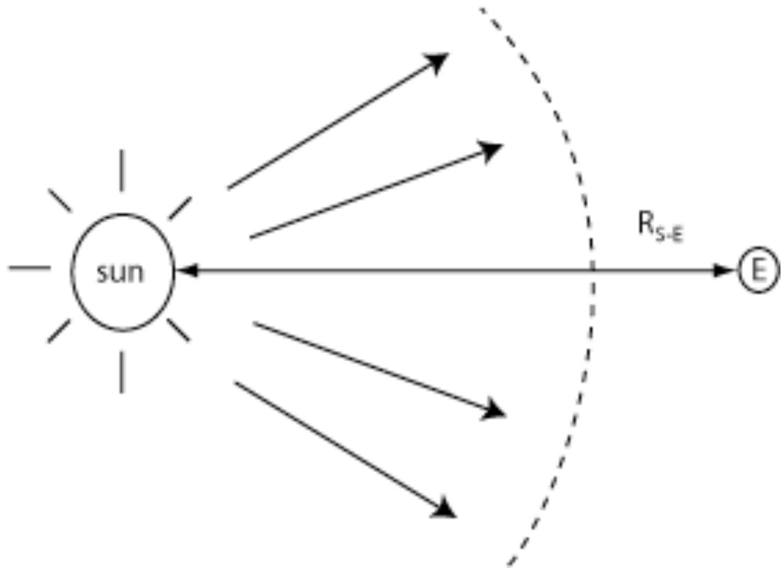
which is quite a lot.

What is the energy flux per unit area at the distance of the Earth is from the sun?

We can get closer to calculating the energy input to the Earth, Q_0, by calculating the energy per unit area at the distance that the Earth is from the Sun ($R_{SE} = 1.5 \times 10^{11}$ m). From geometry, Q_0 is equal to the total energy flux emitted by the sun divided by the area over which it is distributed over at the distance of the Earth from the Sun (see sketch).

That is

$$Q_0 = \frac{I_0}{4\pi R_{SE}^2} \approx 1367 \text{ W m}^{-2}.$$



Solar radiation in emitted essentially uniformly in all directions, and the flux reaching the Earth (in W m^{-2}) is just the total solar output (in W) divided by the area shell whose radius is equal to the distance between the Sun and the Earth.

What is the daily-average solar radiation incident on the surface?

To head towards a calculation of an estimate of the Earths temperature, what we really want is the average heat input per unit area of the Earth's surface. Since the Earth spherical (well nearly anyway), and since it is also spinning, we need to take account of the geometry.

The daily- and global-average radiation per unit area is equal to the total solar radiation intercepted by the Earth divided by the area over which that radiation is distributed. Viewed from the Sun, the Earth presents a disk-shaped area equal to the radius of the Earth, R_E. See the figure below. Total radiation intercepted is $I_0 \times 4\pi\ R_E^2$. Therefore the daily- and global-average radiation per unit area is

$$Q_0 \times \frac{\pi R_E^2}{4\pi R_E^2} = \frac{Q_0}{4} \approx 342 \text{ W m}^{-2}.$$

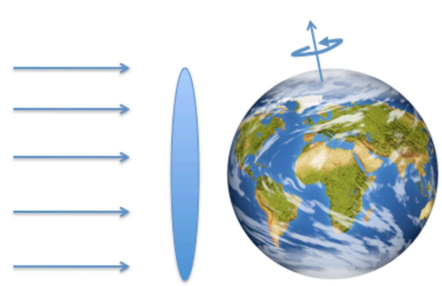


Figure. The sun's rays intercept a circular disc whose radius is equal to that of the Earth. Because the Earth spins, these rays are distributed of the surface area of the sphere (i.e., $4\pi R^2$).

Implications of Wien's law: What is the wavelength of maximum emission for the sun and Earth?

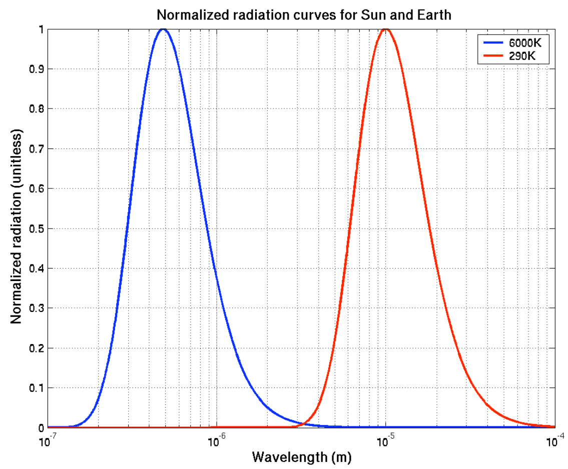


Figure shows approximate (normalized) emissions curves for the sun (blue) and the earth (red). Note that there is only about 1% overlap between the two curves. Normalized means each curve has been divided by its maximum value so both peak at 1.

For the sun, T~6000 K, so from Wien's law, $\lambda_m \approx 0.5$ μm. In other words, solar emission are predominantly in the visible and ultra-violet region of the spectrum (is this a coincidence?). The incoming solar radiation is sometimes called *insolation*, or *shortwave radiation*.

For the Earth, T~300 K, so $\lambda_m \approx 10$ μm. In other words emissions from the Earth are predominantly in the infrared. The outgoing radiation is sometimes known as *terrestrial radiation*, or *longwave radiation*.

An extremely useful consequence of this is that there is very little overlap between the spectra of the solar and terrestrial emissions. In fact ~99% of solar emissions occur at wavelengths less than 5 μm, and 99% of terrestrial emissions occur at wavelengths greater than 5 μm (see figure). Therefore, to a very good approximation, the solar and terrestrial emissions can be treated separately from each other. This is an extremely convenient property of the atmosphere, because it means we can treat solar and terrestrial emissions separately.

Reflectivity of the Earth as a function of wavelength.

The reflectivity of materials depends on the wavelength of radiation. Most common materials absorb and emit radiation in the infrared portion of the spectrum almost perfectly. This means they have a reflectivity close to 1. On the other hand, in visible wavelengths (i.e. those interacting with solar radiation), many materials reflect a substantial fraction of the incident radiation.

Material	Reflectivity (%)
Bare soil	10-25
Sand, desert	25-40
Grass	15-25
Forest	10-20
Snow (clean, dry)	75-75
Snow (wet and/or dirty)	25-75
Sea surface (sun > 25^{o} above horizon)	<10
Sea surface (low sun angle)	10-70

(From W&H, 1978)

Observed from the top-of-the atmosphere, a good global and annual average value for the net reflectivity is about 0.3. The reflectivity is also known as the *albedo*, and the symbol α is commonly used.

The basic energy balance of the planet - an initial estimate.

Averaged over a year or longer the natural system is neither warming up nor cooling down. Therefore the amount of energy flux coming into the system must be equal to the amount of energy flux coming out. In other words the system is in energy balance. We can use this fact to make a basic estimate of the temperature of the planet.

Imagine the simplest model of the Earth. It behaves as a blackbody in the infrared, and absorbs about 70% of the incident solar radiation. We can set up an equation for this model that allows us to get a first estimate for the temperature of the planet, T_E.

Flux in = incident solar radiation = $Q_0/4$ (7)

Flux out = solar flux reflected + terrestrial radiation emitted = $\alpha\ Q_0/4 + \sigma\ T_A^4$; (8)

When the system is in *equilibrium*, energy flux in equals the energy flux out. In other words

$$\frac{Q_0}{4} = \frac{\alpha Q_0}{4} + \sigma T_E^4 . \quad (9)$$

Solving gives

$$T_E = \left(\frac{Q_0(1-\alpha)}{4\sigma}\right)^{\frac{1}{4}} \approx 255 \text{ K}. \quad (10)$$

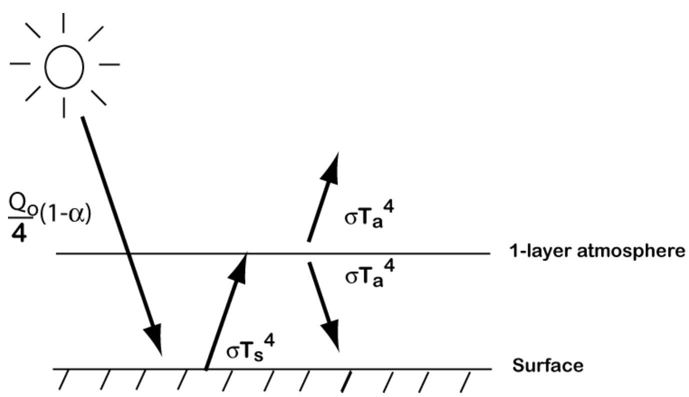
This is a little too cold compared to what we know is the case T~288K, so what have we done wrong?

The greenhouse effect.

We know that the composition of the atmosphere includes gases like water vapor and clouds, and that those gases are effective absorbers of infrared radiation. See the figures in the handout. Such gases are not good absorbers visible and ultra-violet radiation. Thus the atmosphere is largely transparent to solar radiation, but not to terrestrial radiation. Solar energy reaches the Earth's surface where it gets absorbed, and it is remitted at infrared wavelengths. Because of absorption this outgoing radiation is impeded from leaving the system, and so there is a tendency for energy to be accumulation in system. This tendency is known as the greenhouse effect. A simple calculation can estimate the magnitude of this effect by assuming that the atmospheres acts as a single layer that is transparent to solar radiation but perfectly absorbs terrestrial radiation.

The downward-directed (*down-welling*) energy flux at the surface is equal to the absorbed solar radiation, plus the energy emitted downwards from the atmosphere. Let T_A = temperature of the atmospheric layer. The upward-directed (*upwelling*) energy flux is just equal to the surface emissions. Let T_S = temperature of the surface. The energy balance at the surface is given by

$$(1-\alpha)\frac{Q_0}{4} + \sigma T_A^4 = \sigma T_S^4 \tag{11}$$



Simple one-layer model of the greenhouse effect. The solar energy (that is not directly reflected) is absorbed at the surface. The surface radiates to the atmosphere, which is perfectly absorbing in the inra-red. The atmosphere then re-emits this energy out to space and also down to the surface.

At the top of the atmosphere

$$(1-\alpha)\frac{Q_0}{4} = \sigma T_A^4 \tag{12}$$

and for the atmospheric layer, which is transparent to solar radiation, the input of energy comes from the upwelling radiation from the surface. The output of energy is the two-fold. There is a flux upwards to space, and there is a flux downwards to the surface.

Thus

$$\sigma T_S^4 = \sigma T_A^4\Big|_{\text{to space}} + \sigma T_A^4\Big|_{\text{to Earth}} \tag{13}$$

We only need two equations to solve for T_S. From eqn. (13), we can see that

$$T_S = 2^{\frac{1}{4}} T_A. \tag{14}$$

And by comparison of Eqn. (12) and Eqns. (9) and (10), we see that $T_A = T_E = 255\ K$. Hence

$$T_S = 2^{\frac{1}{4}} \times 255\text{K} \approx 303\,\text{K}. \tag{15}$$

This number is a little more than what is seen on the Earth, and so it suggest that the atmosphere is a little less effective than a single perfectly infrared-absorbing layer. Note that the difference between the blackbody planet and the greenhouse planet in only a factor of $2^{1/4}$(=1.19). This seemingly small number encapsulates all of the uncertainty and debate about the effect of changing greenhouse gases on the atmosphere and climate change. Climate changes that are important for ecosystems are a few degrees Centigrade, and relative to absolute zero (which is important for the physics), they are proportionately very small changes. This is stated somewhat facetiously in the maxim “People care about Centigrade, physics cares about Kelvin”.