Physics 328: Statistical Physics
Midterm 1  Friday, 20 May 2005
Professor Marjorie Olmstead

Instructions and Advice

Solve all three problems in the space provided. Use the back if you need to, but make sure it is clear to the grader where to find your answers.

**EXPLAIN YOUR REASONING. A CORRECT ANSWER WITH NO SUPPORTING INFORMATION WILL RECEIVE NO CREDIT.**

You may use any unit system, but you must make it clear what your units are.

One significant figure is sufficient for all numerical answers.

You may find some of the following to be useful: If you think you need a constant or integral that isn’t here, PLEASE ASK.

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\begin{align*}
k_b &= 1.38 \times 10^{-16} \text{ erg/K} = 1.38 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-5} \text{ eV/K}; \\
h &= 2\pi h = 6.63 \times 10^{-27} \text{ erg-s} = 6.63 \times 10^{-34} \text{ J-s} = 4.13 \times 10^{-15} \text{ eV-s}. \\
hc &= 1.24 \times 10^{-6} \text{ eV-m}; \\
c &= 2.998 \times 10^{10} \text{ cm/s} = 2.998 \times 10^{8} \text{ m/s}; \\
e &= 4.80 \times 10^{-10} \text{ esu} = 1.602 \times 10^{-19} \text{ C} \\
m_e &= 9.11 \times 10^{-31} \text{ kg}; \\
1 \text{ amu} &= 1.67 \times 10^{-27} \text{ kg} \\
SB &= \frac{2\pi^2 k_b^4}{15h^3c^2} = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4; \\
1 \text{ Å} &= 10^{-10} \text{ m}; \\
eV &= 1.602 \times 10^{-12} \text{ erg} = 1.602 \times 10^{-19} \text{ J} \\
2 \cosh x &= e^x + e^{-x}; \\
2 \sinh x &= e^x - e^{-x}; \\
\tanh x &= \frac{\sinh x}{\cosh x}; \\
\frac{d}{dx}(\cosh x) &= a \sinh x; \\
\frac{d}{dx}(\sinh x) &= a \cosh x; \\
\frac{d}{dx}(\tanh x) &= a \left[1 - \tanh^2 x\right]; \\
\frac{d}{dx}(\text{ln } f(x)) &= \frac{1}{f(x)} \frac{df}{dx}; \\
\frac{d}{dx}(f(x)^n) &= n f^{n-1} \frac{df}{dx} \\
\lim_{N \to \infty} N! &= (2\pi N)^{1/2} N^N e^{-N}; \\
\lim_{N \to \infty} \text{ln } N! &= N \ln N - N; \\
c(n, r) &= \frac{N!}{(N-r)! r!}; \\
\lim_{N \to \infty} \text{ln } (1 + x) &= x; \\
\lim_{x \to 0} e^x &= 1 + x \\
dU_{1D} &= TdS - pdl; \\
dU_{2D} &= TdS - \gamma dA; \\
dU_{3D} &= TdS - pdV; \\
H &= U + pV; \\
F &= U - TS; \\
G &= U - TS + PV.
\end{align*}
\]

Phys 328 Midterm 2  20 May 05
Professor M. Olmstead
1. **Adsorbed gases [40 pts]**

Consider a crystal surface with $\sigma_0 = 10^{15}$ sites/cm$^2$.

The surface is in equilibrium with Argon, a monoatomic gas with the following properties:

- Atomic mass $m_o = 40$ amu
- Volume density $n = 2.5 \times 10^6$ cm$^3$
- Pressure $p = 1.0 \times 10^{-8}$ N/m$^2$
- Quantum concentration $n_q = 2.5 \times 10^{26}$ cm$^{-3}$
- Temperature $kT = 26$ meV, $T = 300$ K
- Surface binding energy $\epsilon_o$ (to be determined in part D)

Under these conditions, one Ar atom lands on each surface site about once/second. They do not all stick. The atoms that do stick have a binding energy $\epsilon_o$ (lower energy on surface than off).

**A. [8 pts]** Find the chemical potential $\mu$ (in eV) of the Ar molecules on the surface. How does it compare to $\mu_g$, the chemical potential of the Ar gas? Explain your reasoning.

**B. [9 pts]** When an Ar atom leaves the gas phase to stick to the surface, do the following increase, decrease, or stay the same? Explain your reasoning.

i. free enthalpy (Gibbs free energy) $G$

ii. entropy $S$

iii. enthalpy $H$
C. **[8 pts]** What is the grand partition function $\Xi$ for a particular (distinguishable) surface site? Express your answer in terms of fundamental constants and/or parameters given in the problem (e.g., $n$, $n_r$, $p$, $k_B$, $T$, $m$, $h$, $\sigma$) and explain where your terms come from.

D. **[7 pts]** For what value of the binding energy $\varepsilon_o$ (in eV) does the equilibrium coverage of Ar on the surface equal 5% ($f = 0.05$, $\sigma = 5 \times 10^{13}$ cm$^2$)?

E. **[8 pts]** Would the equilibrium coverage increase, decrease, or remain the same relative to the 5% coverage in part D if the following experiments were done? Explain your reasoning.

i. Change the gas from argon (mass = 40 amu) to helium (mass $m = 4$ amu), with the surface treated to keep the binding energy equal to $\varepsilon_o$.

ii. The gas remains the same (Ar at 300 K), but the surface is treated to double the binding energy to $2\varepsilon_o$. 
2. Radiation Box (30 pts)

Consider a cubic box of side $L = 1.0 \text{ m}$ (volume $1.0 \text{ m}^3$) with thermally conducting walls embedded in a thermal reservoir of temperature $T = 300 \text{ K}$ ($kT = 0.026 \text{ eV}$). The box is empty (under vacuum) except for thermal radiation.

A. [5 pts] What is the most probable energy $E_o$ for a photon in the box?

B. [5 pts] What is the total thermal energy $U$ in the box?

C. [8 pts] Estimate the total number of photons of energy $E = 0.12 \pm 0.006 \text{ eV}$ (0.012 eV window, i.e., $\Delta E/E = 10\%$) in the box. Justify your answer.
D. [6 pts] Find the heat capacity of the radiation in the (constant volume) box at 300 K.

E. [6 pts] How do you expect the total number of photons (integrated over all energies) to scale with temperature? (e.g., if $N \sim T^\alpha$, what is $\alpha$, or if it is a non-power law, what is the functional dependence?) Explain your reasoning.
3. **Short Answer [30 pts]**

Choose **any 3** of the following terms. For each term you choose:

(4 pts) Give a definition in words and/or equations. Define all variables in any equations.

(6 pts) Describe an example of how this concept is used *and* how/why it is significant.

[10 pts each, your best three will count if you answer more than three]

A. Grand partition function $\Xi$

B. Chemical potential $\zeta$

C. Internal partition function $Z_{\text{int}}$

D. Ideal gas

E. Quantum concentration $n_Q$