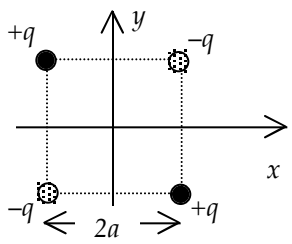
Name_____Student Number_____Score_____
122A (LAST, First Initial)

MULTIPLE CHOICE. MARK ANSWERS ON BOTH THIS PAGE AND THE SCANTRON SHEET

I. Point Charges (25 pts)



Four charges of magnitude q, two positive and two negative, are placed at the corners of a square of side $2a$ as shown at the right. The constant $k = 1/4\pi\varepsilon_o = 8.99\times10^{-12}\ \mathrm{Nm^2/C^2}$

Consider the positive charge in the lower right hand corner, $(x,y) = (a, -a)$.

1. [4 pts] In which direction is the force on this charge?

A. $+\mathbf{x}$ B. $+\mathbf{y}$ C. $+\mathbf{x}+\mathbf{y}$ D. $-\mathbf{x}+\mathbf{y}$ E. $+\mathbf{x}-\mathbf{y}$

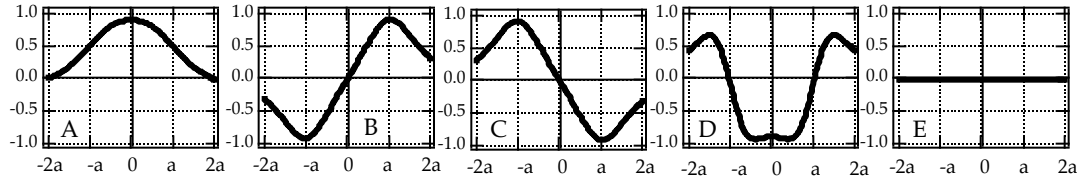
The force is the vector sum of the forces due to the other three charges. The $+q$ in the upper left corner has a force in the $+\mathbf{x}-\mathbf{y}$ direction. The two $-q$ give equal forces along the $-\mathbf{x}$ and $+\mathbf{y}$ directions, for a net $-\mathbf{x}+\mathbf{y}$ for the negative charges. Thus, the answer is either D or E depending on the relative magnitude. The attractive of the two, closer $-q$ is larger than the repulsive of the $+q$, leading to D as the right answer. See below for calculation of the magnitudes.

2. [5 pts] What is the magnitude of the force on this charge?

A. $\frac{kq^2}{4a^2}$ B. $\frac{kq^2}{2\sqrt{2}a^2}$ C. $\frac{kq^2}{a^2}\left(\frac{2\sqrt{2}-1}{8}\right)$ D. $\frac{kq^2}{a^2}\left(\frac{\sqrt{2}-1}{4}\right)$ E. $\frac{3kq^2}{8a^2}$

The repulsive force from $+q$ has magnitude $\frac{kq^2}{(2\sqrt{2}a)^2} = \frac{kq^2}{8a^2}$, while the attractive force from the negative charges is $\frac{kq^2}{(2a)^2}(-\hat{\mathbf{x}}+\hat{\mathbf{y}}) = \frac{kq^2}{4a^2}\sqrt{2}\,\frac{-\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}$, for net magnitude $\frac{kq^2}{a^2}\left(\frac{2\sqrt{2}-1}{8}\right)$.

Consider the following five graphs showing a function along the x axis from $-2a$ to $+2a$ ($x = 0$ is the center of the square; $\pm a$ is the edge of the square. The vertical axis is in arbitrary units.



3. [4 pts] Which of the above graphs depicts the variation of the electrostatic potential V along the x axis?

A. A B. B C. C D. D E. E (V=0)

The net potential $= \frac{+kq}{r_1} + \frac{-kq}{r_2} + \frac{-kq}{r_3} + \frac{+kq}{r_4}$. Everywhere along the x axis, the + and – charge on the left are equal distance from the axis ($r_1 = r_2$) ; likewise for the – and + charges on the right ($r_3 = r_4$). Thus V = 0 all along the axis.

4. **[4 pts]** Which of the above graphs depicts the variation of the y-component of the electric field E_y along the x-axis(positive E_y means the component pointing towards the top of the page)?

A. A B. B C. C D. D E. E (E_y=0)

The field at the origin is zero by symmetry ($+q$'s are symmetric, as are $-q$'s), eliminating A and D. At the left edge of the square, the field from the left dipole is large and in the $-\mathbf{y}$ direction, while the field from the right dipole is small and in the $+\mathbf{y}$ direction. The field has the opposite sign on the right side. Thus the anti-symmetric solution with negative E_y for negative x is the right one.

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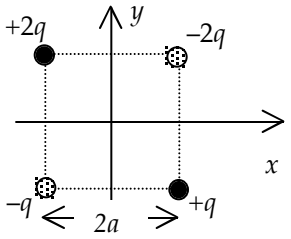
The two top charges are doubled in magnitude to $\pm 2q$.

5. [4 pts] Did the magnitude of the potential V at the center of the square $(x,y) = (0,0)$ increase, decrease or stay the same when the top charges were doubled in magnitude?

A. increase B. decrease $\boxed{\text{C.}}$ stay the same



At the origin, the two new charges are equidistant and opposite sign, leaving the potential at zero.

6. [4 pts] Did the magnitude of the electric field E at the center of the square $(x,y) = (0,0)$ increase, decrease or stay the same when the top charges were doubled in magnitude?

$\boxed{\text{A.}}$ increase B. decrease C. stay the same.

The field was zero before, but now there is a non-zero x component, leading to an increase in magnitude.

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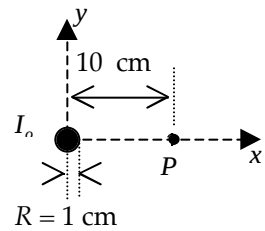
II. Wire Fields and Forces. (25 pts) A long straight wire of radius $R = 1.0$ cm lies on the z-axis and carries a current I_o in the +**z** direction (out of the page). Point P is on the x-axis, a distance $x = 10.0$ cm to the right of the wire. At point P, the magnitude of the magnetic field is $B_P = 2.0 \times 10^{-5}$ T.



7. [3 pts] In which direction is the magnetic field at point P?
A. +**x** B. +**y** C. +**z** D. –**y** E. Some other direction
Right hand rule for current out of the page points up at P.

8-9. How much current I_o is flowing in the wire?

8. [1 pt] units
A) Ampere B) Tesla C) Newton D) Henry E) Volt

9. [4 pts] magnitude
A) 100. B) 4.0 C) 10. D) 31. E) 3.1

$$B = \frac{\mu_o I_o}{2\pi r}\ ;\ I_o = \frac{2\pi(0.1m)(2\times 10^{-5}T)}{4\pi \times 10^{-7} Tm/A} = 10A$$

10. [5 pts] What is the magnitude of the magnetic field at the surface of the wire, at $r = R = 1.0$ cm?
A. 2.0×10^{-5} T B. 2.0×10^{-4} T C. 2.5×10^{-4} T D. 8.0×10^{-4} T E. None of these

$B = \frac{\mu_o I_{enc}}{2\pi r}$. Here $I_{enc} = I_o$ just as for point P. The fields thus have the ratio 1:10 since the radii have the ratio 10:1. $B(r_w) = 10B_P = 2.0 \times 10^{-4}$T.

11. [4 pts] How does the magnitude of the magnetic field B_1 at radius $r_1 = 0.5$ cm (=$R/2$) compare to B_P, the magnitude at point P, at $r = 10.0$ cm?
A. $0 \le B_1 < B_P/2$
B. $B_P/2 \le B_1 < B_P$
C. $B_P \le B_1 < 2\,B_P$
D. $2B_P \le B_1 < 10\,B_P$
E. $B_1 \ge 10\,B_P$
Inside the wire, the field varies linearly, so $B_1 = 0.5\,B_w = 5\,B_P$.

12-14. (8 pts total) A second wire is placed parallel to the first wire through point P. The wire carries an current $I_2 = 5.0$ A in the –z direction (into the page). What is the <u>force per unit length</u> *on the second wire*?

12. **[3 pts]** Direction
A. +**x** B. –**x** C. +**z** D. Some other direction E. no direction **F** = 0

13. **[1 pt]** Units
A. Joule B. Newton C. Volt/m D. kg/s^2 E. Olmstead

14. **[4 pts]** Magnitude
A. 1.0×10^{-4} B. 1.2×10^{-3} C. 0 D. 2.0×10^{-4} E. 5×10^{-5}

Opposing currents repel, so force in +**x** direction. Force per unit length is $\frac{Newton}{meter} = \frac{kgm}{s^2 \bullet m} = \frac{kg}{s^2}$. For magnitude, $\vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$, so $f = F/L = IB = (5\text{ A})(2\times10^{-5}\text{ T}) = 10^{-4}$ N/m.

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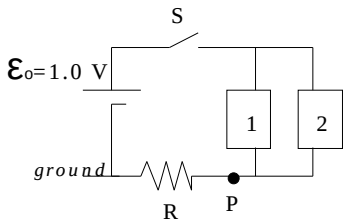
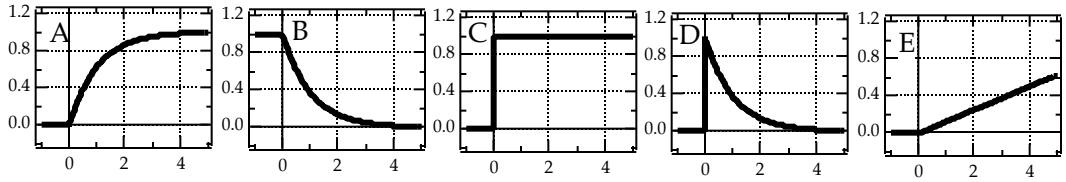
III. Circuit boxes (28 pts)

Consider the circuit at the right, containing an ideal battery $\varepsilon_o = 1.0$ V, a switch, a resistor of resistance R_o, and two boxes that may contain different circuit elements. The low potential side of the battery is grounded, $V = 0$.



The switch has been open for a long time. A student measures V_p, the potential relative to ground at point P, as a function of time, closing the switch at time $t = 0$. She measures one of the following curves, where the vertical axis is in volts and the horizontal axis in seconds.



15. [4 pts] Which of the above curves is measured when box 1 contains an inductor and box 2 contains an open switch?

$\boxed{\text{A.}}$ A B. B C. C D. D E. E

V_a is a measure of the current through the resistor. In an RL circuit, the current starts out at zero and grows to a constant value as $I = I_o\left(1 - e^{-t/\tau}\right)$, or trace A.

16. [4 pts] Which of the above curves is measured when box 1 contains a capacitor and box 2 contains a wire?

A. A B. B $\boxed{\text{C.}}$ C D. D E. E

The wire shorts out the capacitor, and this is just like having a single resistor in the circuit. V_a immediately goes up to ε_o and stays there.

17. [4 pts] Which of the above curves is measured when box 1 contains a capacitor and box 2 also contains a capacitor?

A. A B. B C. C $\boxed{\text{D.}}$ D E. E

Two capacitors in parallel act like a capacitor, so this is like an RC circuit. The current is zero with the switch open, then jumps to a high value and decays exponentially as the capacitor charges.

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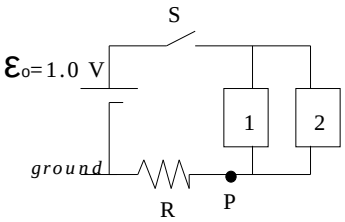
For questions 18-21, box 1 contains an ideal inductor of inductance L_o and box 2 contains a resistor of resistance R_o. The switch has been open for a long time, and is closed at time $t = 0$.



18. [4 pts] What is V_P, the potential at point P just after the switch is closed ($t = 0^+$)?

A. ε_o B. $\varepsilon_o/2$ C. 0 D. R_o/L_o E. None of these

At $t = 0^+$, the inductor acts like an open circuit, so all the current goes through the resistor branch 2, and then the resistor between P and ground. Since the two resistors are identical, and they have the same current through them, the voltage drop is equal across each. Point P is thus at $\varepsilon_o/2$

19. [4 pts] What is V_P a very long time after the switch is closed?

A. ε_o B. $\varepsilon_o/2$ C. 0 D. R_o/L_o E. None of these

After a long time, the inductor acts like a wire, and there is no potential drop across the resistor in box 2. Thus all the potential drop of the battery is across V_p.

After the switch has been closed a very long time, it is then opened at time t_o.

20. [4 pts] Immediately after the switch is opened($t = t_o^+$), what is the current through the battery?

A. ε_o/R_o B. $\varepsilon_o/2R_o$ C. 0 D. ε_o/L_o E. None of these

After the switch is opened, no more current goes through the battery – it is out of the loop.

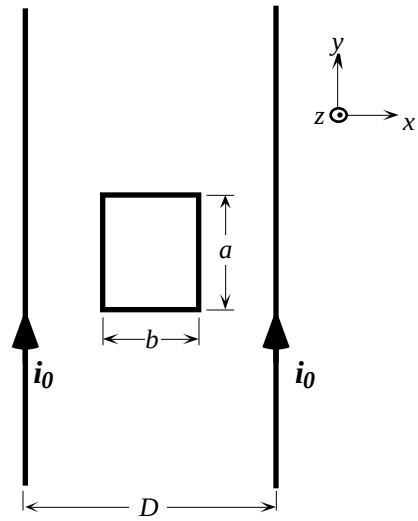
21. [4 pts] Immediately after the switch is opened ($t = t_o^+$), what is the current through the resistor in box 2?

A. ε_o/R_o B. $\varepsilon_o/2R_o$ C. 0 D. $2\varepsilon_o/R_o$ E. None of these

The current through the resistor in box 2 is identical to the current in the inductor, which is identical to that through the inductor before the switch was opened. That was ε_o/R, since there was no potential drop across the inductor.

IV. Wires and Loop [20 points total]

Two parallel, infinitely long wires separated by a distance D each carry current i_0 in the $+y$ direction as shown in the diagram. A conducting loop of height a and width b is placed exactly halfway between the wires and in the same plane as the wires. (As indicated by the coordinate axes to the right of the wires, the $+x$ direction is toward the right of the page, the $+y$-direction is toward the top of the page, and the $+z$ direction is up out of the page.)



22. [5 pts] What is the magnitude of the flux in the loop? Show your work or explain your reasoning.

The flux through the loop due to the current in the left-hand wire has the same absolute value as the flux through the loop due to the right hand wire by symmetry. The magnetic field due to the left hand wire points into the page inside the loop, while the magnetic field due to the right hand wire points out of the page inside the loop. So, the flux contributions from the two wires are opposite in sign. The net flux is the sum of these two contributions, which is equal to zero since it is a sum of equal and opposite quantities.

23. [5 pts] Indicate the direction of the current induced in the loop when the loop is moved in the $+y$ direction. If there is no induced current, state that explicitly. In either case, explain your reasoning.

When the loop is moved in the +y direction, the flux through the loop does not change, because there is no variation in the strength of the magnetic field and there is no change in the area or orientation of the loop. Since there is no change in flux, there is no induced EMF and thus no induced current

24. For each of the following cases, indicate the direction of the current induced in the loop. If there is no induced current, state that explicitly. In either case, explain your reasoning

i. [5 pts] The loop is moved slightly toward the right.

The magnitude of the magnetic field varies as 1/r in the vicinity of the long wire (r is the distance from the wire). If the loop is moved to the right, the magnetic field inside the loop due to the left wire is getting weaker and the magnetic field due to the right wire is getting stronger. If the area vector of the loop is chosen to be out of the page, the flux due to the left wire is negative and its absolute value decreases as the loop approaches the right wire. The flux due to the right wire is positive and its absolute value increases as the loop approaches the right wire. So, the change in net flux is positive as loop is moved to the right. From Lenz' law, the current induced in the loop must be clockwise in order to produce a negative induced flux that opposes the positive change in flux.

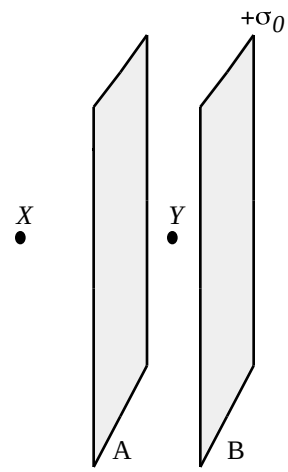
ii. [5 pts] The loop is moved in the $+z$ direction.

From part A, the flux is zero when the loop is centered between the wires. After the loop has moved in the +z direction, the flux due to each wire is still equal in absolute value (again by symmetry) and opposite in sign. The net flux is still zero. Since the flux has not changed, there is no induced EMF in the loop, and no induced current.

IV. Charges and Flux [25 points total]

25. Two very large insulating sheets, each with uniform charge density, are placed parallel to one another. A portion of the sheets far from the edges is shown. The electric field at point *X* points to the right and has a magnitude of $\sigma_0 /2\varepsilon_0$. Sheet B has a positive charge density $+\sigma_0$.

i. [7 pts] Find the charge density (sign and absolute value) of sheet A. Show your work.

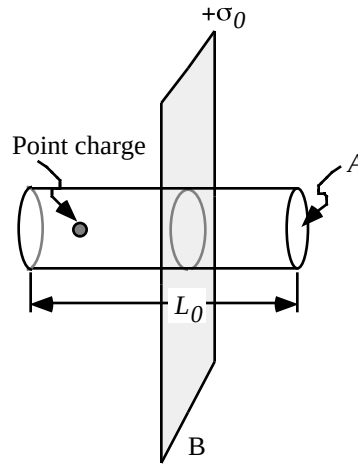


According to the principle of superposition, the electric field at point X is given by the vector sum of the contributions from sheets A and B. The field of a single infinite sheet of charge with positive surface charge density σ has magnitude $\sigma /2\varepsilon_0$ (independent of distance) and points away from the sheet, so the contribution of sheet B to the field at point X is $\sigma_0 /2\varepsilon_0$ to the left. In order for the field at X to be $\sigma_0 /2\varepsilon_0$ to the right, the contribution of sheet A must therefore be σ_0 /ε_0 to the right. Thus the magnitude of the charge density on sheet A must satisfy the relation $\sigma_A/2\varepsilon_0 = \sigma_0 /\varepsilon_0$. Since σ_A must be negative, the charge density of sheet A must be $-2\sigma_0$.

ii. [7 pts] Calculate the electric field (magnitude and direction) at point *Y* between the charged sheets. Show your work.

The contribution to the electric field at point Y from sheet A is σ_0/ε_0 to the left. The contribution to the field at point Y from sheet B is $\sigma_0/2\varepsilon_0$ to the left. By superposition the electric field at point Y has magnitude $3\sigma_0/2\varepsilon_0$ and points to the left.

26. Sheet A is replaced by a point charge. A Gaussian cylinder of length L_0 and end cap area A_0 encloses the point charge and a portion of sheet B as shown. The net flux through the Gaussian cylinder is zero.



i. [6 pts] Find the value (sign and absolute value) of the point charge. Show your work.

By Gauss' Law, the charge enclosed by the Gaussian cylinder must be equal to the net flux through the Guassian cylinder, which is given to be zero in this case. The Gaussian cylinder encloses a portion of the charged sheet which has area A_0 . This portion of the sheet contains an amount of charge $+\sigma_0 A_0$. Thus the point charge is negative and has magnitude $\sigma_0 A_0$.

ii. [5 pts] Is the flux through the curved side surface of the Gaussian cylinder *positive, negative,* or *zero?* Explain.

The contribution of the charged sheet to the flux through the curved side surface of the Gaussian cylinder is zero. The point charge is negative, so the electric field due to the point charge at points on the curved side surface of the Gaussian cylinder has a component into the Gaussian cylinder. Thus the contribution of the point charge to the flux through the curved side surface is negative. Summing the contributions from both the sheet and the point charge, the flux through the curved side surface of the Gaussian cylinder is negative.

Name:_____ **Student #:**_____ **Score:**_____
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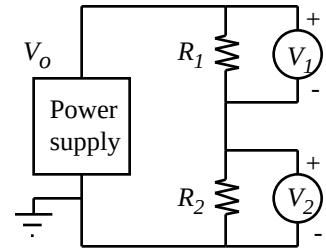
VI. Voltage Divider [27 pts total]
Two resistors, 1 and 2, are connected to a power supply (with negligible internal resistance) as shown. Assume $R_1 = 10\ \Omega$ and $R_2 = 23\ \Omega$. The output voltage, V_o, from the power supply can be varied. The voltages across resistors 1 and 2 are measured by voltmeters reading voltages V_1 and V_2, respectively.

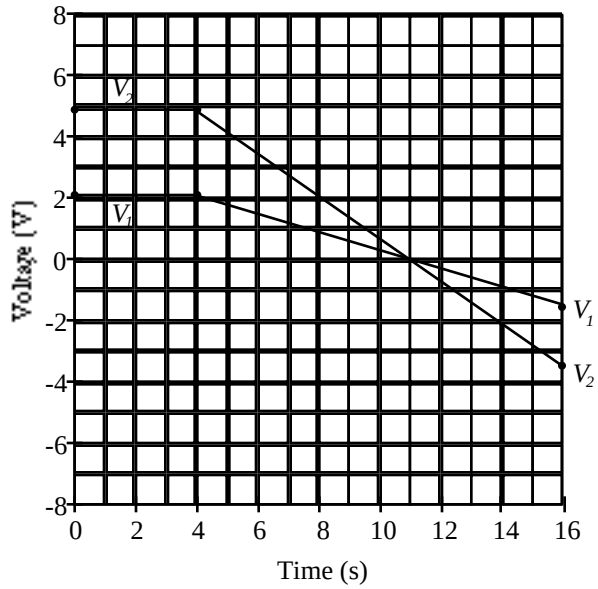


27. [5 pts] If the output voltage $V_o = +7.0$ V, find the voltages V_1 and V_2 across the resistors. Show your work.

The current through both resistors (connected in series) is given by $i_o = \frac{V_o}{R_{eq}} = \frac{V_o}{R_1 + R_2}$. Using Ohm's law and retaining 2 significant figures in each result, $V_1 = i_o R_1 = 2.1\text{V}$ and $V_2 = i_o R_2 = 4.9\text{V}$ when $V_o = +7.0\text{V}$.

V_1 = ____2.1____ V V_2 = ____4.9____ V

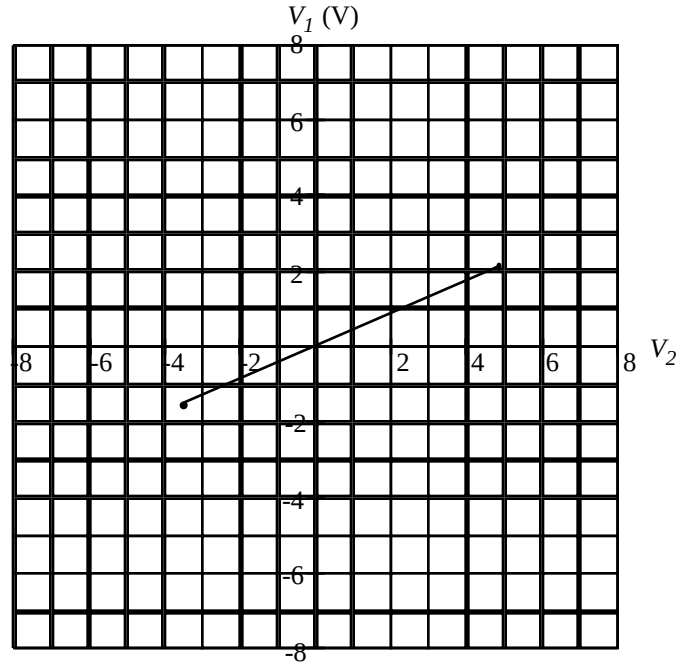
28. [10 pts] If the voltage V_o is equal to +7.0V for the first 4 seconds and then decreases linearly from +7.0V to –5.0V for the following 12 seconds, plot the voltages V_1 and V_2 as a function of time on the graph at right. Label your plots V_1 and V_2.



Since V_o decreases linearly from +7.0V to –5.0V during the final 12 seconds (at a rate of –1.0V/s), V_1 and V_2 must both decrease linearly as well. From the expressions in part A, $V_1 = -1.5\text{V}$ and $V_2 = -3.5\text{V}$ when $V_o = -5.0\text{V}$.

29. [8 pts] Plot the values of the voltages V_1 and V_2 on a graph of V_1 versus V_2 for the same 16 s time period.

30. [4 pts] How do the values of R_1 and R_2 affect your graph of V_1 versus V_2? Explain briefly.



The values of R_1 and R_2 determine the slope of the V_1 versus V_2 line, which is given by $\frac{R_1}{R_2}$. (The ratio of the resistances also affects the end points.) The linearity of the V_1 versus V_2 graph may be attributed to the fact that both resistors are Ohmic devices (constant resistance independent of current).