Homework 1 solutions
ATM/OCN/ESS 589, Spring 2009

April 24, 2009

Solutions

1. The fundamental problem here is that he doesn't explicitly distinguish between atmosphere and ground temperature in the first equation for $F_{out}$ on page 21:

$$F_{out} = A\epsilon\sigma {T_{earth}}^4 \ . \tag{1}$$

Unless there is no absorbing atmosphere (in which case, $\epsilon$ would be the emissivity of the ground, not atmosphere), $T_{earth}$ is some sort of melange of ground and atmosphere temperature (that is not defined).
The second fundamental problem in on pg 25, where he writes the word equation for the**budget for the overall earth**:

$$I_{up,atmosphere} = I_{in,solar} \ . \tag{2}$$

The correct equation must include the energy that is coming upward from the ground ($\sigma {T_g}^4$) that is *not absorbed* by the atmosphere ($\sigma {T_g}^4(1-\epsilon)$):

$$I_{up,atmosphere} + I_{up,ground\ minus\ what\ is\ absorbed \in\ atmosphere} = I_{in,solar} \ . \tag{3}$$

, or (cf, his last equation on this page):

$$\sigma {T_g}^4(1-\epsilon) + \epsilon\sigma {T_a}^4 \ = \frac{I-\alpha}{4} I_{solar}. \tag{4}$$

2. **a** Start from the Energy Balance Equation

$${T_g}^4 = S_o(1-\alpha)/(4\sigma(1-\epsilon/2)) \ . \tag{5}$$

Taking the derivative with respect to $\epsilon$ yields

$${T_g}^3 dT_g = \frac{S_o(1-\alpha)}{4\sigma} \frac{1}{2(1-\epsilon/2)^2} d\epsilon \tag{6}$$

or dividing the LHS (RHS) of (2) by the LHS (RHS) of (1) yields

$$\frac{dT_g}{d\epsilon} = \frac{T_g}{8(1-\epsilon/2)} \ . \tag{7}$$

**b** Recall the sensitivity to changing insolation

$$\frac{dT_g}{dS} = \frac{T_g}{4S_o} \ . \tag{8}$$

So for $T_g = 288k,\ S_o = 1367Wm^{-2}$ we have

$$\Delta T_g = 0.052\frac{k}{Wm^{-2}} \ . \tag{9}$$

So the typical $T$ changes due to the solar cycle are $\pm$0.05k, which is smaller than observed and much smaller than the trend in global temperature over the past century. **c** The equilibration goes as $1 - exp(-t/\tau)$, where $\tau = C_p/B$. Hence, 90% equilibrium temperature is obtained at $t = -\tau ln(0.1)$. For $B = 2.9Wm^{-2}$ and $C_p = \rho_{water}C_{pocean}H = (10^3kg/m^3) * (4 * 10^3J/kg) * (2000m) = 2*10^9J/m^2$, the time to reach 90% of equilibrium is 200 years. Note that the larger the greenhouse gas concentration ($B \rightarrow 0$) and/or the more ocean involved, the longer the time it takes to equilibrate.

3. Recall the equation for sensitivity to $\alpha$:

$$dT_g = -\frac{T_g}{4(1-\alpha)}d\alpha = -100k \tag{10}$$

If we asume the land ice covers an area that had an original albedo that is the same as the average planetary albedo $\alpha_p = 0.30$, then the new albedo is

$$\frac{29}{30}0.30 + \frac{1}{30}0.70 = 0.313 \tag{11}$$

So, the change in albedo is .013, giving a change in temperature of $-1.3k$ in the LGM compared to today.

4. Clearly the albedo changes are a signifcant player in the reduction of the global average surface temperature during the LGM. THe dominant player, however, is elsewhere: the reduction in greenhouse effect due to the reduction in water vapor (and to a smaller degree) to the reduction in carbon dioxide.

5. We apply our energy balance equation (1) locally and assume that $\epsilon$ is uniform in space. Plugging in the numbers given, we find the surface temperature of the tropics $T_{gt} = 299k$ adn the surface temperature in the extratropics is $T_{gp} = 273k$. So the equator-to-pole temperature difference is 26°.

6. **a** In the equatorial box, the atmospheric energy balance is

$$2\epsilon\sigma {T_{at}}^4 = \epsilon\sigma {T_{gt}}^4 - D \ , \tag{12}$$

and the equation for the surface energy balance is unchanged

$$\sigma {T_{gt}}^4 = \epsilon\sigma {T_{at}}^4 + S_{ot}(1-\alpha_t)/4 \ . \tag{13}$$

**b** In the extratropical box, the atmospheric energy balance is

$$2\epsilon\sigma {T_{ap}}^4 = \epsilon\sigma {T_{gp}}^4 + D \ , \tag{14}$$

and the equation for the surface energy balance is unchanged

$$\sigma {T_{gp}}^4 = \epsilon\sigma {T_{ap}}^4 + S_{op}(1 - \alpha_t p)/4 \ . \tag{15}$$

**c** Eliminating $Ta$ from Eq 13, we find

$$\sigma {T_{gt}}^4 = S_{ot}(1 - \alpha_t)/4 + \frac{\epsilon}{2}\sigma {T_{gt}}^4 - D/2 \ , \tag{16}$$

or

$${T_{gt}}^4 = \left\{S_{ot}(1 - \alpha_t)/4 - D/2\right\}/(\sigma(1 - \epsilon/2)) \ . \tag{17}$$

A similar manipulation of Eq 15 yields the equation for the ground temperature in the extratropical regions

$${T_{gp}}^4 = \left\{S_{op}(1 - \alpha_p)/4 + D/2\right\}/(\sigma(1 - \epsilon/2)) \ . \tag{18}$$

**d** The amount of heat removed from the tropics is $2D$ (one $D$ to each extratropical region. The amount delivered into the extratropics is $D$. The area of the extratropics north of 30° latitude is 1/4 of the globe, or $\pi r^2$ where $r$ is the radius of the earth. The tropics cover 1/2 the surface area of the earth. Hence, the heat flux per unit area in the extratropics due to circulation is

$$+\frac{2 * 10^15W}{\pi r^2} = 47W/m^2 \ . \tag{19}$$

The heat flux deducted from the tropics is $-47W/m^2$. The same amount is added to each extratropical region. This is about 1/4 of the absorbed solar in the polar regions: $S_{op}(1 - \alpha_p)/4 = 200W/m^2$.

**e** Plugging in numbers, we find

$$T_{gt} = 19 \quad T_{gp} = 8C^\circ \tag{20}$$

The tropical and polar temperatures are too cold and too warm compared to observations. To some extent, this is due to the gross simplifications in our model. However, another issue is that I forgot to tell you to use representative values of $\alpha$ for the tropics and extratropics of $\alpha_t = 0.25$ and $\alpha_t = 0.25$, respectively. When you do, you find that without transport

$$T_{gt} = 31°C \quad T_{gp} = -5°C \tag{21}$$

and with transport

$$T_{gt} = 23°C \quad T_{gp} = +4°C \ . \tag{22}$$

So without circulation, the tropics are too warm by ∼°C and the extratropics are too cold by ∼ 9°C. Circulation is responsible for the difference, cooling the tropics by ∼ 8°C and warming the extratropics by ∼ $9°C$.